Maximum Spacing's between 2-PrimeFactors Numbers till 1 Trillion

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Abstract—*'k-Prime Factors number' is defined to be a positive integer with precisely k number of prime factors. These divisors need not be always distinct. Particular type, viz., 2-PrimeFactors numbers are already analyzed for their low and high densities of occurrences and minimum spacing between successive 2-PrimeFactors numbers. The present work is about maximum spacings between them. This analysis is also from two perspectives : Within fixed range of 1 trillion, blocks of various sizes like 10, 100 and so on are analyzed and then for blocks of each fixed size like 10, 100 and so on, various ranges are probed. Both times, maximum in-block spacing between 2-PrimeFactors numbers, number of times pairs with maximum spacings occur, first and last starting 2-PrimeFactors numbers having maximum spacing with their successors, and number of blocks accommodating maximum spacing pairs in them are explored.*

Keywords **—***Prime number, k-Prime Factors number, 2-PrimeFactors number, Maximum spacing*
Mathematics Subject Classification (2010) — *11A51, 11N05, 11N80*

I. INTRODUCTION

The primes are studied from quite long in history. The sequence of primes
$$2, 3, 5, 7, 11, 13, 17, \cdots$$
has retained its importance in modern advanced era also, owing to many unique properties, the reasoning of which is yet unknown (one such being their distribution amongst natural numbers [1]). This sequence fails to fit in any known types like arithmetic, geometric, harmonic, any combination of these or other.

One important property of sequence of primes is that the gaps or spacings between successive primes are unpredictable. Their values keep fluctuating up and down [3]. Naturally, special types of primes like twin primes also exhibit this property [4]. In this work we have undertaken such analysis of numbers which are based on primes.

II. k-PRIME FACTORS NUMBERS

Author has come up with a new terminology [6].
Definition (*k*-Prime Factors Number) : For any integer $k \geq 0$, a positive integer having k number of prime factors, which need not be necessarily distinct, is called as *k*-Prime Factors number.

The concept is not altogether new. Factorizing positive integers for getting all prime divisors is a process that has been undertaken from quite long. But categorizing numbers based on number of prime divisors that they have was not done much.

Euclid's two and half millennia old proof about primes being infinite urges to ask about infinitude of every class of numbers based on primes. Except 0-PrimeFactors number, for all positive integers k, *k*-Prime Factors numbers are also infinite in number.

III. 2-PRIMEFACTORS NUMBERS

k-Prime Factors numbers for with value of *k* as 2 give following special type [6].
Definition (2-PrimeFactors Number) : A positive integer having exactly 2 prime divisors, which need not be necessarily distinct, is called as 2-PrimeFactors number.
Initial 2-PrimeFactors numbers are :
$$4, 6, 9, 10, 14, 21, \cdots$$
Their canonical factorizations are : $4 = 2^2$, $6 = 2 \times 3$, $9 = 3 \times 3$, $10 = 2 \times 5$, $14 = 2 \times 7$, $21 = 3 \times 7$, $\cdots$
Just like primes, 2-PrimeFactors numbers are also infinite in number. Like gaps in successive primes keep occurring in quite irregular a fashion.

IV. MAXIMUM SPACING BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZES 10^n

For all analysis of 2-PrimeFactors number done till now in huge ranges, [6], [7], [8], selective algorithms were preferred for determination of primes first [2].

More than 10 computers with multiple cores were running for several months in tandem using Java [5] programming language to get all the analysis done.

Our usual number system is with base 10. So, till 1 trillion, we have decided to takes blocks of sizes of powers of 10, like 10, $10^2 = 100$, $10^3 = 1000$ and so on till 1 trillion itself. Within block of every size, the maximum spacing occurring in pairs of successive 2-PrimeFactors numbers is searched for. The number of such pairs with maximum spacing and the first 2-PrimeFactor number with maximum spacing with its successor as well as last 2-PrimeFactor number with maximum spacing with its successor are looked for. The number of blocks that contain such pairs of number are also considered.

Sr. No.	Block-Size	Maximum In-Block Spacing	Number of Maximum Spacings	First Number with Maximum In-Block Spacing	Last Number with Maximum In-Block Spacing	Number of Blocks with Maximum Spacings
1	10^1	8	2,414,374,134	1,371	999,999,999,311	2,414,374,134
2	10^2	98	128	60,969,002,701	985,832,893,301	128
3	10^3	156	1	927,075,495,233	927,075,495,233	1
4	10^4	163	1	799,980,626,859	799,980,626,859	1
5	10^5	163	1	799,980,626,859	799,980,626,859	1
6	10^6	163	1	799,980,626,859	799,980,626,859	1
7	10^7	163	1	799,980,626,859	799,980,626,859	1
8	10^8	163	1	799,980,626,859	799,980,626,859	1
9	10^9	163	1	799,980,626,859	799,980,626,859	1
10	10^{10}	163	1	799,980,626,859	799,980,626,859	1
11	10^{11}	163	1	799,980,626,859	799,980,626,859	1
12	10^{12}	163	1	799,980,626,859	799,980,626,859	1

