Ordering Of Intuitionistic Fuzzy Numbers Using Centroid Of Centroids Of Intuitionistic Fuzzy Number

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Abstract

This is a novel method of ascertaining the ranking of the Trapezoidal Intuitionistic Fuzzy Number (TIF) and Triangular Intuitionistic Fuzzy Number (TrIF) applying the mean of centroids. A comparative study is conducted about the proposed ranking and other methods of ranking for the Trapezoidal as well as Triangular Intuitionistic Fuzzy Numbers (TIF and TrIF).

Keywords - *Intuitionistic fuzzy set ,Trapezoidal intuitionistic fuzzy number , Triangular intuitionistic fuzzynumber, Ranking of trapezoidal intuitionistic fuzzy number , Centroid of an intuitionistic fuzzy number*.

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I. INTRODUCTION

Fuzzy sets were introduced by Zadeh [29] where as Atanassov [4] broadened their scope and named them as intuitionistic fuzzy sets which include the non-membership function. These intuitionistic fuzzy sets are very handy in expressing the vagueness and then the fuzzy sets themselves. These special kind[1] of fuzzy sets are fuzzy numbers which are substantially useful in solving the problems related to linear programming. Ranking of fuzzy numbers plays a very crucial role in fuzzy set theory. The assessment of the numerical values is not that easy when the numerical values are represented in ambiguity known to be fuzzy numbers. Therefore various methods have been brought forth to order fuzzy numbers. Intuitionistic Fuzzy Numbers (IFN) seem befitting for describing the uncertainty followed by the generalized fuzzy numbers. Methods for ordering IFNs have been presented in the literature. Grzegorzewski [7,8] considered IFNs as two families different metrics and enlarged a ranking method for them .Mitchell[11]proposed the process of ordering triangular intuitionistic fuzzy numbers is by accepting the statistical outlook and thereby interpreting each IFN as a group of ordinary fuzzy numbers. In order to solve the drawback caused by multi-attribute decision making, Li [9] put forth the ranking of TIFs built on the value index to ambiguity index. Very recently, Dubey [6] endeavoured to define TrIF numbers. A ranking function to solve the problems of a class of linear programming was proposed. During the earlier times, Nayagam et al.[14] introduced TrIF numbers alongside a process to rank them. A new method of ranking was introduced for the membership as well as non-membership functions stating that IFNs are fuzzy quantities, as proposed by Nehi [15]. Li [10] applied the same technique and defined the value of uncertainty index for TrIF numbers which is seemingly similar to those of Delgado et al.[5]. These quantities were then used to define the value index and uncertainty index for TIF numbers. Putting into use the same concept, basing on ratio ranking TIF, another method is developed. The shortcomings of many of the current rankings prevailed over until a new ranking process was projected by Amit Kumar [2] which works by modifying an existing ranking process projected for comparing IF numbers. Based on the proposed process of ranking, a new method is planned to find the optimal solution of unbalanced minimum cost flow (MCF) problems, in which all the parameters are represented by IF numbers.Arun[3] presented solution of intuitionistic fuzzy transportation problem. Rezvani [18] defined the values and ambiguities of the membership as well as non-membership degrees for trapezoidal intuitionistic fuzzy numbers. Value index and ambiguity index based ranking approach is also discussed in detail. Nayagam et al.[13] defined new intuitionistic fuzzy scoring method for the intuitionistic fuzzy number in which uncertainty is greater than membership fuzzy number. In contrast, in the intuitionistic fuzzy number, the uncertainty is less than the membership fuzzy number. This novel technique includes the idea of both membership and non-membership function of an intuitionistic fuzzy number. By this defined method, the problems involving uncertainty can be studied easily. In continuation to this, the relation between generalized fuzzy numbers and canonical intuitionistic fuzzy numbers is discussed by Peng et al.[17]. Using the idea of center index and radius index of canonical intuitionistic fuzzy numbers, fuzzy cut sets are introduced. Canonical intuitionistic fuzzy numbers are defined as the ranking index with the degree of optimism of the decision maker. Then a novel ranking method based on the ranking index has been developed and using this method, MADM problems in which the ratings of alternatives on attributes are expressed as TIFNs are solved by the extended additive weighed method by Zhang et al.[30]. Based on membership and non-membership function ranking, triangular intuitionistic fuzzy numbers (TrIF) are obtained by converting two related triangular fuzzy numbers (TrF). Then a new defuzzification for the obtained TFNs using their values and ambiguities was suggested by Salahshour [20]. Mean ranking index method to find out the order relations between two TRIF numbers is introduced by Seikh et al.[24]. A possibility degree method is discussed by Wei et al.[27] to rank intuitionistic fuzzy numbers, which is used to rank the alternatives in multi-criteria decision-making problems. .[24]. A possibility degree method is discussed by Wei et al.[27] to rank intuitionistic fuzzy numbers, which is used to rank the alternatives in multi-criteria decision-making problems. Ye [28] developed an approach of cosine similarity measure and extended it to rank alternatives. Seikh et al.[24] gave another practical example of a modified approach to select the investment alternatives and they also put forth the basic arithmetic operations of Generalized Triangular Intuitionistic Fuzzy Numbers (GTFINs) and the notion of (a, b)-cut sets. Yet another closest interval estimate method is described to estimate a GTFIN to a nearest interval number. In addition, the average ranking index is introduced to discover the order relations between two GTFINs. Intuitionistic Trapezoidal Fuzzy Weighted arithmetic averaging operator and weighted geometric averaging operator are introduced by Wang et al.[25][26]. The definitions of the expected values, score function and the precision function of intuitionistic trapezoidal fuzzy numbers are also given by them. A ranking of the whole alternative set has been obtained by comparing the score function and the precision function valies of integrated fuzzy numbers. A ranking procedure for TrIF numbers by means of a, b-cut, score function and precision function has been proposed by Nagoorgani et al.[12]. The method is calibrated by applying the idea to solve the intuitionistic fuzzy variable linear programming problem. Suresh[21][22] solving the intuitionistic fuzzy linear programming by using ranking function and assignment problem.Seikh et al. in [23]. A method is described to approximatea TIFN to a nearly approximated interval number.Applying this result and using interval arithmetic, a boundunconstrained optimization problem is solved whose coefficients are fixed TIFNs.

