# A Note on Invariance for One Form on Smooth Manifolds

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**ABSTRACT**: In this paper we have considered the natural invariance associated with one forms on smooth manifolds and developed proposition on orthonormal frames transformation from one frame to the other.

**Keywords:** orthonormal frames, Euclidean framework, smooth manifolds.

**INTRODUCTION**

In this note we observe that there is a natural invariance associated with the 1-forms on smooth manifolds which arise and present itself under the ambiance of Euclidean frame work. Before we get into the problem thus considered we recapitulate the algebraic background for the formulation of the problem. The classical frame work if the Euclidean n-dimensional space which serve as prototypes for smooth manifolds.A non trivial case is a general linear group GL(n,ℝ ),whose elements are $n \times n$ non singular matrices with entries in ℝ .Regarded as smooth manifolds admitting $\mathbb{R}^{n^2}$ atlas(differentiable structure).

A finite dimensional vector space over ℝ or ℂ will enable us to understand GL(n,ℝ ) which is a group and at the same time a topological space .We will not go into these details because they themselves the kind of information we intend to gather does not warrant us to know them.

As a thumb rule, we assume that the vector space V with a basis B given by $\{e_1, e_2 \ldots\ldots e_n\}$ nlinearly independent vectors spanning V form an orthonormal frame.

Proposition: If $\{e_1, e_2 \ldots\ldots e_n\}$ is an orthonormal frame of V. Then there is a transformation.

Let us denote it by $\mathcal{F} = \{e_1, e_2 \ldots\ldots e_n\}$ as an orthonormal frame ,then there is a transformation of $\mathcal{F}$ to another frame $\mathcal{F}^1 = \{e_1^1, e_2^1 \ldots\ldots e_n^1\}$ . Then there exists A $\epsilon$ GL (n,ℝ) such that $\mathcal{F}$ and $\mathcal{F}^1$ relate the basis elements of 1-frame with the basis elements of another frame by scalar multiplication given by $e_i^1 = a_i^j e_j$.

$a_i^j$ and $n^2$ entries which are real and they are the entries of the matrix $A = a_i^j$ as

$e_1^1 = a_1^1 e_1 + a_1^2 e_2 + \ldots\ldots\ldots + a_1^n e^n$

$e_2^1 = a_2^1 e_1 + a_2^2 e_2 + \ldots\ldots\ldots + a_2^n e^n$

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$e_n^1 = a_n^1 e_1 + a_n^2 e_2 + \ldots\ldots\ldots + a_n^n e^n$

In the above expression the Einstein's summation convection is used over the repeated indices. This proposition enables us to formulate the required invariants .To this end let us consider the family of such frames on V .Let ℝ(V) denote this family of frames in V.Following proposition gives us the invariance .In the light of above proposition the following result follows.

Proposition: If $\mathcal{F}_0$ is a frame in V and $\mathcal{F}_0$ is fixed and $\mathcal{F}$(V) be a frame in V such that $\mathcal{F}$(V) is located from $\mathcal{F}_0$.Then $\mathcal{F}$(V) = $\mathcal{F}_a$(V), v $\epsilon\ V$.

In fact this $a_i^j$'s are functions on $\mathbb{R}$(V) which are $n^2$ in number and since determinant of $a_i^j \neq 0$ in a way $\mathbb{R}$(V) a manifold of dimension $n^2$.

Thus $\mathcal{F}_a$ is a differentiable function which is linear with this parameters that is a $=a_i^j$.

Regarding $\mathcal{F}_a$ as such a differentiable function on $\mathbb{R}$(V), the following translation will result into a nice invariance.

So , $\mathcal{F}_{a+da} = \mathcal{F}_0 \cdot (a + da)$

$= \mathcal{F}_0 \cdot a + \mathcal{F}_0$da ( $\mathcal{F}_0$ is linear )

$= \mathcal{F}_a( a^{-1})( a + da)$

$= \mathcal{F}_a$ ( I + $a^{-1}$da)

$= \mathcal{F}_a$ ( I+ $\omega_a$) where $\omega_a = a^{-1}$da is 1-form.

$\omega_a$ is a differential 1-form.

d$\mathcal{F}_a = \mathcal{F}_a \, . \omega_a$

$\mathcal{F}_{a+da} = \mathcal{F}_a + \mathcal{F}_a \omega_a$

$\mathcal{F}_{a+da}$ - $\mathcal{F}_a$= I $\omega_a$

d$\mathcal{F}_a = \mathcal{F}_a \, . \, \omega_a$ ( $\omega_a$ is the left invariant of 1 –form) .

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