SUPRA α-OPEN SETS AND SUPRA α-SEPARATION AXIOMS IN SUPRA BITOPOLOGICAL SPACES

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Abstract

The aim of this paper is to introduce and study the concept of supra α-open (briefly $S_{\tau_{ij}}$-α-open) (res.Closed) sets and supra α-separation axioms (briefly $S_{\tau_{ij}}$-α-separation) in supra bitopological spaces. Also we study some of their basic properties in supra bitopological spaces.

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Keywords: $S_{\tau_{ij}}$-pre-open set, $S_{\tau_{ij}}$-semi-open set, $S_{\tau_{ij}}$-α-open set, $S_{\tau_{ij}}$-α-T_0-space, $S_{\tau_{ij}}$-α-T_1-space, $S_{\tau_{ij}}$-α-T_2-space.

1 Introduction

In 1988, Aray and Nour[2] was introduced separation axioms in bitopological spaces. The supra topological spaces have been introduced by Mashhour [14] in 1983. In topological space the arbitrary union condition is enough to have a supra topological space. Here every topological space is a supra topological space but the converse is not always true. Sreeja and Janaki [18] has discussed a new type of separation axioms in topological spaces in 2012. In 1992, Kar, Bhattacharyya[11] have been discussed pre open sets in bitopological spaces. In 1981, Bose[3], has introduced semi-open sets, semi-continuity and semi-open mappings in bitopological spaces. In 2008, Devi, Sampathkumar and Caldas[6] introduced the supra- α-open set. Gowri and Rajayal[8] introduced supra bitopological spaces and discussed basic properties of supra pairwise separation axioms in supra bitopological spaces. In this paper we introduce and investigate $S_{\tau_{ij}}$-pre-open, $S_{\tau_{ij}}$-semi-open, $S_{\tau_{ij}}$-α-open (res. closed) sets and $S_{\tau_{ij}}$-α-separation axioms. Also we study the relationship between supra pairwise separation axioms and $S_{\tau_{ij}}$-α-separation axioms in supra bitopological spaces.

2 Preliminaries

Definition 2.1 *[14] (X, S_τ) is said to be a supra topological space if it is satisfying these conditions:*
(1) $X, \emptyset \in S_\tau$
(2) The union of any number of sets in S_τ belongs to S_τ.

Definition 2.2 *[14] Each element $A \in S_\tau$ is called a supra open set in (X, S_τ), and its complement is called a supra closed set in (X, S_τ).*

Definition 2.3 *[14] If (X, S_τ) is a supra topological spaces, $A \subseteq X$, $A \neq \emptyset$, S_{τ_A} is the class of all intersection of A with each element in S_τ, then (A, S_{τ_A}) is called a supra subspace of (X, S_τ).*

Definition 2.4 *[14] The supra closure of the set A is denoted by S_τ-cl(A) and is defined as S_τ-cl(A)=$\cap\{B : B$ is a supra closed and $A \subseteq B\}$.*

Definition 2.5 *[14] The supra interior of the set A is denoted by S_τ-int(A) and is defined as S_τ-int(A)=$\cup\{B : B$ is a supra open and $B \subseteq A\}$.*

Definition 2.6 *[6] The set A of X is called a supra α - open set, if $A \subseteq$ supra int(supra cl(supra int(A))). The complement of a supra α - open set is a supra α - closed set.*

Definition 2.7 *[7] If S_{τ_1} and S_{τ_2} are two supra topologies on a non-empty set X, then the triplet $(X, S_{\tau_1}, S_{\tau_2})$ is said to be a supra bitopological space.*

Definition 2.8 *[7] Each element of S_{τ_i} is called a supra τ_i-open sets in $(X, S_{\tau_1}, S_{\tau_2})$. Then the complement of S_{τ_i}-open sets are called a supra τ_i-closed sets, for i = 1,2.*

Definition 2.9 *[7] If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, $Y \subseteq X$, $Y \neq \emptyset$ then $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is a supra bitopological subspace of $(X, S_{\tau_1}, S_{\tau_2})$ if*
$S_{\tau_1^*} = \{U \cap Y; U \text{ is a } S_{\tau_1} - \text{open in } X\}$ *and*
$S_{\tau_2^*} = \{V \cap Y; V \text{ is a } S_{\tau_2} - \text{open in } X\}$.