The maximum spacing between two successive 2-PrimeFactors numbers between blocks of initial sizes increases till 10^4 and later remains same. For higher spacing in higher blocks, there is unique pair with such large spacing.
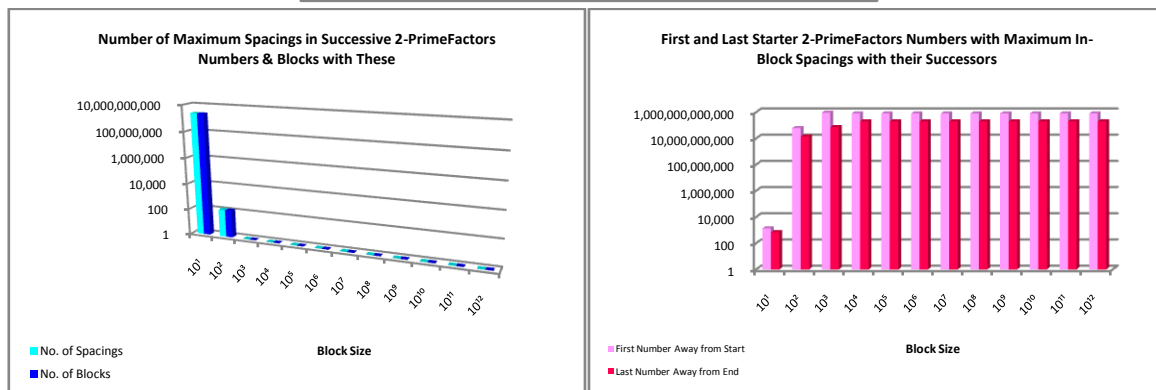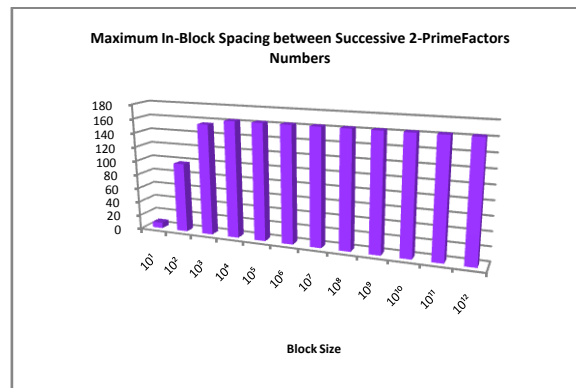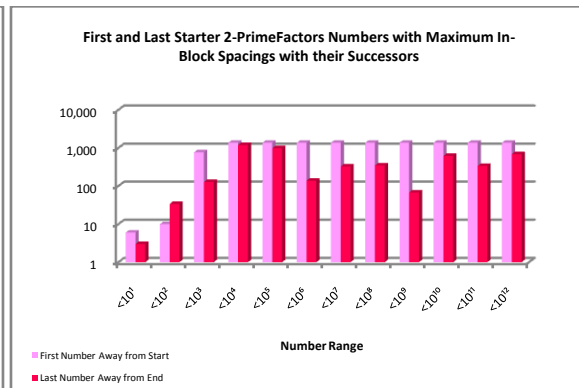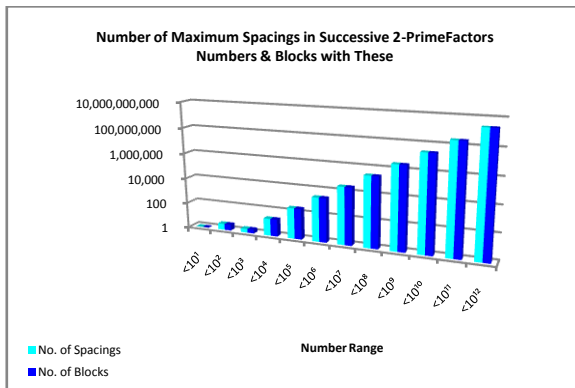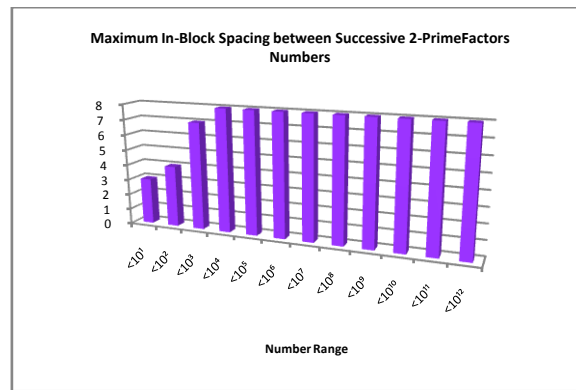

IVA. MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10

After working in fixed range of 1 trillion for blocks of different sizes, we reverse our approach. For fixed sized blocks, beginning with block size of 10, different higher ranges areanalysed. Same parameters as in the earlier section are determined.

Here, block 0 will mean number range 0 to 9, block 10 the range 10 to 19 and so on.

