The formula form of the points of Parabolic tangents

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Abstract—*Finding the formula form of the points of the tangents from the point to the parabola. Given a parabola and a point in its exterior.*

Keywords—*Parabola, Tangents, Exterior region.*

I. INTRODUCTION

Finding the points of the tangents from the given point to the parabola.

In 2002, David R. Duncan and Bonnie H. Litwiller have studied finding the points of tangents to parabola $y = kx^2$ with k > 0 through the given point see more detail in [1].

In this paper has been improved to find the formula form of the points of the tangents from the parabola $y = Ax^2 + Bx + D$ where A, B and D are constants and A ≠0 through the given point.

If the point (a, b) is in the exterior region area of the parabola $y = Ax^2 + Bx + D$, $A > 0$ then it is represented by $y < Ax^2 + Bx + D$. If the points (a, b) is in the interior region area of the parabola $y = Ax^2 + Bx + D$, $A > 0$ then it is represented by $y > Ax^2 + Bx + D$, and the set of points on parabola is represented by $y = Ax^2 + Bx + D$ such that $A > 0$. Similarly, if the point (a, b) is in the exterior region area of the parabola $y = -Ax^2 + Bx + D$, $A > 0$ then it is represented by $y > -Ax^2 + Bx + D$. If the points (a, b) is in the interior region area of the parabola $y = -Ax^2 + Bx + D$, $A > 0$ then it is represented by $y < -Ax^2 + Bx + D$, and the set of points on parabola is represented by $y = -Ax^2 + Bx + D$ such that $A > 0$.

II. MAIN RESULTS

Let $P(a, b)$ be a point not on the parabola $y = Ax^2 + Bx + D$, $A > 0$ and let $Q(c, d)$ be a point on the parabola, the line PQ is the tangent to the parabola, $y = Ax^2 + Bx + D$ at Q see Figure 1.

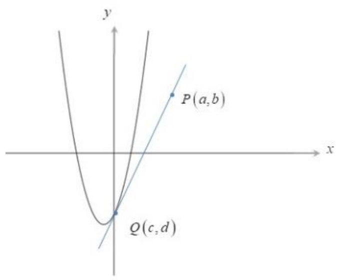


Figure 1: The line PQ.

By the definition of the slope, $m = \dfrac{d-b}{c-a}$ and by the calculus, the slope of the line through Q, tangent to the parabola is the derivative of the function $y = Ax^2 + Bx + D$, evaluated at $x = c$. Since $y' = 2Ax + B$, the slope is $2Ax + B$. Thus $m = 2Ac + B$. This give us $m = \dfrac{d-b}{c-a} = 2Ac + B$. Since $Q(c, d)$ lies on the parabola, the equation $d = Ac^2 + Bc + D$ holds. Substituting into the equation $\dfrac{d-b}{c-a} = 2Ac + B$. We have

$$\frac{Ac^2 + Bc + D - b}{c - a} = 2Ac + B$$
$$Ac^2 - 2Aac + (b - Ba - D) = 0$$

thus

$$c = \frac{2Aa \pm \sqrt{(2Aa)^2 + 4A(Ba + D - b)}}{2A}.$$

To evaluate this quadratic equation has solutions for c, evaluate its discriminant

$$W = 4A^2a^2 + 4A(Ba + D - b)$$
$$= 4A(Aa^2 + Ba + D - b).$$

If $P(a, b)$is in the interior region of the parabola, then by the inequality describing the interior $b > Aa^2 + Ba + D$. Thus $Aa^2 + Ba + D - b < 0$ and $W > 0$. There are no real solutions to the critical equation. Hence no tangents to the parabola pass through point$P(a, b)$.

If $P(a, b)$ is in the exterior region of the parabola, then by the inequality describing the exterior $b < Aa^2 + Ba + D$. Thus $Aa^2 + Ba + D - b > 0$ and $W > 0$. There are two solutions c. Hence two tangents to the parabola pass through the point $P(a, b)$.

Similarly, let $P(a, b)$ be a point not on the parabola $y = -Ax^2 + Bx + D$, $A > 0$ and $Q(c, d)$ be a point on the parabola. We get

$$c = \frac{2Aa \pm \sqrt{(2Aa)^2 + 4A(b - Ba - D)}}{2A},$$

and the discriminant

$$W = 4A^2a^2 + 4A(b - Ba - D)$$
$$= 4A(Aa^2 + b - Ba - D).$$

The $P(a, b)$ is in the exterior region of the parabola when $-Aa^2 + Ba + D < b$ and $W > 0$. There are two tangents to the parabola pass through the point $P(a, b)$.

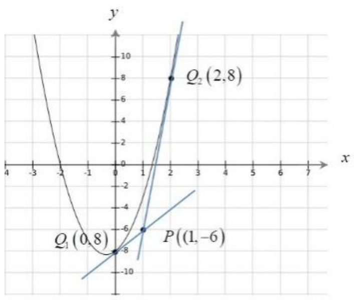
III.NUMERICAL EXAMPLE

Example 3.1. Find a points of the tangents from the parabola $y = 3x^2 + 2x - 8$ through the point $P(1, -6)$.

Solution: $A = 3$, $B = 2$, $D = -8$, $a = 1$, $b = -6$ thus $Aa^2 + Ba + D - b = 1 > 0$ then there are two solutions c,

$$c = \frac{2Aa \pm \sqrt{(2Aa)^2 + 4A(Bb + D - b)}}{2A} = 1 \pm 1$$

so, $Q_1(0, -8)$ and $Q_2(2, -8)$ are two points of the tangents to the parabola $y = 3x^2 + 2x - 8$ through the point $P(1, -6)$ as Figure 2.



Example 3.2. Find a points of the tangents from the parabola $y = -x^2 + 2x - 3$ through the point $P(1, 2)$.

Solution: $A = 1$, $B = 2$, $D = -3$, $a = 1$, $b = 2$ thus $-Aa^2 + Ba + D - b = -4 < 0$ then there are two solutions c,

$$c = \frac{2Aa \pm \sqrt{(2Aa)^2 + 4A(b - Ba - D)}}{2A} = 1 \pm 2$$

so, $Q_1(-1, -6)$ and $Q_2(3, -6)$are two points of the tangents to the parabola $y = -x^2 + 2x - 3$ through the point $P(1, 2)$ as Figure 3.

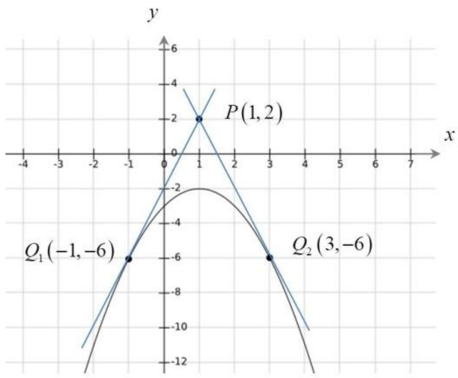


Figure 3: picture of example 3.2.

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REFERENCES

[1] David R. Duncan and Bonnie H. Litwiller, Alabama Journal of Mathematics, fall 2002.

Figure 2: picture of example 3.1.