Effect of Fractional Order in Vertical Pole Motion

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Abstract *The frequency response methods are most powerful in the conventional control system. The impulse response characteristics are related to the location of poles of F(s). In this paper, we discuss impulse response and frequency response of transfer function* $\{1/(s^2+as+b)^q\}$ *for different fractional values of q where* $0.5 < q < 1$, $q = 1$, $1 < q < 1.5$ *in vertical pole motion in LHP. The different characters of the impulse response and frequency response are shown in numerical examples. The numbers of figures are presented to explain the concepts.*
Keywords: Transfer Function. Impulse response. Frequency response. Mat-lab

I. INTRODUCTION

The transfer function G(s) is given by

$$G(s) = \frac{L_o}{L_i} \qquad (1)$$

where L denotes the Laplace transform. The frequency response function and the transfer function are interchangeable by the substitution $s = j\omega$ [1].

The frequency response function $G(j\omega)$ is

$$G(j\omega) = \frac{F_O}{F_i} \qquad (2)$$

where F denotes the Fourier transform.

TF has been used in many applications. One important application among them is monitoring the mechanical integrity of transformer windings (during testing and while in service).Mechanical deformation arise mainly due to short circuit forces, unskilled handling and rough transportation. Information related to winding deformation is embedded in the TF. Hence the first step should ensure accurate diagnosis, a correct interpretation of TF [2]. TF can be used to describe a variety of filter or to express solution of linear differential equation accurately [3]. The TF of system is analyzed and response curves are simulated [4].The location of poles and zeros gives idea regarding response characteristics of a system. Transfer function is mainly used in control system and signal processing.

II. SYSTEM STABILITY

If poles are in LHP, the system is stable; if poles are in RHP, the system is unstable and poles on imaginary axis then system is marginally stable or limitedly stable [5, 6, 7].

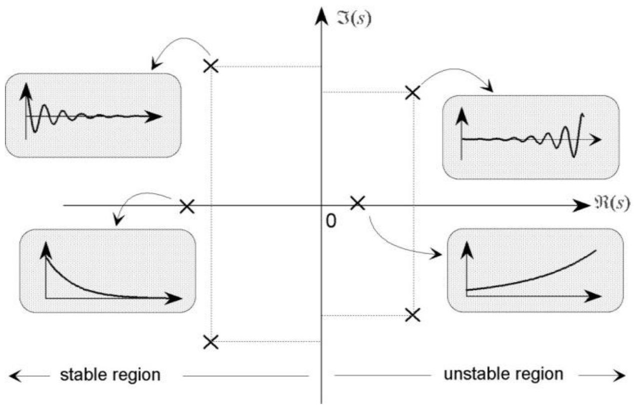

Fig.1. Stable and unstable region according to pole position

III. FRACTIONAL CALCULUS

Fractional calculus is a part of mathematics dealing with derivative of arbitrary order. The fractional order integral of an integral function f (t) with r €R. Thus the uniform formula of fractional order integral is defined as follows:

$$ {}_0D_t^{-q}x(t) = D^{-q}x(t) = \frac{1}{\Gamma(q)}\int_0^t (t-u)^{q-1}x(u)du \qquad (3) \qquad \text{where } q > 0 $$

, f(t) is an arbitrary integrable function [8,9,10]

Laplace transforms for fractional – order integral operator is: [11]

$$ L\{D^{-q}f(t)\} = s^{-q}F(s) \qquad (4) $$

Fractional calculus has a 300-year-old history, and for a long time, it was considered as a purely theoretical subject with nearly no applications. In recent decades, the theory of fractional order calculus is developed greatly as well as the application of fractional controller attracts increasing attention due to the higher degree of freedom provided [12]. In mathematics, fractional calculus permits to convert the differential or integral operator with integer order to the fractional order [13].

IV. IMPULSE RESPONSE ANALYSIS

Impulse signals and their responses, Frequency response are commonly used in control systems analysis and

Design Both responses are a function of frequency and apply only to the steady state sinusoidal response of the system [5]. One of the most useful representations of a transfer function is a logarithmic plot which consists of two graphs, one giving magnitude and the other phase angle both plotted against frequency in logarithmic scale i.e. Bode plots[6,7].

The inverse Laplace transform of transfer function

$$ F(s) = \frac{1}{(s^2 + as + b)^q} \qquad (5) $$

is derived by using the complex integral

$$f(t) = \Gamma(1-q)\, t^{q-1} e^{pt} \frac{\sin(q\pi)}{\Gamma(q)} = \frac{t^{q-1} e^{pt}}{\Gamma(q)} \quad (6)$$

Theorem 1: For TF F(s) = $\{1/(s^2+as+b)^q\}$,

(i)When both of the poles lie in the open LHP, $\lim_{t\to\infty} f(t) = 0$ with q > 0. [11]

(ii)When both the poles lie on the imaginary axis, $f_1(t)$has damped oscillation with o < q < 1; has undamped oscillation with q = 1; and has diverging oscillation with q > 1,

(iii) When both of the poles lie in the open RHP, $\lim_{t\to\infty} f_1(t) = \infty$ with q > 0

An idea of fractional order transfer function is analogical to an idea of integer order transfer function [14]. Consider transfer function $\{1/(s^2+as+b)^q\}$where a, b > = 0 with different fractional order in vertical pole motion in LHP then analyze impulse response and frequency response of poles and MATLAB-based evaluation of impulse responses are given.