Definition 2.10 *[7] The supra τ_i-closure of the set A is denoted by S_{τ_i}-cl(A) and is defined as S_{τ_i}-cl(A)=$\cap\{B : B \text{ is a } S_{\tau_i} - \text{closed and } A \subseteq B, for\ i = 1, 2\}$.*

Definition 2.11 *[7] The supra τ_i-interior of the set A is denoted by S_{τ_i}-int(A) and is defined as S_{τ_i}-int(A)=$\cup\{B : B \text{ is a } S_{\tau_i} - \text{open and } B \subseteq A, for\ i = 1, 2\}$.*

Definition 2.12 *[7] A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is called a supra pairwise T_0 if for each pair of distinct points x,y $\in X$, $x \neq y$, there is a S_{τ_1}-open set U and S_{τ_2}-open set V such that $x \in U$, $y \notin U$ or $y \in V$, $x \notin V$.*

Definition 2.13 *[7] A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is called a supra pairwise T_1 if for each pair of distinct points x,y $\in X$, $x \neq y$,there is a S_{τ_1}-open set U and S_{τ_2}-open set V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$.*

Definition 2.14 *[7] A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is called a supra pairwise T_2 if for each pair of distinct points x,y $\in X$, $x \neq y$, there is a S_{τ_1}-open set U and S_{τ_2}-open set V such that $x \in U$ and $y \in V$, $U \cap V = \emptyset$.*

Proposition 2.15 *[8] Let $(X, S_{\tau_1}, S_{\tau_2})$ be supra bitopological spaces, if U and V are S_{τ_i}-open sets then $U \cup V$ also S_{τ_i}-open, for i = 1,2.*

Proposition 2.16 *[8] Let $(X, S_{\tau_1}, S_{\tau_2})$ be supra bitopological spaces, if U and V are S_{τ_i}-closed sets then $U \cap V$ also S_{τ_i}-closed, for i = 1,2.*

3 Supra α-open sets in supra bitopological spaces

Definition 3.1 *Let $(X, S_{\tau_1}, S_{\tau_2})$ be a supra bitopological space, $A \subseteq X$, A is said to be $S_{\tau_{ij}}$-pre-open (briefly $S_{\tau_{ij}}$-p-open), if $A \subseteq S_{\tau_i}$-int(S_{τ_j}-cl(A)), where $i \neq j$, i,j = 1,2.*

Definition 3.2 *Let $(X, S_{\tau_1}, S_{\tau_2})$ be a supra bitopological space, $A \subseteq X$, A is said to be $S_{\tau_{ij}}$-semi-open (briefly $S_{\tau_{ij}}$-s-open), if $A \subseteq S_{\tau_j}$-cl(S_{τ_i}-int(A)), where $i \neq j$, i,j = 1,2.*

Definition 3.3 *Let $(X, S_{\tau_1}, S_{\tau_2})$ be a supra bitopological space, $A \subseteq X$, A is said to be $S_{\tau_{ij}}$-α-open, if $A \subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(A))), where $i \neq j$, i,j = 1,2.*

Remark 3.4 *The family of all $S_{\tau_{ij}}$-p-open (res.$S_{\tau_{ij}}$-s-open and $S_{\tau_{ij}}$-α-open) sets of X is denoted by $S_{\tau_{ij}}$-PO(X) (res. $S_{\tau_{ij}}$-SO(X) and $S_{\tau_{ij}}$-αO(X)), where $i \neq j$, i,j = 1,2.*

Example 3.5 *Let $X = \{a, b, c\}$*
$S_{\tau_1} = \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{b\}, \{a, c\}, \{b, c\}\}$,
$S_{\tau_{12}}$-PO(X) $= \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$.
$S_{\tau_{12}}$-SO(X) $= \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$.
$S_{\tau_{12}}$-αO(X) $= \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$.

Definition 3.6 *The complement of $S_{\tau_{ij}}$-p-open (res. $S_{\tau_{ij}}$-s-open and $S_{\tau_{ij}}$-α-open) set is said to be $S_{\tau_{ij}}$-p-closed (res. $S_{\tau_{ij}}$-s-closed and $S_{\tau_{ij}}$-α-closed).*
The family of all $S_{\tau_{ij}}$-p-closed (res. $S_{\tau_{ij}}$-s-closed and $S_{\tau_{ij}}$-α-closed) sets of X is denoted by $S_{\tau_{ij}}$-PC(X) (res. $S_{\tau_{ij}}$-SC(X) and $S_{\tau_{ij}}$-αC(X)), where $i \neq j$, i,j = 1,2.