A new ranking method for intuitionistic fuzzy numbers is introduced in this paper. A deep comparative study is presented on the existing and the new ranking methods. The rest of the paper is structured as follows; SECTION-2 discusses the basic definitions of intuitionistic fuzzy sets and intuitionistic fuzzy numbers. SECTION-3 discusses the ranking of intuitionistic fuzzy number based on centroid of centroids of intuitionistic fuzzy number. SECTION-4 presents a numerical example for the proposed ranking method and SECTION-5 discusses the comparative and conclusions.

II. PRELIMINARIES

In this section some basic definitions related to Intuitionistic Fuzzy set and Intuitionistic numbers are revived

2.1.1 *Intuitionistic Fuzzy set[4]*

Let X be the universal set and the membership function, non membership functions defined on X by $\mu_{\tilde{A}} : X \to [0,1]$, $v_{\tilde{A}} : X \to [0,1]$ .Degree of membership function and non membership functions are $\mu_{\tilde{A}}(x)$, $v_{\tilde{A}}(x)$ which always satisfies the condition $\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1 \ \forall x \in X$,then the set $\tilde{A} = \{x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) / x \in X\}$ is an Intuitionistic fuzzy set (IFS).

2.1.2 $\alpha -$ *cut of* $\tilde{A}$

An Intuitionistic fuzzy set (IFS) $\tilde{A}$ and $\alpha \in [0,1]$, then the set

$\tilde{A}_\alpha = \{x \in X / \mu_{\tilde{A}}(x) \ge \alpha, v_{\tilde{A}}(x) \le (1-\alpha) / x \in X\}$ is said to be $\alpha -$ cut of $\tilde{A}$.

2.1.3 *Intuitionistic fuzzy normal*[15]

An Intuitionist fuzzy set (IFS) $\tilde{A}_\alpha$ is Intuitionist fuzzy normal if there

Exist points $x_0, x_1 \in X$ such that $\mu_{\tilde{A}}(x_0) = 1, v_{\tilde{A}}(x_1) = 1$.