Numerical example 1: To observe the impulse response and frequency response of transfer functions $\{1/(s^2+as+b)^q\}$ where a, b > 0 for the following positive values of a and b for q = 0.7, 1.0, 1.3 [9,10].when a_1 = 2, b_1 = 2 then impulse response of $\lim_{t\to\infty} f(t) = 0$ is demonstrated in figures 4, 5 respectively [15,16].

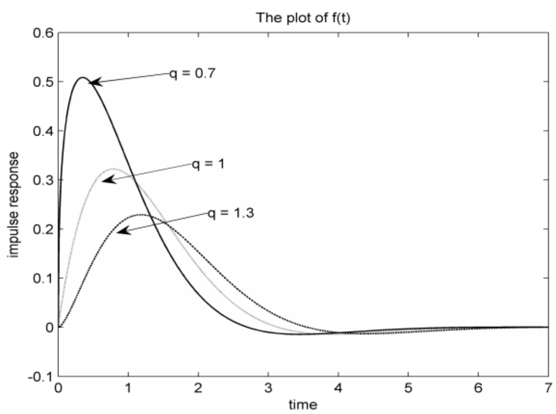

Fig 2 Impulse response of Numerical 2

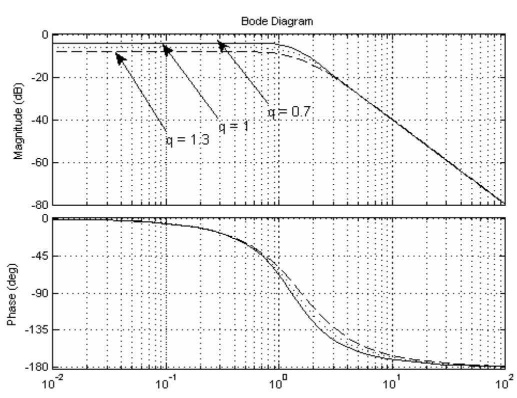

Fig.3 Frequency response of Numerical 1

For all values of q, impulse response tends to zero as t tends to infinity. As on increasing value of q from q = 0.7 to q = 1.3, it is observed that peak values goes on decreasing and rise time , settling time goes on increasing . For fractional q values, Magnitude with different straight lines below zero db line then coincide and decreases with negative slope. Phase angle coincide with zero line then decreases at different frequencies and again coincide at -180 degree line.

Numerical example 2: To observe the impulse response and frequency response of transfer functions $1/(s^2+as+b)^q\}$ for the following different values of a and b where poles moves away from origin in LHP for q =0.7,

1.0, 1.3 [9, 10].When $a_2 = 4$, $b_2 = 8$ then impulse response is demonstrated in figures 4, 5 respectively [15].

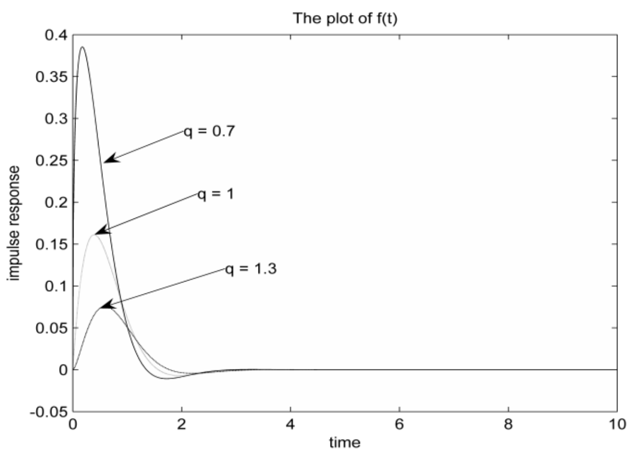

Fig. 4 Impulse response of Numerical 2

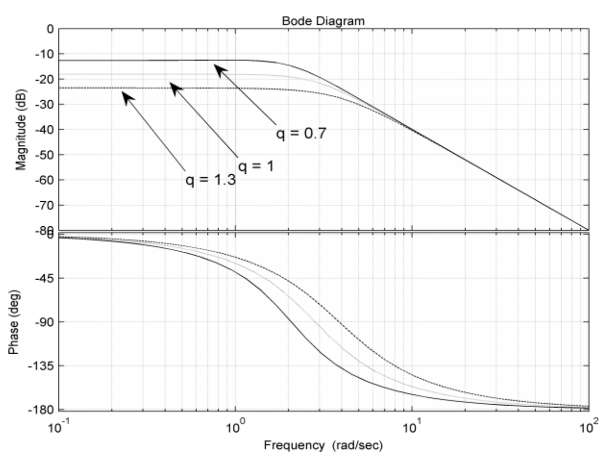

Fig. 5 Frequency response of Numerical 2

For all values of q, impulse response tends to zero as t tends to infinity. As on increasing value of q from q = 0.7 to q = 1.3, it is observed that peak values goes on decreasing and rise time , settling time goes on increasing . For fractional q values, Magnitude with different straight lines below -10db line then coincide and decreases with negative slope. Phase angle coincide with zero line then decreases at different frequencies and again coincide at -180 degree line. Frequency difference is greater than numerical 1.