Sr. No.	Range	*Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10*				
		Maximum In-Block Spacing	*Number of Maximum Spacings*	*First Number with Maximum In-Block Spacing*	*Last Number with Maximum In-Block Spacing*	*Number of Blocks with Maximum Spacings*
1	$<10^1$	3	1	6	6	1
2	$<10^2$	4	3	10	65	3
3	$<10^3$	7	2	771	871	2
4	$<10^4$	8	22	1,371	8,801	22
5	$<10^5$	8	249	1,371	99,011	249
6	$<10^6$	8	2,699	1,371	999,861	2,699
7	$<10^7$	8	27,773	1,371	9,999,671	27,773
8	$<10^8$	8	277,711	1,371	99,999,651	277,711
9	$<10^9$	8	2,718,328	1,371	999,999,931	2,718,328
10	$<10^{10}$	8	26,311,768	1,371	9,999,999,381	26,311,768
11	$<10^{11}$	8	252,648,637	1,371	99,999,999,661	252,648,637
12	$<10^{12}$	8	2,414,374,134	1,371	999,999,999,311	2,414,374,134

The maximum spacing starts with 30% in initial range and reaches as high as 80% for higher ones. The count of pairs keeps increasing with range size.
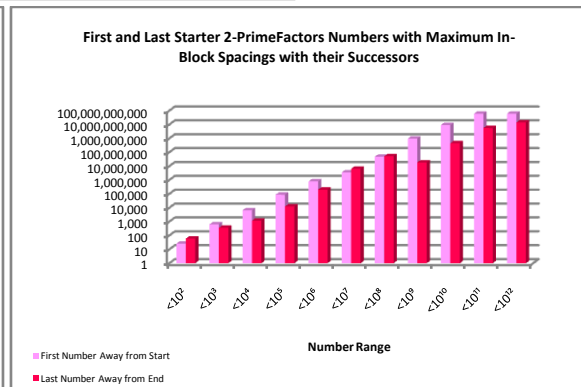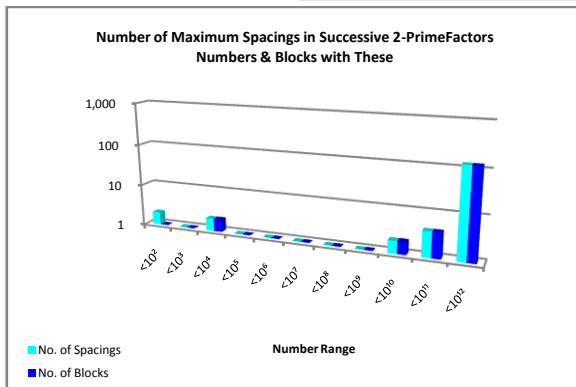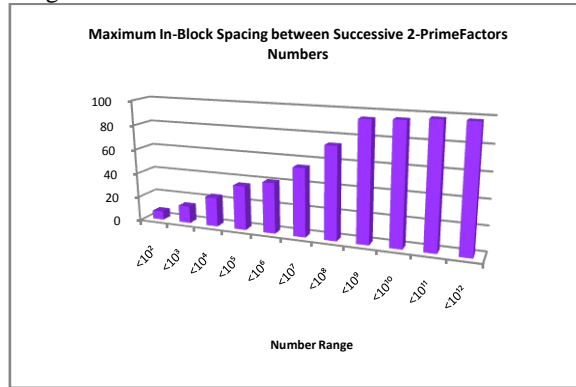

Maximum In-Block Spacing between Successive 2-PrimeFactors Numbers

Number of Maximum Spacings in Successive 2-PrimeFactors Numbers & Blocks with These

First and Last Starter 2-PrimeFactors Numbers with Maximum In-Block Spacings with their Successors

IVB.   MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^2

Second block size in queue is 10^2, i.e., 100. Block 0 will denote range 0 to 99, block 100, 100 to 199 & so on.

Sr. No.	Range	*Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^2*				
		Maximum In-Block Spacing	*Number of Maximum Spacings*	*First Number with Maximum In-Block Spacing*	*Last Number with Maximum In-Block Spacing*	*Number of Blocks with Maximum Spacings*
1	$<10^2$	7	2	26	39	1
2	$<10^3$	14	1	635	635	1
3	$<10^4$	24	2	6,559	8,818	2
4	$<10^5$	36	1	87,627	87,627	1
5	$<10^6$	41	1	797,542	797,542	1
6	$<10^7$	55	1	3,513,418	3,513,418	1
7	$<10^8$	74	1	48,200,909	48,200,909	1
8	$<10^9$	95	1	981,270,902	981,270,902	1
9	$<10^{10}$	96	2	9,219,014,603	9,550,995,901	2
10	$<10^{11}$	98	4	60,969,002,701	94,392,784,901	4
11	$<10^{12}$	98	128	60,969,002,701	985,832,893,301	128

Steady increase in max spacing is seen till it has become 98%.
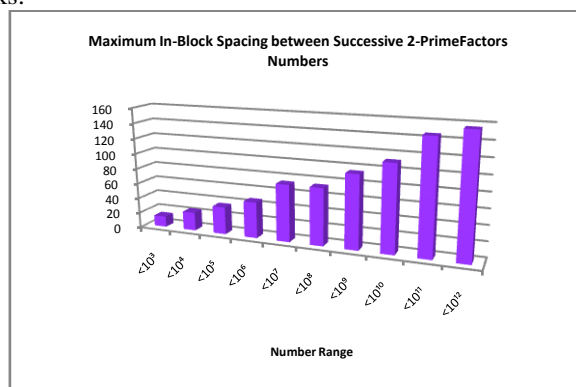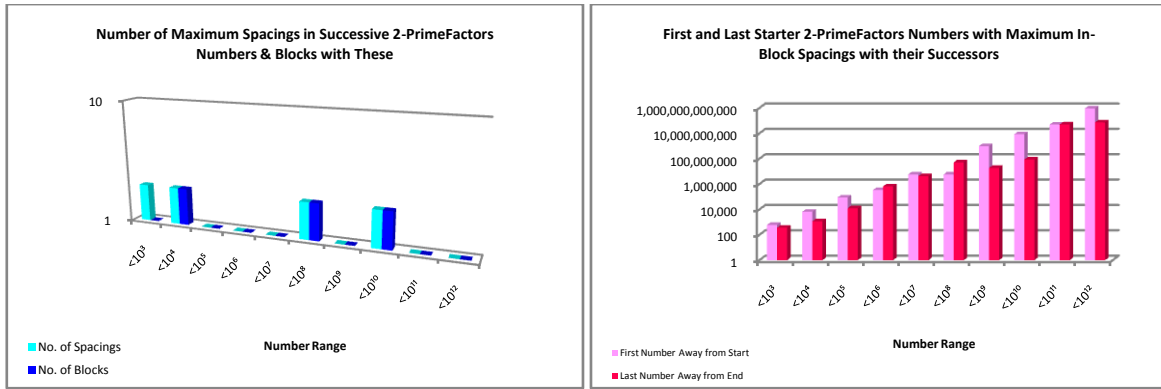