Definition 3.7 *The $S_{\tau_{ij}}$-α-closure of a set A is denoted by $S_{\tau_{ij}}$-α-cl(A) and defined as, $S_{\tau_{ij}}$-α-cl(A) $= \cap \{B : B$ is a $S_{\tau_{ij}}$-α-closed set and $A \subseteq B\}$.*

Definition 3.8 *The $S_{\tau_{ij}}$-α-interior of a set A is denoted by $S_{\tau_{ij}}$-α-int(A) and defined as, $S_{\tau_{ij}}$-α-int(A) $= \cup \{B : B$ is a $S_{\tau_{ij}}$-α-open set and $A \supseteq B\}$.*

Theorem 3.9 *Every S_{τ_i}-open set is $S_{\tau_{ij}}$-α-open set, for i = 1,2.*

Proof: Let A be a S_{τ_i}-open set in $(X, S_{\tau_1}, S_{\tau_2})$. since A $\subseteq S_{\tau_j}$-cl(A), thenS_{τ_i}-int(A). Hence A $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(A))). Therefore A is $S_{\tau_{ij}}$-α-open set. □

Theorem 3.10 *Every S_{τ_i}-open set is $S_{\tau_{ij}}$-p-open set, for i = 1,2.*

Proof: Obvious. □

Theorem 3.11 *Every S_{τ_i}-open set is $S_{\tau_{ij}}$-s-open set, for i = 1,2.*

Proof: The proof of the theorem is similar to the proof of Theorem, 3.9. □

Theorem 3.12 *Every $S_{\tau_{ij}}$-α-open set is $S_{\tau_{ij}}$-p-open set.*

Proof: Let A be a $S_{\tau_{ij}}$-α-open set in $(X, S_{\tau_1}, S_{\tau_2})$. Therefore A $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(A))) . It is obvious that S_{τ_i}-int(S_{τ_j}-cl(S_{τ_i}-int(A))) $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(A)). Hence A $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(A)). Therefore A is $S_{\tau_{ij}}$-p-open set. □

Theorem 3.13 *Every $S_{\tau_{ij}}$-α-open set is $S_{\tau_{ij}}$-s-open set.*

Proof: Obvious. □

Theorem 3.14 *Finite union of $S_{\tau_{ij}}$-α-open sets is always a $S_{\tau_{ij}}$-α-open set.*

Proof: Let A and B be two $S_{\tau_{ij}}$-α-open sets.Then A $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(A))) and B $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(B))). By Proposition 2.15, this implies A $\cup$ B $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(A$\cup$B))). Therefore A $\cup$ B is a $S_{\tau_{ij}}$-α-open set. □

Remark 3.15 *Finite intersection of $S_{\tau_{ij}}$-α-open sets may fail to be a $S_{\tau_{ij}}$-α-open set as shown in the following example.*

Example 3.16 *Let $X = \{a, b, c\}$*
$S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\}$,
$S_{\tau_{12}}$-$\alpha O(X) = \{\emptyset, X, \{a, b\}, \{b, c\}\}$.
Here $\{a, b\}, \{b, c\}$ are $S_{\tau_{12}}$-$\alpha O(X)$ but their intersection is not a $S_{\tau_{12}}$-α-open set.

Theorem 3.17 *Finite intersection of $S_{\tau_{ij}}$-α-closed sets is always a $S_{\tau_{ij}}$-α-closed set.*

Proof: Let A and B be two $S_{\tau_{ij}}$-α-closed sets. Then A $\subseteq S_{\tau_i}$-cl(S_{τ_j}-int(S_{τ_i}-cl(A))) and B $\subseteq S_{\tau_i}$-cl(S_{τ_j}-int(S_{τ_i}-cl(B))). By Proposition 2.16, this implies A $\cap$ B $\subseteq S_{\tau_i}$-cl(S_{τ_j}-int(S_{τ_i}-cl(A$\cap$B))). Therefore A $\cap$ B is a $S_{\tau_{ij}}$-α-closed set. □

