I- sets and C- sets

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Abstract: In this paper we observe the relationship between interiors and closures of an intuitionistic fuzzy set in an intuitionistic fuzzy topological space and its co-ordinate fuzzy sets in its corresponding fuzzy topological spaces and in this context we define I- sets and C- sets.

Key words: Fuzzy closure, fuzzy interior, intuitionistic fuzzy closure, intuitionistic fuzzy interior, I- sets, C- sets.

1. Introduction.

We have that an intuitionistic fuzzy topological space can be associated with two fuzzy topological spaces and vice versa [1]. . If (X, τ) is an IFTS and τ_1= { μ_a / ∃ $\gamma_a \in I^x$ such that (μ_a , γ_a) ∈ τ }, τ_2 = { 1- γ_a / ∃ $\mu_a \in I^x$ such that (μ_a, γ_a) ∈ τ},then (X, τ_1) and (X, τ_2) are fuzzy topological spaces. Similarly if (X, τ_1) and (X, τ_2) are two fuzzy topological space, τ = {(u,1- v)/ u∈ τ_1, v∈ τ_2 and u ⊆ v} is an intuitionistic fuzzy topology and (X, τ) is an intuitionistic fuzzy topological spaces. We study some relationships connecting the closures and interiors of an intuitionistic fuzzy set in an intuitionistic fuzzy topological space and the closures and interiors of its co-ordinate fuzzy sets in its corresponding fuzzy topological spaces. .

2. Preliminaries.

Now we introduce some basic definitions needed.

Definition 2.1[3] Let I^x denote the set of all fuzzy sets defined on X. A family T ⊆ I^x of fuzzy sets is called a ***fuzzy topology*** for X if it satisfies.

(i) 0, 1 ∈ T

(ii) ∀ A, B ∈ T ⇒ A∩B ∈T

(iii) ∀ $\{A_j\}_{j\in J}$ ∈T ⇒$\bigcup_{j\in J} A_j$ ∈ T

The pair (X, T) is called a ***fuzzy topological space***. The elements of T are called ***fuzzy open sets***. A fuzzy set K is called ***fuzzy closed*** set if K^c ∈T.

Definition 2.2 : [2]

Let an ordinary non fuzzy set X be given. An ***intuitionistic fuzzy set*** (IFS in short) A in X has the form A = {< x, $\mu_A(x)$, $\gamma_A(x)$ >: $x \in X$}, where $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ are functions defining membership and non membership, respectively of each element in the set X. Moreover for each $x \in X$, the inequality $\mu_A(x) + \gamma_A(x) \leq 1$ is fulfilled.

Definition 2.3 : [2]

Let Y be a nonempty set. Let A and B be intuitionistic fuzzy sets. Let $\{A_\alpha : \alpha \in \wedge\}$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then

(i) $A \subseteq B$ if $\forall\, y \in Y\, [\mu_A(y) \leq \mu_B(y)$ and $\gamma_A(y) \geq \gamma_B(y)]$.

(ii) A = B if $A \subseteq B$ and $B \subseteq A$.

(iii) $A^C = \{ < y, \gamma_A(y), \mu_A(y) > : y \in Y \}$.

(iv) $0 = \{ < y, 0, 1 > : y \in Y \}$.

(v) $1 = \{ < y, 1, 0 > : y \in Y \}$.

(vi) $\cap A_\alpha = \{ < y, \cap \mu_A(y), \cup \gamma_{A(y)} > : y \in Y \}$.

(vii) $\cup A_\alpha = \{ < y, \cup \mu_A(y), \cap \gamma_{A(y)} > : y \in Y \}$.

Definition 2.4:[4] An ***intuitionistic fuzzy topology*** (IFT in short) on a nonempty set X is a family T of intuitionistic fuzzy sets (IFS in short) in X which satisfy the following axioms.

(i) 0, 1 $\in$ T

(ii) $A_1 \cap A_2 \in T$ for any $A_1, A_2 \in T$.

(iii) $\cup A_\alpha$ for any arbitrary family $\{ A_\alpha : \alpha \in \wedge \} \in T$.