IVC.   MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^3

Third block size is 10^3, i.e., 1000, where block 0 will be understood to be number range 0 to 999, block 1000 to be range 1000 to 1999 and so on.

Sr. No.	Range	Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^3				
		Maximum In-Block Spacing	Number of Maximum Spacings	First Number with Maximum In-Block Spacing	Last Number with Maximum In-Block Spacing	Number of Blocks with Maximum Spacings
1	$<10^3$	14	2	597	635	1
2	$<10^4$	24	2	6,559	8,818	2
3	$<10^5$	36	1	87,627	87,627	1
4	$<10^6$	47	1	340,894	340,894	1
5	$<10^7$	74	1	5,835,191	5,835,191	1
6	$<10^8$	74	2	5,835,191	48,200,909	2
7	$<10^9$	95	1	981,270,902	981,270,902	1
8	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
9	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
10	$<10^{12}$	156	1	927,075,495,233	927,075,495,233	1

There is slow increase in maximum spacing compared to blocks of previous size. The pairs with maximum spacing come in late in blocks.
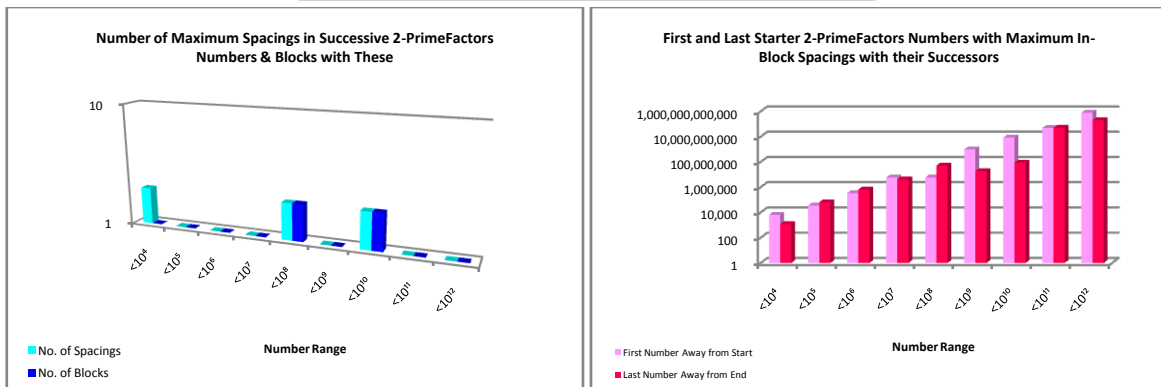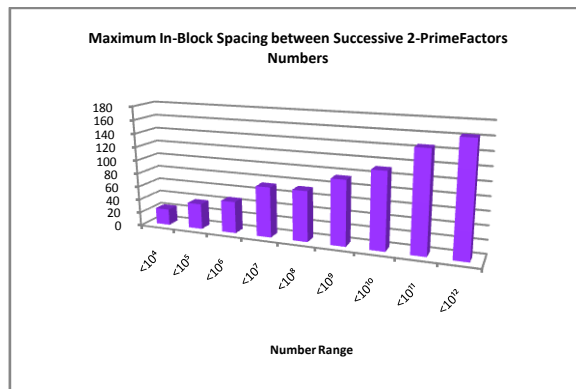
IVD. **MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^4**

Fourth turn is of block size 10^4, i.e., 10000 with block 0 standing for number range 0 to 9999, block 10000 standing for number range 10000 to 19999 and so on.

Sr. No.	Range	Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^4				
		Maximum In-Block Spacing	Number of Maximum Spacings	First Number with Maximum In-Block Spacing	Last Number with Maximum In-Block Spacing	Number of Blocks with Maximum Spacings
1	$<10^4$	24	2	6,559	8,818	1
2	$<10^5$	38	1	35,981	35,981	1
3	$<10^6$	47	1	340,894	340,894	1
4	$<10^7$	74	1	5,835,191	5,835,191	1
5	$<10^8$	74	2	5,835,191	48,200,909	2
6	$<10^9$	95	1	981,270,902	981,270,902	1
7	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
8	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
9	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