Remark 3.18 *Finite union of $S_{\tau_{ij}}$-α-closed sets may fail to be a $S_{\tau_{ij}}$-α-closed set as shown in the following example.*

Example 3.19 *Let $X = \{a, b, c, d\}$*
$S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\}$.
$S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\}$.
$S_{\tau_{12}}$-$\alpha C(X) = \{\emptyset, X, \{c, d\}, \{a, d\}\}$.
Here $\{a, d\}, \{c, d\}$ are $S_{\tau_{12}}$-$\alpha C(X)$ but their union is not a $S_{\tau_{12}}$-α-closed set.

Theorem 3.20 *A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-open set if and only if A is $S_{\tau_{ij}}$-p-open set.*

Proof: Let A be a subset of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$. Let A be a $S_{\tau_{ij}}$-α-open set. Therefore A $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(S_{τ_i}-int(A))). It is obvious that S_{τ_i}-int(S_{τ_j}-cl(S_{τ_i}-int(A))) $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl((A)). Hence A $\subseteq S_{\tau_i}$-int(S_{τ_j}-cl(A)). Therefore A is $S_{\tau_{ij}}$-p-open set. □

Theorem 3.21 *A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-open set if and only if A is $S_{\tau_{ij}}$-s-open set.*

Proof: The proof of the theorem is similar to the proof of Theorem, 3.21. □

4 Supra α - separation axioms in supra bitopological spaces

Definition 4.1 *If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, for all x,y $\in$ X, $x \neq y$, then there exist a $S_{\tau_{ij}}$-α-open sets U and V such that $x \in U$, $y \notin U$ or $y \in V$, $x \notin V$. Then $(X, S_{\tau_1}, S_{\tau_2})$ is called a $S_{\tau_{ij}}$-α-T_0-space.*

Theorem 4.2 *If $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_0-space and $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is a supra bitopological subspace of $(X, S_{\tau_1}, S_{\tau_2})$ then $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is also a $S_{\tau_{ij}}$-α-T_0-space.*

Proof: Suppose x, y $\in$ X, x $\neq$ y. Since Y $\subseteq$ X, x, y $\in$ X. Since $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_0-space, there exist a $S_{\tau_{ij}}$-α-open sets U and V such that x $\in$ U, y $\notin$ U or y $\in$ V, x $\notin$ V. Then U $\cap$ Y, V $\cap$ Y are $S_{\tau_{ij}}$-α-open sets in Y repectively. Such that x $\in U \cap Y$, y $\notin U \cap Y$ or y $\in V \cap Y$, x $\notin V \cap Y$. Hence $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is also a $S_{\tau_{ij}}$-α-T_0-space. □

Theorem 4.3 *Let $(X, S_{\tau_1}, S_{\tau_2})$, $(X, S_{\tau_1'}, S_{\tau_2'})$ be two supra bitopological spaces. $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-α-T_0-space and f is a supra open bijective function then $(X, S_{\tau_1'}, S_{\tau_2'})$ is also a $S_{\tau_{ij}}$-α-T_0-space.*

Proof: Suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-α-T_0-space. Let $x', y' \in X'$,$x' \neq y'$, since f is bijective function then there exist x,y $\in$ X such that $x' = f(x), y' = f(y)$ and x $\neq$ y. Since $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-α-T_0-space, there exist a $S_{\tau_{ij}}$-α-open sets U and V such that x $\in$ U, y $\notin$ U or y $\in$ V, x $\notin$ V. Clearly f(U)$\subseteq X'$ and f(V)$\subseteq X'$, since f is a supra open function, f(U) and f(V) are $S_{\tau_{ij}}$-α-open sets in X'. Also f(x)$\in$ f(U), f(y) $\notin$ f(U) or f(y) $\in$ f(V), f(x) $\notin$ f(V). Hence $(X, S_{\tau_1'}, S_{\tau_2'})$ is also a $S_{\tau_{ij}}$-α-T_0-space. □

Definition 4.4 *If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, for all x, y $\in$ X, $x \neq y$, then there exist a $S_{\tau_{ij}}$-α-open sets U and V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Then $(X, S_{\tau_1}, S_{\tau_2})$ is called a $S_{\tau_{ij}}$-α-T_1-space.*