2.1.4 *Intuitionistic fuzzy convex*[15]

An Intuitionist fuzzy set (IFS) $\tilde{A} = \{(x, \mu_{\tilde{A}}, v_{\tilde{A}}, x \in X)\}$ is Intuitionist fuzzy convex if there exist points $x_0, x_1 \in X$, $\lambda \in [0,1]$ such that

$$\mu_{\tilde{A}}\left(\lambda x_0+(1-\lambda)x_1\right)\geq\mu_{\tilde{A}}(x_0)\wedge\mu_{\tilde{A}}(x_1)$$
$$v_{\tilde{A}}\left(\lambda x_0+(1-\lambda)x_1\right)\geq v_{\tilde{A}}(x_0)\wedge v_{\tilde{A}}(x_1)$$

2.1.5 *Intuitionist fuzzy number*[15]

An Intuitionist fuzzy set (IFS) $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x),v_{\tilde{A}}(x)\right)/x\in X\right\}$ is Intuitionist Fuzzy number if

$\tilde{A}$ is Intuitionistic fuzzy normal

$\tilde{A}$ is Intuitionistic fuzzy convex $\mu_{\tilde{A}}$, $v_{\tilde{A}}(x)$ are upper and lower semi continuous, the set $\tilde{A}=\left\{x\in X/v_{\tilde{A}}(x)<1\right\}$ is bounded

2.1.6 *Arithmetic operations between TIF numbers*[16]

Let R be the universal set of real numbers the arithmetic operation between TIF numbers are presented.

TIF numbers addition

TwoTIF $\tilde{A}=\left(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4:w_1,w_2\right)$ and

$\tilde{B}=\left(b_1',a_1',b_2',a_2',a_3',b_3',a_4',b_4':w_1',w_2'\right)$ defined on universal set R then

$$\tilde{A}\oplus\tilde{B}=\begin{pmatrix}b_1+b_1',a_1+a_1',b_2+b_2',a_2+a_2',a_3+a_3',\\ b_3+b_3',a_4+a_4'+b_4+b_4';\\ \max\left\{w_1,w_2,w_1',w_2'\right\}\\ 0<w_1,w_2,w_1',w_2'\leq 1\end{pmatrix}$$

TIF numbers multiplication

TwoTIF $\tilde{A}=\left(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4:w_1,w_2\right)$ and

$\tilde{B}=\left(b_1',a_1',b_2',a_2',a_3',b_3',a_4',b_4':w_1',w_2'\right)$ defined on universal set R then

$$\tilde{A}\otimes\tilde{B}=\begin{pmatrix}b_1b_1',a_1a_1',b_2b_2',a_2a_2',a_3a_3',b_3b_3',a_4a_4',b_4\,b_4';\\ 0<w_1w_2w_1'w_2'\leq 1\end{pmatrix}$$

TIF numbers multiplied by constant λ

Two TIF $\tilde{A}=\left(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4:w_1,w_2\right)$ and defined on universal set R then

$$\lambda\tilde{A}=\begin{pmatrix}\lambda b_{11},\lambda a_1,\lambda b_2,\lambda a_2,\lambda a_3,\lambda b_3,\lambda a_4,\lambda b_4,\\ 0<w_1,w_2\leq 1,\lambda>0\end{pmatrix}$$

2.1.7GeneralizedTrapezoidal(TIF) and Triangular(TrIF) Intuitionist fuzzy number [15]

A trapezoidal intuitionistic fuzzy number with parameters $b_1\leq a_1\leq b_2\leq a_2\leq a_3\leq b_3\leq a_4\leq b_4$ is denoted by $\tilde{A}=\left(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4\right)$ in this the membership and non membership function defined as

