Semi Pre Zc-Connected Spaces In General Topology.

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Abstract — *In this paper, we introduce the concept of spZc-connectedness and separate sets using Zc-open sets. Some properties and theorems using spZc-continuous, spZc-irresolute through spZc –open sets are also discussed.*

Keywords — *spZc -open, spZc –closed, spZc-connected space, spZc-separated space, spZc-continuous, spZc-irresolute.*

I. INTRODUCTION

Connectedness is one of the principal property in the study of topological spaces. Weierstrass introduced the notion of arcwise connectedness. However Cantor introduced the notion of Connectedness which we are using today. Connectedness is mainly used to distinguish topological spaces.

2.Preliminaries

Throughout this paper (X,τ) represents a topological space on which no separation axiom is assumed unless otherwise stated. Let A $\subseteq$ (X,τ) ,then cl(A) and int(A) denotes the closure of A and the interior of A respectively.

Definition 2.1 [2]: A subset A of a space X is Zc-open`if for each x $\in$ A$\in$ ZO(X) ,there exists a closed set F such that x $\in$ F $\subset$ A. A subset A of a space X is Zc-closed if X$-$A is Zc-open. The family of all Zc-open(resp. Zc-closed) subsets of a topological space (X,τ) is denoted by ZcO (X,τ) or ZcO(X) (resp. ZcC (X,τ) or ZcC(X)).

Definition 2.2 [3]: A function f : X$\rightarrow$Y is Zc-continuous if f^{-1}(V) is Zc-open in X for every open set V in Y.

Definition 2.3 [3]: A function f : X$\rightarrow$Y is

(i) Zc-irresolute if for every Zc-open set V in Y, f^{-1}(V) is Zc-open in X.

(ii) strongly Zc-continuous if f^{-1}(V) is open in X ,for every Zc-open set V in Y.

(iii) faintly Zc-continuous if f^{-1}(V) is Zc-open in X for every θ-openset V in Y.

Definition 2.4 [4] : Let (X,τ) be a topological space. Then
(i) Zc-interior of A is union of all Zc-open sets contained in A and is denoted by Zc -Int(A).
(ii) Zc-closure of A is the intersection of all Zc-closed sets containing A and is denoted by Zc-cl(A).

Definition 2.5 [1]: A topological space X is said to be connected if X cannot be written as the disjoint union of two non-empty open sets in X.

3. spZc separated and spZc connected spaces

Definition 3.1: A subset A of (X,τ) is called
(i) spZc open if A $\subseteq$Zccl(Zc int(Zccl(A))) and is denoted by spZcO(X)
(ii) spZc closed if X$-$A is spZc open and is denoted by spZcC(X) .

Definition 3.2: (i) The semi pre Zc interior of a subset A of X is the union of all semi pre Zc open sets contained in A and is denoted by spZcInt (A).

(ii) The semi pre Zc closure of a subset A of X is the intersection of all semi pre Zc closed sets containing A and is denoted by spZcCl(A).

Example 3.3: `Let X ={a,b,c,d} with τ={$\emptyset$,X,{a},{c,d},{a,c,d}} then the family of Zc-open sets are ZcO(X) = {X, $\emptyset$,{a,b}{b,c,d}} and
spZcO(X)={X, $\emptyset$,{b},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d},{a,c,d},{c},{d}}.

Definition 3.4: Let f : X→Y is called
(i) spZc continuous if f^{-1}(V) is spZc open in X for every open set V in Y.
(ii) spZc irresolute if f^{-1}(V) is spZc open in X for each open set V in Y.
(iii) contra spZc-continuous if f^{-1}(V) is spZc-closed in X, for every open set in Y. .

Definition 3.5: Let (X,τ) be a topological space. X is spZc-connected if X cannot be written as the disjoint union of two non-empty spZc open sets in X.

Definition 3.6: X =A∪B is said to be a spZc separation of X if A and B are non-empty, disjoint, spZc open sets in X.

Remarks 3.7: If there is no spZc-separation of X, then X is said to be spZc-connected. Otherwise it is said to be spZc-disconnected.

Example 3.8: X = {a,b,c,d}with τ={$\emptyset$,X,{a},{b},{a,b},{a,b,c}}.Then the family of Zc-open sets are ZcO(X) = {X, $\emptyset$,{a,c,d},{b,c,d}} and
spZcO(X)={X, $\emptyset$,{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d}, {a,b,c},{a,c,d},{b,c,d},{a,b,d}}. The subsets {c}and {a,b,d} are spZc-separated but not Zc-separated.