Theorem 4.5 *If $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_0-space and $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is a supra bitopological subspace of $(X, S_{\tau_1}, S_{\tau_2})$ then $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is also a $S_{\tau_{ij}}$-α-T_1-space.*

Proof: The theorem will be proved by using the definition of $S_{\tau_{ij}}$-α-T_1-space with the idea of the proof of Theorem, 4.2. □

Theorem 4.6 *If $(X, S_{\tau_1}, S_{\tau_2})$, $(X, S_{\tau_1'}, S_{\tau_2'})$ are two supra bitopological spaces, $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-α-T_1-space and f is a supra open bijective function then $(X, S_{\tau_1'}, S_{\tau_2'})$ is also a $S_{\tau_{ij}}$-α-T_1-space.*

Proof: The proof is similar to the way of Theorem, 4.5. □

Definition 4.7 *If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, for all x, y $\in$ X, $x \neq y$, then there exist a $S_{\tau_{ij}}$-α-open sets U and V such that $x \in U$, $y \in V$, $U \cap V = \emptyset$. Then $(X, S_{\tau_1}, S_{\tau_2})$ is called a $S_{\tau_{ij}}$-α-T_2-space.*

Example 4.8 *Let* $X = \{a, b, c\}$
$S_{\tau_1} = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$,
$S_{\tau_{12}}\text{-}\alpha O(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
Let $a,b \in X$, *then there exist* $S_{\tau_{12}}$-α-*open sets of* X, $U = \{a\}$ *and* $V = \{b, c\}$,
$U \cap V = \emptyset$.
Let $b,c \in X$, *then there exist* $S_{\tau_{12}}$-α-*open sets of* X, $U = \{a, b\}$ *and* $V = \{c\}$,
$U \cap V = \emptyset$.
Let $a,c \in X$, *then there exist* $S_{\tau_{12}}$-α-*open sets of* X, $U = \{a\}$ *and* $V = \{b, c\}$,
$U \cap V = \emptyset$.

Theorem 4.9 *If* $(X, S_{\tau_1}, S_{\tau_2})$ *is* $S_{\tau_{ij}}$-α-T_2-*space and* $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ *is a supra bitopological subspace of* $(X, S_{\tau_1}, S_{\tau_2})$ *then* $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ *is also a* $S_{\tau_{ij}}$-α-T_2-*space.*

Proof: Suppose x, y $\in$ X, x $\neq$ y. Since Y $\subseteq$ X, x, y $\in$ X. Since $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_2-space, then there exist a $S_{\tau_{ij}}$-α-open sets U and V such that x$\in$U, and y$\in$V, U$\cap$V = $\emptyset$. Then U$\cap$Y, V$\cap$Y are $S_{\tau_{ij}}$-α-open sets in Y respectively, such that x $\in U \cap Y$, y $\in V \cap Y$, U$\cap$V = (U$\cap$Y) $\cap$ (V$\cap$Y)
= (U$\cap$Y) $\cap$ Y
= $\emptyset \cap$ Y
= $\emptyset$.
Hence $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is also $S_{\tau_{ij}}$-α-T_2-space. □

Theorem 4.10 *If* $(X, S_{\tau_1}, S_{\tau_2})$, $(X, S_{\tau_1'}, S_{\tau_2'})$ *are two supra bitopological spaces,* $(X, S_{\tau_1}, S_{\tau_2})$ *is a* $S_{\tau_{ij}}$-α-T_2-*space and f is a supra open bijective function then* $(X, S_{\tau_1'}, S_{\tau_2'})$ *is also* $S_{\tau_{ij}}$-α-T_2-*space.*

Proof: The proof is similarly to the proof of Theorem, 4.3. □

5 Some relationships between two Classes supra pairwise separation axioms and $S_{\tau_{ij}}$-α-separation axioms

Theorem 5.1 *Every supra pairwise* T_2-*space is* $S_{\tau_{ij}}$-α-T_2-*space.*

Proof: Let $(X, S_{\tau_1}, S_{\tau_2})$ be supra pairwise T_2-space and let x, y $\in$ X, x $\neq$ y, then there exist U and V S_{τ_i}-open sets in X such that x $\in$ U and y $\in$ V, U$\cap$V = $\emptyset$, for i = 1,2. Since every S_{τ_i}-open set is $S_{\tau_{ij}}$-α-open set. Then U and V are $S_{\tau_{ij}}$-α-open sets in X such that x $\in$ U and y $\in$ V, U$\cap$V = $\emptyset$. Hence $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_2-space. □