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & x<a_1\\ \dfrac{x-a_1}{a_2-a_1} & a_1\leq x<a_2\\ 1 & a_2\leq x\leq a_3\\ \dfrac{x-a_4}{a_3-a_4} & a_3\leq x<a_4\\ 0 & a_4<x\end{cases}\qquad v_{\tilde{A}}(x)=\begin{cases}0 & x<b_1\\ \dfrac{x-b_2}{b_1-b_2} & b_1\leq x<b_2\\ 0 & b_2\leq x<b_3\\ \dfrac{x-a_4}{a_3-a_4} & b_3\leq x<b_4\\ 1 & b_4<x\end{cases}$$

In the above definition if $b_1\leq a_1\leq\left(b_2=a_2=a_3=b_3\right)\leq a_4\leq b_4$ then the trapezoidal(TIF) become Triangular(TrIF) Intuitionist fuzzy number is denoted by $\tilde{A}=\left(b_1,a_1,b_2,a_4,b_4\right)$

2.1.8*Symmetric trapezoidal Intuitionistic fuzzynumber*

An Intuitionistic fuzzy number A symmetric trapezoidal Intuitionistic fuzzy number if there exists real numbers a_1, a_2, h, h' where $a_1 \le a_2, h \le h'$ and $h, h' > 0$ such that membership and non membership functions as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(a_1-h)}{h} & x \in [a_1-h, a_1] \\ 1 & x \in [a_1, a_2] \\ \dfrac{(a_2+h)-x}{h} & x \in [a_2, a_2+h] \\ 0 & \text{other wise} \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_1-x}{h'} & x \in [a_1-h', a_1] \\ 0 & x \in [a_1, a_2] \\ \dfrac{x-a_2}{h} & x \in [a_2, a_2+h'] \\ 1 & \text{otherwise} \end{cases}$$

Where $\tilde{A} = (a_1, a_2, h, h; a_1, a_2, h', h')$

III. RANKING OF INTUITIONISTIC FUZZY NUMBER BASED ON CENTROID OF CENTROIDS OFINTUITIONISTIC FUZZY NUMBER

In this section, ranking of Intuitionistic fuzzy numbers based on centroid of centroids point of the trapezoidal intuitionistic fuzzy numbers and triangular intuitionistic fuzzy numbers using membership and non membership functions are discussed. The method of ranking of trapezoidal intuitionistic fuzzy numbers with centroid of centroids index uses the geomentric center of a trapezoidal intuitionistic fuzzy number.The Geomentric center corresponds to $\overline{x}(\tilde{A})$ value on the horizontal axis and $\overline{y}(\tilde{A})$ vertical axis.In this trapezoidal intuitionistic fuzzy number is devided in to two trapezoids and are corresponds to membership and nonmembersip functions respectively. The trapezoid divides into three plane figures which are three triangles ARC, RCS, CSD. The centroids of these three triangles are G_1, G_2, and G_3 respectively. $G_{\tilde{A}}$ is the centroid of the centroids G_1, G_2, and G_3. Using this centroid as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers could be achieved. Generally, the centroid of the trapezoid is considered to be the balancing point of the trapezoid. Pick the centroid point of centroids as a point of reference to rank fuzzy numbers. Each centroid point G_1 of triangle ARC, G_2 of triangle RCS, G_3 of triangle CSD is a balancing point of each plane and the centroid of these centroids is a better balancing point of generalized trapezoidal fuzzy number. Centroid of triangle ARC formed by the points $A = (a_1, 0)$ $R = (a_2, w_1)$ $C = (a_3, 0)$ is $G_1 = \left(\dfrac{a_1 + a_2 + a_3}{3}, \dfrac{w_1}{3} \right)$

centroid of triangle RCS formed by the points and $R = (a_2, w_1)$ $C = (a_3, 0)$ and $S = (a_3, w_1)$ is

$$G_2 = \left(\frac{a_2 + a_3 + a_3}{3}, \frac{2w_1}{3} \right) = \left(\frac{a_2 + 2a_3}{3}, \frac{2w_1}{3} \right)$$

centroid of triangle CSD formed by the points $C = (a_3, 0)$ $S = (a_3, w_1)$ and $D = (a_4, 0)$ is

$$G_3 = \left(\frac{a_3 + a_3 + a_4}{3}, \frac{w_1}{3} \right) = \left(\frac{2a_3 + a_4}{3}, \frac{w_1}{3} \right)$$