Numerical example 3: To observe the impulse response and frequency response of transfer functions

$\{1/(s^2+as+b)^q\}$ for the following different values of a and b where poles moves away from origin in vertical direction in LHP for q = 0.7, 1.0, 1.3 [9,10].When $a_3 = 6$, $b_3 = 18$ then impulse response of poles which lies in LHP for vertical pole motion is demonstrated in figures 6, 7 respectively.

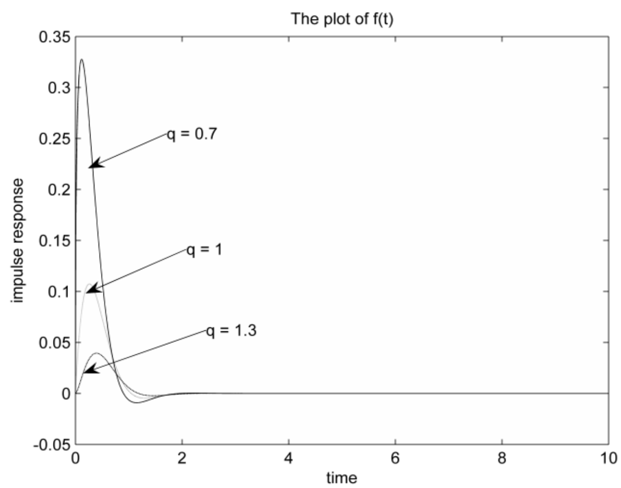

Fig. 6 Impulse response of Numerical 3

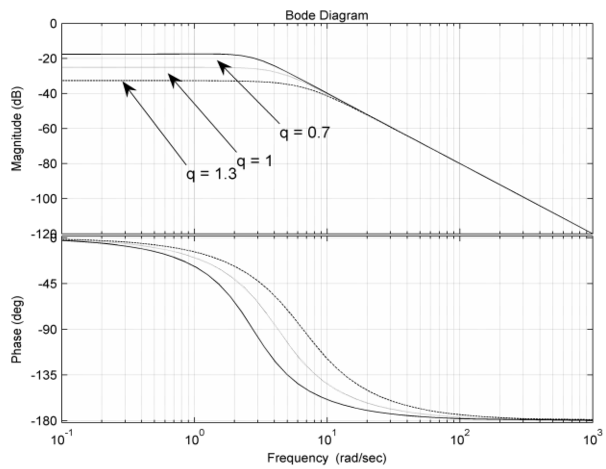

Fig. 7 Frequency response of Numerical 3

For all values of q, impulse response tends to zero as t tends to infinity. As on increasing value of q from q = 0.7 to q = 1.3, it is observed that peak values goes on decreasing and rise time , settling time goes on increasing. For fractional q values, Magnitude with different straight lines below -15 db line then coincide and decreases with negative slope. Phase angle coincide with zero line then decreases at different frequencies and again coincide at -180 degree line.

Fractional order transfer function used in Control system, power system and filter designing. An application of elementary fractional order transfer functions to effective modeling of high order control plants [14]. Automatic Generation Control (AGC) is one of the most important problems in control and design of an electrical power system. Its role is to control the active power, generators output in response to load variation… etc. [17]**.** For this reason, several control strategies have been described in the literature. The most widely used is based on classical controller such as PI, PD, ID and PID… etc. The main features of these controllers are their ability to maintain a zero steady-state error to a step change in reference [18]. The Tilt Integral Derivative controller is a kind of Fractional Order Controllers that offers simpler tuning, better disturbance rejection ratio [12]. In most cases, an objective of using fractional calculus is to apply the fractional-order control to enhance the system control performance [19].

V. CONCLUSIONS

When poles lie in LHP and poles move away from origin in vertical direction, impulse response tends to zero as time tends to infinity. When pole moves away from origin in vertical direction, peak time goes on decreasing and rise time goes on increasing as q increases.

When poles move away from origin in vertical direction, similar types of frequency responses observed. Frequency response for all values i.e. q = 0.7, 1, 1.3 are below zero db then coincide and decreases with negative slopes and magnitude coincide with different straight lines below zero db line moves downward i.e. below to – 0db and then goes down to below -15db and then decreases with same negative slope and phase angle coincide with zero line then decreases with different slopes at different frequencies and again coincide at -180 degree line.

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