Theorem 5.2 *Every supra pairwise* T_1-*space is* $S_{\tau_{ij}}$-α-T_1-*space.*

Proof: Let $(X, S_{\tau_1}, S_{\tau_2})$ be supra pairwise T_1-space, and let x, y $\in$ X, x $\neq$ y, then there exist U and V S_{τ_i}-open sets in X such that x $\in$ U, y $\notin$ U and y $\in$ V, x $\notin$ V, for i = 1,2.
Since every S_{τ_i}-open set is a $S_{\tau_{ij}}$-α-open set. Then U and V are $S_{\tau_{ij}}$-α-open sets in X such that x $\in$ U, y $\notin$ U and y $\in$ V, x $\notin$ V.
Hence $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_1-space. □

Theorem 5.3 *Every supra pairwise T_0-space is a $S_{\tau_{ij}}$-α-T_0-space.*

Proof: The way of the proof is similar to the proof of Theorem, 5.2. □

Theorem 5.4 *Every $S_{\tau_{ij}}$-α-T_2-space is a $S_{\tau_{ij}}$-α-T_1-space.*

Proof: Let $(X, S_{\tau_1}, S_{\tau_2})$ be $S_{\tau_{ij}}$-α-T_2-space and let x,y ∈ X, x ≠ y, then there exist $S_{\tau_{ij}}$-α-open sets U and V such that x ∈ U, and y ∈ V, U ∩ V = ∅.
Since U ∩ V = ∅, this implies x ∈ U, y ∉ U and y ∈ V, x ∉ V. Hence $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-α-T_1-space. □

Theorem 5.5 *Every $S_{\tau_{ij}}$-α-T_1-space is a $S_{\tau_{ij}}$-α-T_0-space.*

Proof: Let $(X, S_{\tau_1}, S_{\tau_2})$ be $S_{\tau_{ij}}$-α-T_1-space, and let x,y ∈ X, x ≠ y, and there exist $S_{\tau_{ij}}$-α-open sets U and V in X such that x ∈ U, y ∉ U and y ∈ V, x ∉ V. This means that there exist U and V $S_{\tau_{ij}}$-α-open sets in X such that x ∈ U, y ∉ U or y ∈ V, x ∉ V. Hence $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-α-T_0-space. □

Theorem 5.6 *Every $S_{\tau_{ij}}$-α-T_2-space is a $S_{\tau_{ij}}$-α-T_0-space.*

Proof: Let $(X, S_{\tau_1}, S_{\tau_2})$ be $S_{\tau_{ij}}$-α-T_2-space, and let x,y ∈ X, x ≠ y, then there exist $S_{\tau_{ij}}$-α-open sets U and V such that x ∈ U, and y ∈ V, U ∩ V = ∅.
Since U ∩ V = ∅, this implies x ∈ U, y ∉ U or y ∈ V, x ∉ V. Hence $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-α-T_0-space. □

Remark 5.7 *From the above results, we conclude that the relation between two classes of supra pairwise separation axioms and $S_{\tau_{ij}}$-α-separation axioms.*

$$\begin{array}{ccc}
\text{Supra pairwise } T_2\text{-space} & \Rightarrow & S_{\tau_{ij}}\text{-}\alpha\text{-}T_2\text{-space} \\
\Downarrow & & \Downarrow \\
\text{Supra pairwise } T_1\text{-space} & \Rightarrow & S_{\tau_{ij}}\text{-}\alpha\text{-}T_1\text{-space} \\
\Downarrow & & \Downarrow \\
\text{Supra pairwise } T_0\text{-space} & \Rightarrow & S_{\tau_{ij}}\text{-}\alpha\text{-}T_0\text{-space}
\end{array}$$

6 Conclusion

In this paper, we introduced notion of $S_{\tau_{ij}}$-α-open set and $S_{\tau_{ij}}$-α-separation axioms. Also investigate the relationship between supra pairwise separation axioms and $S_{\tau_{ij}}$-α-separation axioms.

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