In this case the pair (X, T) is called an ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in T is known as ***intuitionistic fuzzy open set*** in X. An IFS K is called IF closed if $K^c \in T$

Definition2.5: [4]

Let (X, T) be an IFTS and A be an IFS in X. Then the ***intuitionistic fuzzy interior*** and ***intuitionistic fuzzy closure*** of A are defined by

Cl (A) = $\cap$ {K: K is an IFCS in X and $A \subseteq K$}

Int (A) = $\cup$ {G: G is an IFOS in X and $G \subseteq A$}.

Definition 2.6 : [5]

Let A be a fuzzy set in FTS (X, T). The ***closure*** $\bar{A}$ and ***interior*** $\mathbf{A^o}$ of A are defined respectively by

$\bar{A} = \cap \{B : B \supseteq A, B^c \in T\}$.

$A^o = \cup \{B : B \subseteq A, B \in T\}$.

Result 2.1 : [4]

Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space (X, T). Then

(i) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl (A) = A.

(ii) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int (A) = A.

(iii) $cl(A^c) = (int(A))^c$.

(iv) $int(A^c) = (cl(A))^c$.

(v) $A \subseteq B \Longrightarrow int(A) \subseteq int(B)$.

(vi) $A \subseteq B \Longrightarrow cl(A) \subseteq cl(B)$.

(vii) $cl(A \cup B) = cl(A) \cup cl(B)$.

(viii) $int(A \cap B) = int(A) \cap int(B)$.

3. In this section we study study some relationships connecting the closures and interiors of an intuitionistic fuzzy set in an intuitionistic fuzzy topological space and the closures and interiors of its co-ordinate fuzzy sets in its corresponding fuzzy topological spaces.

Notations: $C_T(U)$ denote the collection of closed sets containing the set U under the topology T. $I_T(U)$ denote the collection of open sets contained in U under the topology T.

Theorem 3.1: If $Int_T(U_1,U_2) = (\mu_k, \gamma_k)$, then $\mu_k \le Int_{T1} U_1$ and $\gamma_k \ge Cl_{T2} U_2$.

Proof: $Int_T(U_1,U_2) = \max\{ (\mu_i, \gamma_i) / (\mu_i, \gamma_i) \le (U_1, U_2) \}$

$=.\max \{ (\mu_i, \gamma_i) / \mu_i \le U_1$ and $\gamma_i \ge U_2 \}$ $= \max I_T(U_1, U_2) .= (\mu_k, \gamma_k)$.

$Int_{T1} U_1 = \max \{ \mu_i / \mu_i \le U_1\} = \max I_{T1}(U_1)$.

Hence $(\mu_k, \gamma_k) \in I_T(U_1, U_2) \Longrightarrow \mu_k \in I_{T1} U_1$. Therefore $\mu_k \le Int_{T1} U_1$.

Also $Cl_{T2} U_2 = \min \{\gamma_i / U_2 \le \gamma_i\} = \min C_{T2}(U_2)$.

Therefore $(\mu_k, \gamma_k) \in I_T(U_1, U_2) \Longrightarrow \gamma_k \in C_T(U_2)$.

Hence $\gamma_k \ge Cl_{T2}(U_2)$.

Theorem 3 .2: If $Cl_T(U_1,U_2) = (\gamma_k, \mu_k)$. Then $\mu_k \le (Cl_{T1} U_1)^C$ and $\gamma_k \le (Int_{T2}U_2)^C$.

Proof:

$Cl_T(U_1,U_2) = \min\{(\gamma_i , \mu_i) / (U_1,U_2) \le (\gamma_i , \mu_i)\}$.

$=\min \{(\gamma_i , \mu_i) / U_1 \le \gamma_i$ and $U_2 \ge \mu_i \}$

$.= \min C_T(U_1,U_2) .= (\gamma_k , \mu_k)$.

$Cl_{T1}(U_1) = \min\{1- \mu_i / U_1 \le 1- \mu_i \}$.

$= \min C_{T1}(U_1)$.Now $(\gamma_i, \mu_i) \in C_T(U_1,U_2) \Longrightarrow U_2 \ge \mu_i$ $\Longrightarrow 1-U_2 \le 1- \mu_i. \Longrightarrow U_1 \le 1- \mu_i$.

(Since (U_1, U_2) is an IFS $U_1 \le 1-U_2$). Hence $(\gamma_i, \mu_i) \in C_T(U_1,U_2) \Longrightarrow 1- \mu_i \in C_{T1}(U_1)$.