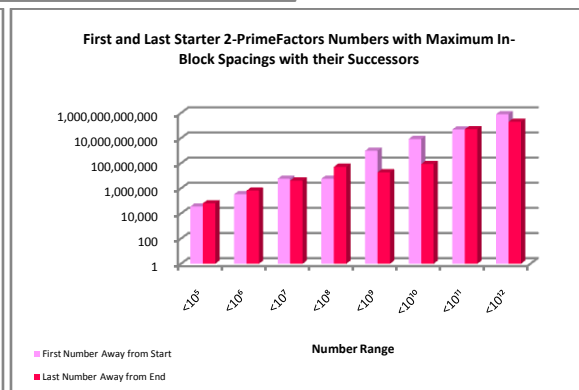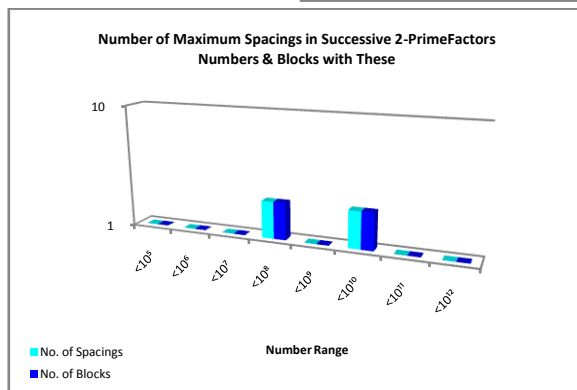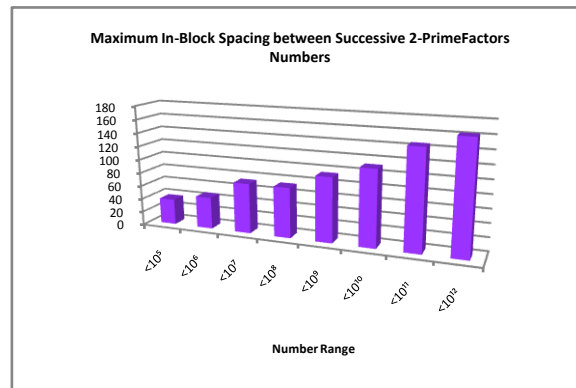The patterns of parameters under consideration are as shown below.

IVE. MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^5

Block size of 10^5, i.e., 100000 takes next turn with, block 0 indicating range 0 to 99999, block 100000 indicating number range 100000 to 199999 and so on.

Sr. No.	Range	*Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^5*				
		Maximum In-Block Spacing	*Number of Maximum Spacings*	*First Number with Maximum In-Block Spacing*	*Last Number with Maximum In-Block Spacing*	*Number of Blocks with Maximum Spacings*
1	$<10^5$	38	1	35,981	35,981	1
2	$<10^6$	47	1	340,894	340,894	1
3	$<10^7$	74	1	5,835,191	5,835,191	1
4	$<10^8$	74	2	5,835,191	48,200,909	2
5	$<10^9$	95	1	981,270,902	981,270,902	1
6	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
7	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
8	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

In most of the cases, there is unique pair of successive numbers of our type accommodating maximum possible spacing in-between.

Maximum In-Block Spacing between Successive 2-PrimeFactors Numbers

Number of Maximum Spacings in Successive 2-PrimeFactors Numbers & Blocks with These

First and Last Starter 2-PrimeFactors Numbers with Maximum In-Block Spacings with their Successors
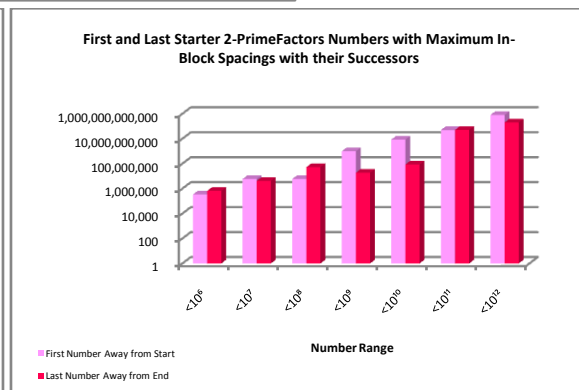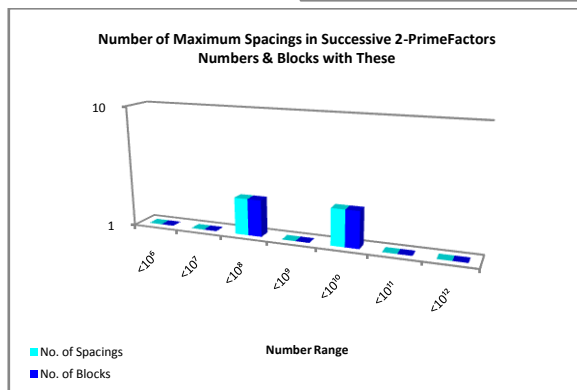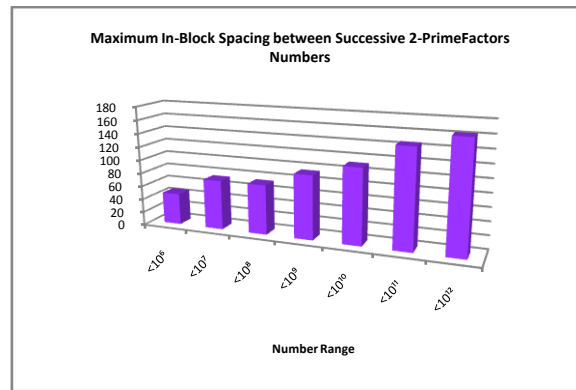
IVF. MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^6

Next number is of block of size 10^6, i.e., 1000000. Here 0 means number range 0 to 999999, block 1000000 means number range 1000000 to 1999999 and so on.