Observe that the centroid points G_1 G_2 and G_3 are non-collinear points, because the point G_2 does not lie on $y=\frac{w_1}{3}$ of the line $\overleftrightarrow{G_1G_3}$, Therefore they form a triangle. Now the cetroid of the triangle $\Delta G_1G_2G_3$ with vertices are $G_1=\left(\frac{a_1+a_2+a_3}{3},\frac{w_1}{3}\right)$ $G_2=\left(\frac{a_2+2a_3}{3},\frac{2w_1}{3}\right)$ and $G_3=\left(\frac{2a_3+a_4}{3},\frac{w_1}{3}\right)$ is $G=\left(\frac{a_1+2a_2+5a_3+a_4}{9},\frac{4w_1}{9}\right)$

Similarly, the centroid of centroids of trapezoidal fuzzy number using non membership function is $G'_{\tilde{A}}=\left(\frac{b_1+5b_2+2b_3+b_4}{9},\frac{5w_2}{9}\right)$

The mean of centroid of centroids of trapezoidal intuitionistic fuzzy number using membership function and non -membership is

$$\left(\bar{x}_{\mu v},\bar{y}_{\mu v}\right)=\left(\frac{a_1+b_1+2\left(a_2+b_3\right)+5\left(a_3+b_2\right)+\left(a_4+b_4\right)}{18},\frac{4w_1+5w_2}{18}\right)$$

The mean of centroid of centroids of triangular trapezoidal intuitionistic fuzzy number using membership function and non –membership $b_1\le a_1\le\left(b_2=a_2=a_3=b_3\right)\le a_4\le b_4$ is

$$\left(\bar{x}_{\mu v},\bar{y}_{\mu v}\right)=\left(\frac{\left(a_1+b_1\right)+7a_2+\left(a_4+b_4\right)}{18},\frac{4w_1+5w_2}{18}\right)$$

Now the ranking function is defined for Trapezoidal and triangular Intuitionistic fuzzy number as

$$R\left(\tilde{A}\right)=\left(\frac{a_1+b_1+2\left(a_2+b_3\right)+5\left(a_3+b_2\right)+\left(a_4+b_4\right)}{18}\right)\left(\frac{4w_1+5w_2}{18}\right)$$

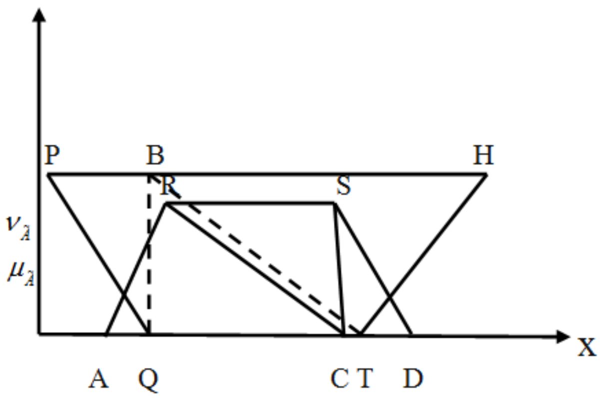

Figure 1: Trapezoidal intuitionistic fuzzy number

The properties of ranking function for any two intuitionistic fuzzy numbers fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are $R\left(\tilde{A}\right)<R\left(\tilde{B}\right)\Rightarrow\tilde{A}<\tilde{B}$; $R\left(\tilde{A}\right)>R\left(\tilde{B}\right)\Rightarrow\tilde{A}>\tilde{B}$; $R\left(\tilde{A}\right)=R\left(\tilde{B}\right)\Rightarrow\tilde{A}\approx\tilde{B}$ in this the discrimination of fuzzy numbers is not possible. In such case define the index associated with the ranking as

$$I_{\alpha,\beta}\left(\tilde{A}\right)=\beta S_M\left(\tilde{A}\right)+(1-\beta)I_\alpha\left(\tilde{A}\right)$$

where $\beta\in[0,1]$. $S_M\left(\tilde{A}\right)$ is the mode associated with the triangular Intuitionistic fuzzy number which is equal to 'a_2' for a triangular Intuitionistic fuzzy number and the average value of the area of stability for a trapezoidal Intuitionistic fuzzy number and we define the index associated with the ranking as

$$I_\alpha\left(\tilde{A}\right)=\alpha\bar{y}_0+(1-\alpha)\bar{x}_0$$

where $\alpha\in[0,1]$ is the index of optimism which represents the degree of optimism of a decision maker. The ranking is defined as