$\mu_k = \max \{ \mu_i /(\gamma_i , \mu_i) \in C_T(U_1,U_2)\}.Cl_{T1}(U_1)$ $= \min\{1- \mu_i / 1- \mu_i \in C_T(U_1)\}$.

$\max \mu_i = 1-\min (1- \mu_i)$. Hence $\mu_k \le (Cl_{T1} U_1)^C$.

$Int_{T2}(U_2) = \max \{1- \gamma_i / 1- \gamma_i \le U_2\} = \max I_{T2}(U_2)$. $1-\gamma_i \in I_{T2}(U_2) \Longrightarrow U_2 \ge 1- \gamma_i \ge \mu_i$.

Also $U_2 \ge 1- \gamma_i \Longrightarrow 1- U_2 \le \gamma_i \Longrightarrow U_1 \le \gamma_i$. (Since (U_1,U_2) is an IFS $U_1 \le 1-U_2$)

Hence $1- \gamma_i \in I_{T2}(U_2) \Longrightarrow (\gamma_i , \mu_i) \in C_T(U_1,U_2)$

$Int_{T2}(U_2) = \max \{1- \gamma_i / 1- \gamma_i \in I_{T2}(U_2)\}$.

$Min\ \gamma_i = 1-\max (1- \gamma_i)$. Therefore $\gamma_k \le [Int_{T2}(U_2)]^C$.

Theroem 3.3: If $Cl_T(U_1, U_2) = (\gamma_k, \mu_k)$ then $\gamma_k \ge Cl_{T2} U_1$ and $\mu_k \le Int_{T1}U_2$.

Proof: $Cl_T(U_1,U_2) = (Int_T(U_2,U_1))^C$. (By result 1.1, iii)

$\Longrightarrow (Int_T(U_2 ,U_1))^C = (\gamma_k , \mu_k)$. $\Longrightarrow Int_T(U_2 ,U_1) =(\mu_k\ \gamma_k)$.

Hence by theorem 3.1, $\mu_k \le Int_{T1}(U_2)$ and $\gamma_k \ge Cl_{T2}U_1$.

Definition 3.1: Let (X, T) be an intuitionistic fuzzy topological space. Let (X, T_1) and (X, T_2) be the first and second coordinate fuzzy topological spaces . An IF set (A_1, A_2) is said to be an ***I-set*** if $Int_T(A_1,A_2) = (Int_{T1} A_1, Cl_{T2}A_2)$.

Theorem 3.4: Intersection of two I- sets is an I- set.

Proof: Let (X, T) be an IFTS. Let (A_1, A_2), (B_1, B_2) be two I-sets defined on X. Then

$Int_T(A_1, A_2) = (Int_{T1}A_1, Cl_{T2}A_2)$ and $Int_T(B_1, B_2) = (Int_{T1}B_1, Cl_{T2}B_2)$

Let $(D_1, D_2) = (A_1, A_2) \cap (B_1, B_2) = (A_1 \cap B_1, A_2 \cup B_2)$.

Hence $D_1 = A_1 \cap B_1$ and $D_2 = A_2 \cup B_2$

Now $Int_T(D_1, D_2) = Int [(A_1,A_2) \cap (B_1,B_2)] = Int (A_1, A_2) \cap Int_T(B_1,B_2)$

$= (Int_{T1}A_1, Cl_{T2} A_2) \cap (Int_{T1}B_1 , Cl_{T2}B_2) = (Int_{T1}A_1 \cap Int_{T1}B_1, Cl_{T2}A_2 \cup Cl_{T2}B_2)$

$= (Int_{T1}(A_1 \cap B_1), Cl_{T2} (A_2 \cup B_2))$ (By result 1.1, (vii) and (viii)) $= (Int_{T1} D_1, Cl_{T2} D_2)$.

Remark3.1 : However, union of two I-sets need not be an I-set.

Consider the following example.

Example:

Let X= {a, b, c} .Let T = {$0_\sim$, (μ_i, γ_i); i=1 to 5, $1_\sim$} where (μ_i, γ_i)s are given below.