Sr. No.	Range	*Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^6*				
		Maximum In-Block Spacing	*Number of Maximum Spacings*	*First Number with Maximum In-Block Spacing*	*Last Number with Maximum In-Block Spacing*	*Number of Blocks with Maximum Spacings*
1	$<10^6$	47	1	340,894	340,894	1
2	$<10^7$	74	1	5,835,191	5,835,191	1
3	$<10^8$	74	2	5,835,191	48,200,909	2
4	$<10^9$	95	1	981,270,902	981,270,902	1
5	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
6	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
7	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

Values are same as for previous block-size except that range 10^5, being smaller than block-size, cannot be considered here.
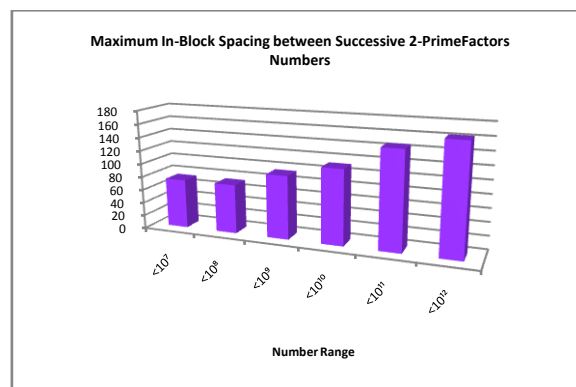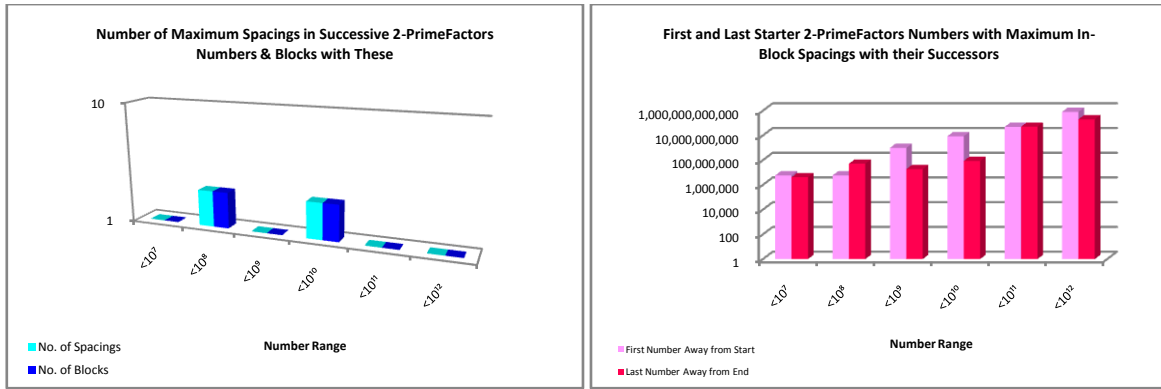

Maximum In-Block Spacing between Successive 2-PrimeFactors Numbers

Number of Maximum Spacings in Successive 2-PrimeFactors Numbers & Blocks with These

First and Last Starter 2-PrimeFactors Numbers with Maximum In-Block Spacings with their Successors

IVG.   **MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^7**

Seventh block size is 10^7, i.e., 10000000. Block 0 represents range 0 to 9999999, block 10000000 indicates number range 10000000 to 19999999 and so on.

Sr. No.	Range	*Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^7*				
		Maximum In-Block Spacing	*Number of Maximum Spacings*	*First Number with Maximum In-Block Spacing*	*Last Number with Maximum In-Block Spacing*	*Number of Blocks with Maximum Spacings*
1	$<10^7$	74	1	5,835,191	5,835,191	1
2	$<10^8$	74	2	5,835,191	48,200,909	2
3	$<10^9$	95	1	981,270,902	981,270,902	1
4	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
5	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
6	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

Now also, except omittance of range 10^6, values continue to be same.

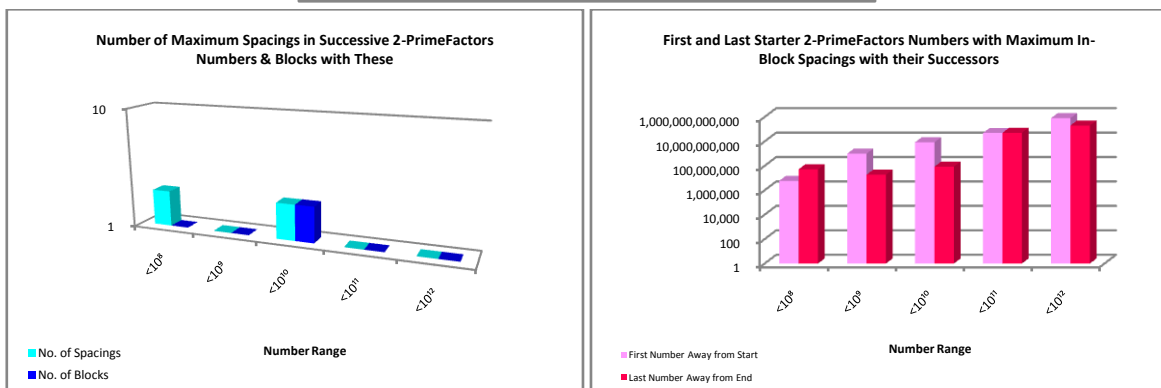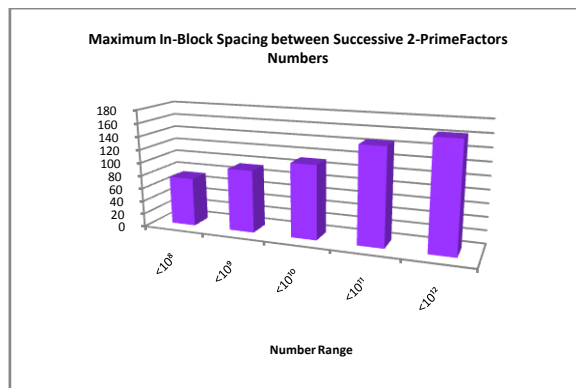Maximum In-Block Spacing between Successive 2-PrimeFactors Numbers