If $I_{\alpha,\beta}\left(\tilde{A}_i\right)>I_{\alpha,\beta}\left(\tilde{A}_j\right)\Rightarrow\tilde{A}_i>\tilde{A}_j$

If $I_{\alpha,\beta}\left(\tilde{A}_i\right)<I_{\alpha,\beta}\left(\tilde{A}_j\right)\Rightarrow\tilde{A}_i<\tilde{A}_j$

Theorem -1

Let $\tilde{A}=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ and $\tilde{B}=(b_1',a_1',b_2',a_2',a_3',b_3',a_4',b_4')$ be two trapezoidal intuitionistic fuzzy number and then $\bar{x}_{\mu\nu}\left(\tilde{A}+\tilde{B}\right)=\bar{x}_{\mu\nu}\left(\tilde{A}\right)+\bar{x}_{\mu\nu}\left(\tilde{B}\right)$

Proof:

Consider normal trapezoidal intuitionistic fuzzy number $\tilde{A}=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ and $\tilde{B}=(b_1',a_1',b_2',a_2',a_3',b_3',a_4',b_4')$

$$\tilde{A}+\tilde{B}=\begin{pmatrix}b_1+b_1',a_1+a_1',b_2+b_2',a_2+a_2'\\ ,a_3+a_3',b_3+b_3',a_4+a_4'+b_4+b_4'\end{pmatrix}$$

$$\bar{x}_{\mu\nu}\left(\tilde{A}+\tilde{B}\right)=\left(\frac{\begin{matrix}(b_1+b_1'+a_1+a_1')+2(a_2+a_2'+b_3+b_3')+5(a_3+a_3'+b_2+b_2')\\+(a_4+a_4'+b_4+b_4')\end{matrix}}{18}\right)$$

$$\bar{x}_{\mu\nu}\left(\tilde{A}+\tilde{B}\right)=\left(\frac{a_1+b_1+2(a_2+b_3)+5(a_3+b_2)+(a_4+b_4)}{18}\right)$$

$$+\left(\frac{a_1'+b_1'+2(a_2'+b_3')+5(a_3'+b_2')+(a_4'+b_4')}{18}\right)$$

$$\bar{x}_{\mu\nu}\left(\tilde{A}+\tilde{B}\right)=\bar{x}_{\mu\nu}\left(\tilde{A}\right)+\bar{x}_{\mu\nu}\left(\tilde{B}\right)$$

Theorem -2

Let $\tilde{A}=\begin{pmatrix}b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4\\ :0<w_1,w_2\le 1\end{pmatrix}$ and

$\tilde{B}=(b_1',a_1',b_2',a_2',a_3',b_3',a_4',b_4':0<w_1',w_2'\le 1)$ be two normal trapezoidal intuitionistic fuzzy number and then $\bar{y}_{\mu\nu}\left(\tilde{A}+\tilde{B}\right)=\bar{y}_{\mu\nu}\left(\tilde{A}\right)+\bar{y}_{\mu\nu}\left(\tilde{B}\right)$

Proof:

Consider normal trapezoidal intuitionistic fuzzy number

$\tilde{A}=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4:0<w_1,w_2<1)$

and $\tilde{B}=(b_1',a_1',b_2',a_2',a_3',b_3',a_4',b_4':0<w_1',w_2'<1)$

Choose $w_1^*=\frac{1}{4}(4w_1+5w_2)$, $w_2^*=\frac{1}{5}(4w_1'+5w_2')$ and $0<w_1^*<1, 0<w_2^*<1$