	(μ_1, γ_1)	(μ_2, γ_2)	(μ_3, γ_3)	(μ_4, γ_4)	(μ_5, γ_5)
a	(.3,.4)	(.5,.2)	(.5,.2)	(.3,.4)	(.6,.05)
b	(.6,.2)	(.7,.1)	(.7,.1)	(.6,.2)	(.8,.01)
c	(.5,.4)	(.3,.3)	(.5,.3)	(.3,.4)	(.6,.02)

Let (A_1, A_2), (B_1, B_2) be two IF-sets and (C_1, C_2) = (A_1, A_2) ∪(B_1,B_2) as given below.

	(A_1,A_2)	(B_1,B_2)	(C_1,C_2)
a	(.4,.3)	(.6,.1)	(.6,.1)
b	(.8,.1)	(.75,.05)	(.8,.05)
c	(.6,.2)	(.4,.2)	(.6,.2)

$Int_T(A_1, A_2) = (\mu_1, \gamma_1)$, $Int_{T1}(A_1) = \mu_1$, $Cl_{T2}(A_2) = \gamma_1$. $Int_T(B_1,B_2) = (\mu_2, \gamma_2)$, $Int_{T1}(B_1) = \mu_2$, $Cl_{T2}(B_2) = \gamma_2$.Therefore (A_1, A_2) and (B_1, B_2) are I-sets.

But $Int_T(C_1, C_2) = (\mu_3, \gamma_3)$ and $Int_{T1}(C_1) = \mu_5 \neq \mu_3$.Hence ($C_1$, C_2) is not an I- set.

Theorem 3.5: Every open set is an I-set.

Proof: Let (X, T) be an IFTS. Let (A_1, A_2) be an IF open set under T. Then A_1 is open under T_1 and A_2 is closed under T_2 which implies $Int_{T1}A_1 = A_1$ and $Cl_{T2}A_2 = A_2$.

Hence (A_1, A_2) is open $\Longrightarrow Int_T(A_1, A_2)= (A_1, A_2)= (Int_{T1} A_1, Cl_{T2}A_2) \Longrightarrow$ (A_1, A_2) is an I-set.

Definition 3.2: Let (X, T) be an intuitionistic fuzzy topological space. Let (X, T_1) and (X, T_2) be the first and second coordinate fuzzy topological spaces. An IF Set (A_1, A_2) is said to be a ***C- set*** if $Cl_T(A_1,A_2) = (Cl_{T2}A_1 ,Int_{T1} A_2)$.

Theorem 3.6: Union of two C-sets is a C-set.

Proof: Let (X, T) be an IFTS. Let (A_1, A_2), (B_1, B_2) be two C-sets.

Hence $Cl_T(A_1, A_2) = (Cl_{T2} A_1 ,Int_{T1} A_2)$ and $Cl_T(B_1, B_2) = (Cl_{T2} B_1, Int_{T1}B_2)$.

Let $(C_1, C_2) = (A_1, A_2) \cup (B_1, B_2) = (A_1 \cup B_1, A_2 \cap B_2)$.

Hence $C_1 = A_1 \cup B_1$, $C_2 = A_2 \cap B_2$.

$Cl_T(C_1,C_2) = Cl_T(A_1,A_2) \cup Cl_T(B_1,B_2) = (Cl_{T2}A_1, Int_{T1} A_2) \cup (Cl_{T2} B_1, Int_{T1} B_2)$.

$= (Cl_{T2} A_1 \cup Cl_{T2} B_1, Int_{T1} A_2 \cap Int_{T1} B_2)$.$= (Cl_{T2} A_1 \cup B_1, Int_{T1} A_2 \cap B_2))$

(By result 1.1, (vii) and (viii)) $= (Cl_{T2}C_1, Int_{T1}C_2)$.

Theorem 3.7: (A_1, A_2) is an I- set iff it complement is a C- set.

Proof: Let (X, T) be an IFTS.

(A_1, A_2) is an I-set in X $\Leftrightarrow$ $Int_T (A_1, A_2) = (Int_{T1} A_1, Cl_{T2} A_2)$

$\Leftrightarrow [Cl_{T2}(A_2,A_1)]^C = (Int_{T1} A_1, Cl_{T2} A_2)$ (By result 1.1, iv)

$\Leftrightarrow C_{l\,T}(A_2, A_1) =(Cl_{T2}A_2, Int_{T1} A_1) \Leftrightarrow (A_1, A_2)$ is a C-set.