IVH. MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^8

Further we consider block-size 10^8, i.e., 100000000. For this block 0 stands for range 0 to 99999999, block 100000000 stands for number range 100000000 to 199999999 and so on.

Sr. No.	Range	Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^8				
		Maximum In-Block Spacing	Number of Maximum Spacings	First Number with Maximum In-Block Spacing	Last Number with Maximum In-Block Spacing	Number of Blocks with Maximum Spacings
1	$<10^8$	74	2	5,835,191	48,200,909	1
2	$<10^9$	95	1	981,270,902	981,270,902	1
3	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
4	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
5	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

For this block size either, the values are same as those of previous ones.
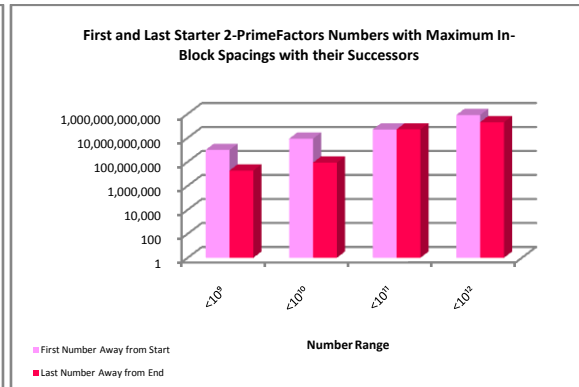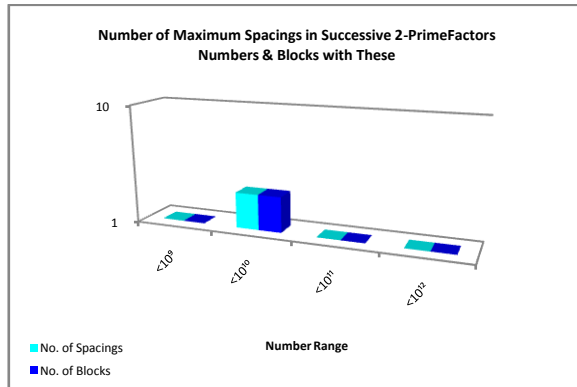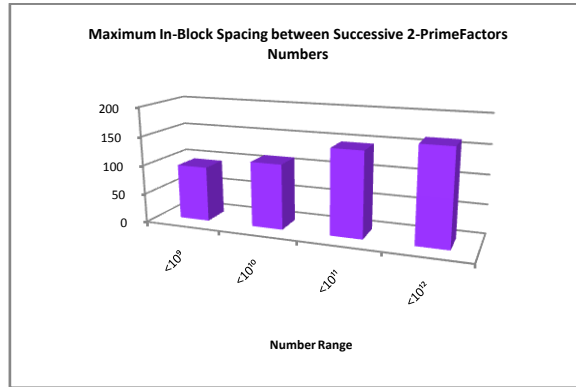

IVI. MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^9

Block-size 10^9, i.e., 1000000000 finds its number for analysis at ninth place as per its size. In this case, 0 symbolizes number range 0 to 999999999, block 1000000000 symbolizes number range 1000000000 to 1999999999 and so on.

Sr. No.	Range	Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^9				
		Maximum In-Block Spacing	Number of Maximum Spacings	First Number with Maximum In-Block Spacing	Last Number with Maximum In-Block Spacing	Number of Blocks with Maximum Spacings
1	$<10^9$	95	1	981,270,902	981,270,902	1
2	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	2
3	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
4	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

Earlier trend of identicalness with previous size continues.

IVJ. MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^{10}

At number 10 is block size 10^{10}, i.e., 10000000000, for which block 0 yields range 0 to 9999999999, block 10000000000 yields number range 10000000000 to 19999999999 and so on.