$$\tilde{A}+\tilde{B}=\begin{pmatrix} b_1+b_1', a_1+a_1', b_2+b_2', a_2+a_2', a_3+a_3', b_3+b_3', a_4+a_4'+b_4+b_4' \\ : \max\left\{w_1^*, w_2^*\right\}, 0<w_1^*, w_2^*<1, \end{pmatrix}$$

$$\bar{y}_{\mu v}\left(\tilde{A}+\tilde{B}\right)=\left(\frac{4w_1^*+5w_2^*}{18}\right)$$

$$\bar{y}_{\mu v}\left(\tilde{A}+\tilde{B}\right)=\left(\frac{4\left(\frac{1}{4}\left(4w_1+5w_2\right)\right)+5\left(\frac{1}{5}\left(4w_1'+5w_2'\right)\right)}{18}\right)$$

$$\bar{y}_{\mu v}\left(\tilde{A}+\tilde{B}\right)=\left(\frac{4w_1+5w_2}{18}\right)+\left(\frac{4w_1'+5w_2'}{18}\right)$$

$$\bar{y}_{\mu v}\left(\tilde{A}+\tilde{B}\right)=\bar{y}_{\mu v}\left(\tilde{A}\right)+\bar{y}_{\mu v}\left(\tilde{B}\right)$$

Theorem -3

Let $\tilde{A}=\left(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4\right)$ and $-\tilde{A}=\left(-b_4,-a_4,-b_3,-a_3,-a_2,-b_2,-a_1,-b_1\right)$ be two normal trapezoidal intuitionistic fuzzy number and then $R\left(-\tilde{A}\right)=-R\left(\tilde{A}\right)$

Proof:

Let $\tilde{A}=\left(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4\right)$ and $-\tilde{A}=\left(-b_4,-a_4,-b_3,-a_3,-a_2,-b_2,-a_1,-b_1\right)$ be two normal trapezoidal intuitionistic fuzzy number

$$R\left(-\tilde{A}\right)=\left(\frac{\left(-b_4-a_4\right)+2\left(-a_2-b_3\right)+5\left(-a_3-b_2\right)+\left(-a_1-b_1\right)}{18}\right)\left(\frac{9}{18}\right)$$

$$=\left(\frac{\left(-b_4-a_4\right)+2\left(-a_2-b_3\right)+5\left(-a_3-b_2\right)+\left(-a_1-b_1\right)}{18}\right)\left(\frac{9}{18}\right)$$

$$=-\left(\frac{\left(a_1+b_1\right)++2\left(a_2+b_3\right)+5\left(b_2+a_3\right)+\left(b_4+a_4\right)}{18}\right)\left(\frac{9}{18}\right)$$

$$=-R\left(\tilde{A}\right)$$

IV.NUMARICAL EXAMPLES

In this section some numerical examples are presented for proposed method

Example4.1

Consider two trapezoidal intuitionistic fuzzy number

$$\tilde{A}=\left(0.2,0.4,0.6,0.8,1,1.2,1.4,1.6:0.39,0.57\right)$$

and

$$\tilde{B}=\left(0.1,0.3,0.5,0.7,0.9,1.1,1.3,1.5:0.39,0.57\right)$$

$$R\left(\tilde{A}\right)=\left(\frac{a_1+b_1+2\left(a_2+b_3\right)+5\left(a_3+b_2\right)+\left(a_4+b_4\right)}{18}\right)\left(\frac{4w_1+5w_2}{18}\right)$$

$$R\left(\tilde{A}\right)=\left(\frac{0.2+0.4+2\left(0.8+1.2\right)+5\left(1.0+0.6\right)+\left(1.4+1.6\right)}{18}\right)\left(\frac{4\left(0.39\right)+5\left(0.57\right)}{18}\right)$$

$$R\left(\tilde{A}\right)=0.212, R\left(\tilde{B}\right)=0.18$$

Therefore $R\left(\tilde{A}\right) > R\left(\tilde{B}\right)$

Example4.2

Consider two triangular intuitionistic fuzzy numbers

$\tilde{A} = (0.2, 0.4, 0.6, 0.8, 1, : 0.33, 0.67)$ and $\tilde{B} = (0.1, 0.3, 0.5, 0.7, 0.9 : 0.33, 0.67)$