Remark 3.2: Intersection of two C-sets need not be a C-set. For, as in the example in the remark 3.1, (A_1, A_2) and (B_1, B_2) are I-sets and $(A_1, A_2) \cup (B_1, B_2) = (C_1, C_2)$

$\Rightarrow [(A_1, A_2) \cup (B_1, B_2)]^C = (C_2, C_1) \Rightarrow (A_2, A_1) \cap (B_2, B_1) = (C_2, C_1)$.

(A_1, A_2) and (B_1, B_2) are I-sets $\Rightarrow (A_2, A_1)$ and (B_2, B_1) are C-Sets. Hence (C_2, C_1) is an intersection of two C-sets.(C_1, C_2) is not an I-Set $\Rightarrow (C_2, C_1)$ is not a C-set.

Therefore intersection of two C-sets need not be a C-set.

Theorem 3.8: Every closed set is a C-set.

Proof: Let (X, T) be an IFTS.

An IFS (A_1, A_2) is closed in (X, T) $\Rightarrow (A_1, A_2)^C$ is open.

$\Rightarrow (A_1, A_2)^C$ is an I-set (by theorem 3.5) $\Rightarrow ((A_1, A_2)^C)^C$ is a C-set (by theorem 3.7)

$\Rightarrow (A_1, A_2)$ is a C-set.

Theorem 3.9: Let (X, T) be an IFTS. Let T_1, T_2 be the co-ordinate fuzzy topologies of T.

(i). An IFS (A_1, A_2) is an I-set iff $(Int_{T1} A_1, Cl_{T2} A_2)$ is open under T.

(ii). An IFS (A_1, A_2) is a C-set iff $(Cl_{T2} A_1 , Int_{T1} A_2)$ is a closed under T.

Proof: (i) (A_1, A_2) is an I-set

$\Rightarrow Int_T(A_1, A_2) = (Int_{T1} A_1, Cl_{T2} A_2)$.

Hence $(Int_{T1} A_1, Cl_{T2} A_2)$ is open under T.

Conversely assume $(Int_{T1} A_1, Cl_{T2} A_2)$ is open under T.

By theorem 3.1, $Int_T (A_1, A_2) \leq (Int_{T1} A_1, Cl_{T2} A_2)$.Also $(Int_{T1} A_1, Cl_{T2} A_2) \leq (A_1, A_2)$ and $(Int_{T1} A_1, Cl_{T2} A_2)$ is open $\Rightarrow Int_T(A_1, A_2) \geq (Int_{T1} A_1, Cl_{T2} A_2)$.

Hence $Int_T(A_1, A_2) = (Int_{T1} A_1, Cl_{T2} A_2)$.

(ii). (A_1, A_2) is a C-set $\Leftrightarrow (A_2, A_1)$ is an I-set

(by theorem 3.7)

$\Leftrightarrow (Int_{T1} A_2, Cl_{T2} A_1)$ is open (by (i)) $\Leftrightarrow (Cl_{T2}A_1, Int_{T1} A_2)$ is closed.

Theorem 3.10: Let (X, T) be an IFTS where T= $\{(\mu_i, \gamma_i) / i \in I\}$.Let (A_1, A_2) be IFS such that $\mu_i \leq A_1 \Leftrightarrow \gamma_i \geq A_2$, $\forall i \in I$, then Int $_T (A_1, A_2)$ = (Int$_{T1}$ A_1, Cl $_{T2}$ A_2).

Proof: $\mu_i \in I_{T1} A_1 \Leftrightarrow \gamma_i \in C_{T2} A_2 \Leftrightarrow (\mu_i, \gamma_i) \in I_T (A_1, A_2)$.

Therefore Int$_T(A_1, A_2)$ = (Int$_{T1}$ A_1, Cl $_{T2}$ A_2).

Remark 3.3: The converse of the above theorem is not true.

Let X = {a, b, c}. Let (X, T) be an IFTS, where T = { 0~, τ_1, τ_2 , τ_3 , τ_4 , τ_5 ,τ_6 , ,τ_7 ,τ_8 ,τ_9,τ_{10}, τ_{11},1~ } and τ_i = (μ_i, γ_i) are given below (μ_i, γ_i) are given below.