Sr. No.	Range	Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^{10}				
		Maximum In-Block Spacing	Number of Maximum Spacings	First Number with Maximum In-Block Spacing	Last Number with Maximum In-Block Spacing	Number of Blocks with Maximum Spacings
1	$<10^{10}$	112	2	8,512,915,573	9,913,934,497	1
2	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
3	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

The remarks for earlier block size are equally applicable to this one also.
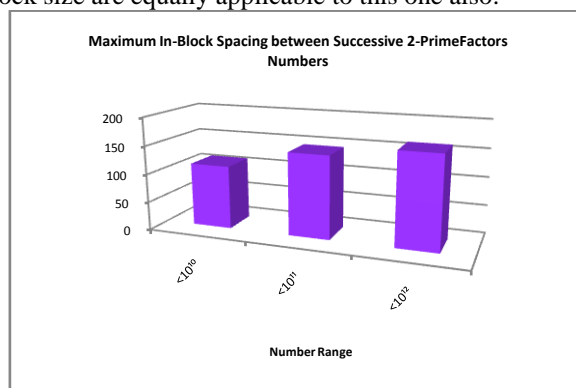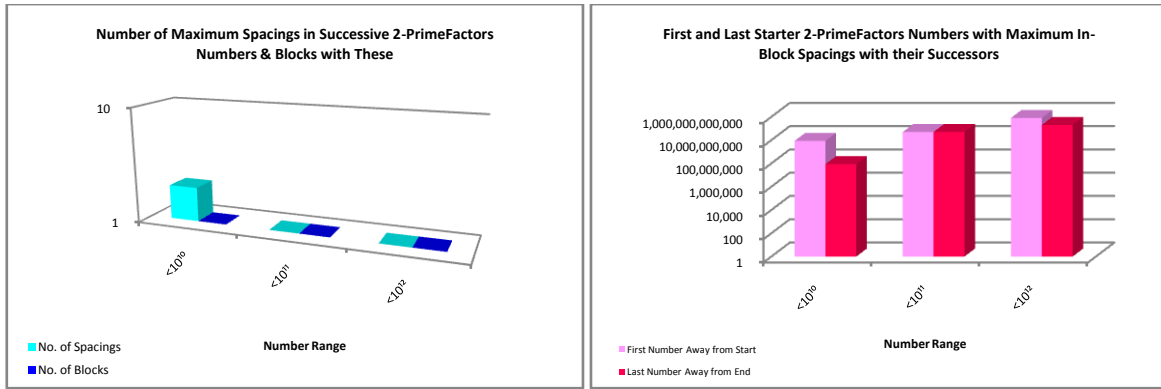
IVK.   MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^{11}

The eleventh block size is of 10^{11}, i.e., 100000000000, in which 0 specifies number range 0 to 99999999999, block 100000000000 specifies number range 100000000000 to 199999999999 and so on.

Sr. No.	Range	*Maximum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^{11}*				
		Maximum In-Block Spacing	*Number of Maximum Spacings*	*First Number with Maximum In-Block Spacing*	*Last Number with Maximum In-Block Spacing*	*Number of Blocks with Maximum Spacings*
1	$<10^{11}$	146	1	49,167,925,231	49,167,925,231	1
2	$<10^{12}$	163	1	799,980,626,859	799,980,626,859	1

There is no change, except increase in base block size and consequently next starter range, in the values for this case.
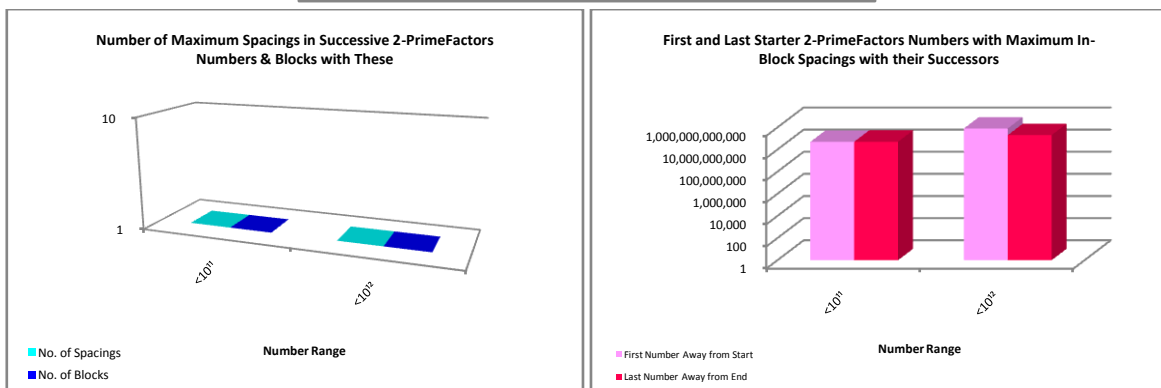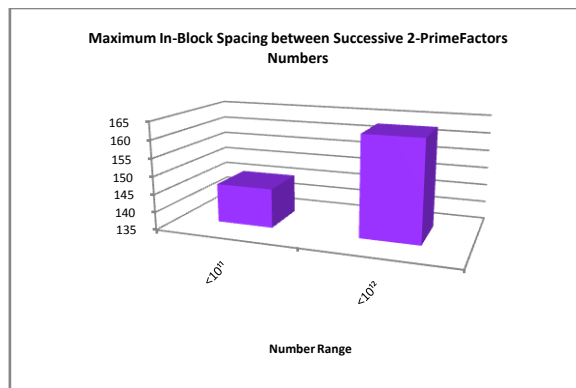
IVL.   MAXIMUM SPACINGS BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZE 10^{12}

Twelfth and last block size is the complete range 10^{12}, i.e., trillion itself. So analysis of this block size within the same range gives unique values, which are observations for entire range.

163 is maximum spacing between successive 2-PrimeFactors numbers which occurs only once between 799980626859 and immediate next 2-PrimeFactors number 799980627022.

This work has shown that the maximum spacing between consecutive 2-PrimeFactors numbers doesn't raise much in higher ranges. It signifies that 2-PrimeFactors numbers don't tend to be much apart from theneighbours of their own kind.

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