$$R\left(\tilde{A}\right) = \left(\frac{(a_1 + b_1) + 14a_2 + (a_4 + b_4)}{18}\right)\left(\frac{4w_1 + 5w_2}{18}\right)$$

$$R\left(\tilde{A}\right) = \left(\frac{(0.2 + 0.4) + 14(0.6) + (0.8 + 1)}{18}\right)\left(\frac{4(0.33) + 5(0.67)}{18}\right)$$

$$R\left(\tilde{A}\right) = 0.156, R\left(\tilde{B}\right) = 0.129$$

Therefore $R\left(\tilde{A}\right) > R\left(\tilde{B}\right)$

Example4.3

Consider two symmetric trapezoidal intuitionistic fuzzy numbers

$\tilde{A} = (1, 1, 3, 3, 23, 23, 25, 25 : 1, 1)$ and $\tilde{B} = (2, 2, 4, 4, 5, 5, 7, 7 : 1, 1)$

$$R\left(\tilde{A}\right) = \left(\frac{a_1 + b_1 + 2(a_2 + b_3) + 5(a_3 + b_2) + (a_4 + b_4)}{18}\right)\left(\frac{4w_1 + 5w_2}{18}\right)$$

$$R\left(\tilde{A}\right) = \left(\frac{1 + 1 + 2(3 + 23) + 5(23 + 3) + (25 + 25)}{18}\right)\left(\frac{4(1) + 5(1)}{18}\right)$$

$$R\left(\tilde{A}\right) = 6.5, R\left(\tilde{B}\right) = 2.25$$

Therefore $R\left(\tilde{A}\right) > R\left(\tilde{B}\right)$

Example 4.4

Consider trapezoidal intuitionistic fuzzynumber $\tilde{A} = (0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6 : 0.39, 0.57)$ and triangular intuitionistic fuzzy number $\tilde{B} = (0.2, 0.4, 0.6, 0.8, 1, : 0.33, 0.67)$

$$R\left(\tilde{A}\right) = 0.212, R\left(\tilde{A}\right) = 0.156$$

Therefore $R\left(\tilde{A}\right) > R\left(\tilde{B}\right)$

V.COMPARITIVE STUDY

A comparative study is furnished on existing ranking and new ranking method for trapezoidal intuitionistic and Triangular intuitionistic fuzzy numbers

S .no	Methods → / Examples	Wei's Method [27]	Wang and Zhong Method[25]	Rezvani's Method [18]	Li's Method [9]	Dubey and meherMethod [6]	Sagaya's Method [19]	Proposed Method
1	$\tilde{A}=(0.57,0.57,0.73,0.83,0.83:0.73,0.2)$ $\tilde{B}=(0.58,0.58,0.74,0.819,0.819:0.72,0.2)$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}>\tilde{B}$
2	$\tilde{A}=(-9,-9,1.5,3,3:0.6,0.2)$ $\tilde{B}=(-9,-9,1.5,3,3:0.7,0.3)$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}<\tilde{B}$
3	$\tilde{A}=(3,3,4,5,5:0.8,0.2)$ $\tilde{B}=(6,6,8,10,10:0.7,0.3)$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$
4	$\tilde{A}=(1,1,2,3,3:0.6,0.4)$ $\tilde{B}=(2,2,4,6,6:0.3,0.7)$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$
5	$\tilde{A}=(4,4,5.5,5.5,6,6,8,8:1,0)$ $\tilde{B}=(3.5,3.5,5,5,7,7,7.5,7.5:1,0)$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$
6	$\tilde{A}=(0.55,0.55,0.75,0.75,0.8,0.8,0.9,0.9:0.5,0.5)$ $\tilde{B}=(0.5,0.5,0.7,0.7,0.85,0.85,0.95,0.95:0.5,0.5)$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}<\tilde{B}$

VI.Conclusion

In this paper a new ranking method is established for trapezoidal intuitionistic and triangular intuitionistic fuzzy numbers. A comparative study is presented on existing ranking methods and new ranking methods. The advantage of this method is very easy to order the intuitionistic fuzzy numbers when compare with the other methods.

VII.Reference

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