	(μ_1, γ_1)	(μ_2, γ_2)	(μ_3, γ_3)	(μ_4, γ_4)
a	(0.15,0.5)	(0.1,0.3)	(0.3,0.6)	(0.1,0.5)
b	(0.3,0.3)	(0.2,0.4)	(0.2,0.5)	(0.2,0.4)
c	(0.2,0.35)	(0.1,0.2)	(0.4,0.4)	(0.1,0.35)

	(μ_5, γ_5)	(μ_6, γ_6)	(μ_7, γ_7)	(μ_8, γ_8)
a	(0.1,0.6)	(0.15,0.6)	(0.1,0.6)	(0.15,0.3)
b	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	(0.3,0.3)
c	(0.1,0.4)	(0.2,0.4)	(0.1,0.4)	(0.2,0.2)

	(μ_9, γ_9)	(μ_{10}, γ_{10})	(μ_{11},γ_{11})
a	(0.3,0.3)	(0.3,0.5)	(0.3,0.3)
b	(0.2,0.4)	(0.3,0.3)	(0.3,0.3)
c	(0.4,0.2)	(0.4,0.35)	(0.4,0.2)

Let (A1,A2) be IFS define by

	(A_1, A_2)
a	(0.2,0.4)
b	(0.4,0.2)
c	(0.3,0.3)

Int $_T (A_1, A_2) = \tau_1$, Int $_{T1}$ $A_1 = \mu_1(=\mu_8)$, Cl $_{T2}$ $A_2 = \gamma_1(=\gamma_{10})$.

Hence Int$_T(A_1, A_2)$ = (Int$_{T1}$ A_1, Cl $_{T2}$ A_2).

However, $\mu_2 \leq A_1$ and $\gamma_2 \ngeq A_2$. $\mu_8 \leq A_1$ and $\gamma_8 \ngeq A_2$. $\gamma_3 \geq A_2$ and $\mu_3 \nleq A_1$.

$\gamma_{10} \geq A_2$ and $\mu_{10} \nleq A_{10}$..

Theorem 3.11: Let (X, T) be an IFTS . Let T_1 and T_2 be the co- ordinate fuzzy topologies of T. Let (A_1, A_2) be IFS. Let Int $_{T1}$ $A_1 = \mu_k$ and Cl $_{T2}$ $A_2 = \gamma_s$. Then Int $_T (A_1, A_2)$ = (Int$_{T1}$ A_1, Cl $_{T2}$ A_2) iff $\gamma_k \geq A_2$ and $\mu_s \leq A_1$

Proof: Assume that $Int_T(A_1, A_2) = (Int_{T1} A_1, Cl_{T2} A_2)$ then $\gamma_k = \gamma_s$ and $\mu_s = \mu_k$.

$\gamma_s \geq A_2$ and $\mu_k \leq A_1 \Longrightarrow \gamma_k \geq A_2$ and $\mu_s \leq A_1$.

Conversely, assume $\gamma_k \geq A_2$ and $\mu_s \leq A_1$.Now $\mu_k \leq A_1$ and $\gamma_k \geq A_2$

$\Longrightarrow (\mu_k, \gamma_k) \leq (A_1, A_2)$. $\mu_s \leq A_1$ and $\gamma_s \geq A_2 \Longrightarrow (\mu_s, \gamma_s) \leq (A_1, A_2)$. Hence $(\mu_k \cup \mu_s, \gamma_k \cap \gamma_s)$ is open and $\leq (A_1, A_2)$.

$\mu_k = Int_{T1} A_1$ and $\mu_s \leq A_1 \Longrightarrow \mu_s \leq \mu_k$.$\gamma_s = Cl_{T2} A_2$ and $\gamma_k \geq A_2 \Longrightarrow \gamma_k \geq \gamma_s$.

Hence $\mu_k \cup \mu_s = \mu_k$ and $\gamma_k \cap \gamma_s = \gamma_s$. Hence (μ_k, γ_s) is open and $\leq (A_1, A_2)$

If (μ_i, γ_i) is any open set such that$(\mu_i, \gamma_i) \leq (A_1, A_2)$, $\mu_k = Int_{T1} A_1 \Longrightarrow \mu_i \leq \mu_k$

$\gamma_s = Cl_{T2}(A_2) \Longrightarrow \gamma_i \geq \gamma_s$. Hence $(\mu_i, \gamma_i) \leq (\mu_k, \gamma_s)$.

Therefore $Int_T(A_1, A_2) = (\mu_k, \gamma_s) = (Int_{T1} A_1, Cl_{T2} A_2)$.

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