Theorem 3.9: The sets A and B of a space X are spZc-separated if and only if there exist U and V in spZcO(X) such that A ⊆U, B ⊆V, A∩V =$\emptyset$, B ∩U = $\emptyset$.
Proof: Let A and B be spZc-separated sets. Let V = X− spZcCl(A) and U = X− spZcCl(B).Then U,V are spZc open sets such that A ⊆U, B ⊆V, A∩V =$\emptyset$, B ∩U = $\emptyset$.Conversely let U,V are spZc open sets such that A ⊆U, B ⊆V, A∩V =$\emptyset$, B ∩U = $\emptyset$.Clearly X −V and X −U are spZc closed, then spZcCl(A) ⊆ X −V ⊆ X −B and spZcCl(B) ⊆ X −U ⊆ X −A.Thus spZcCl(A)∩B =$\emptyset$ and spZcCl(B)∩A =$\emptyset$.Hence A and B are spZc-separated.

Theorem 3.10: Let (X,τ) be a topological space. X is connected if
(i) X cannot be decomposed into two disjoint,non-empty spZc-open sets.
(ii) There does not exists a spZc-separation of X .
(iii) There does not exist a proper non empty subset of X which is both spZc open and spZc closed in X .

Lemma 3.11: Let (X,τ) be a topological space. X is spZc-connected if and only if the only subsets of X that remains both open and closed in X are X itself and the empty set.
Proof: Let X be spZc-connected and A ⊆X. By definition A is non-empty proper subset. Let A be both spZc-open and spZc-closed in X .Thus X and X −A forms a spZc separation of X which implies they are disjoint, non-empty and union is X itself. Then X will be a spZc-disconnected space which is a contradiction. Conversely, Let X be a spZc-disconnected space. Let A and B form a separation of X. Then A is non-empty and it is both spZc open and spZc closed in X which is a contradiction.

Lemma 3.12: (i) Every indiscrete space is spZc-connected.
(ii) Every singleton set is spZc-connected.
(iii) Every discrete space which contain more than one point is spZc-connected.
Proof: (i) Let X be an indiscrete space. Since X and $\emptyset$ are the only subsets of X ,which is both spZc-open and spZc-closed in X .Thus X is spZc-connected.
(ii) Let {x}∈ (X,τ) .Clearly {x} cannot be written as the union of two non-empty disjoint sets which implies {x} has no spZc-separation and thus spZc-connected.

(iii) Let (X,τ) be a discrete space and $x \in X$. $\{x\}$ is a non-empty proper subset of X. Also it is both spZc open and spZc closed in X and thus X is spZc-disconnected.

Theorem 3.13: Let (X,τ) be a topological space and $A \subseteq X$; $B \subseteq X$ such that $A \subseteq B \subseteq$ spZcCl(A). If A is spZc-connected, then B is spZc –connected.
Proof: If B is spZc –disconnected then by definition there exists two spZc separated sets U and V such that $B= U \cup V$.Then $A \subseteq U$ or $A \subseteq V$.Without loss of generality , let $A \subseteq U$.Then $A \subseteq U \subseteq B$, $spZcCl_B(A) \subseteq spZcCl_B(U) \subseteq spZcCl(U)$. Also $spZcCl_B(A) = B\cap spZcCl(A) = B \supseteq spZcCl(U)$. Thus B= spZcCl (U).Thus U and V are not spZc-separated and so B is spZc-connected.

Theorem 3.14: Let A and B be non-empty sets in a space X. Then the following holds :
(i) If $A\cap B = \emptyset$ such that each of A and B are both spZc-closed and spZc-open, then A and B are spZc-separated.
(ii) If A and B are spZc-separated and $A_1 \subseteq A$ and $B_1 \subseteq B$ then A_1 and B_1 are spZc-separated. (iii) If A and B are both spZc-closed and spZc-open and if $U = A\cap(X-B)$ and $V = B\cap(X-A)$,then U and V are spZc-separated.
Proof: (i) Since A= spZccl(A) and B = spZccl(B) and $A\cap B= \emptyset$, then spZccl(A)$\cap B = \emptyset$ and spZccl(B)$\cap A = \emptyset$,which implies A and B are spZc-separated. If A and B are spZc-open, then $(X-A)$ and $(X-B)$ are spZc-closed.
(ii) Let $A_1 \subseteq A$.Clearly spZccl$(A_1) \subseteq$ spZccl(A).Then $B\cap$ spZccl(A) = $\emptyset$ which implies $B_1\cap$spZccl(A) = $\emptyset$ and $B_1\cap$spZccl$(A_1) = \emptyset$.Similarly $A_1\cap$ spZccl$(B_1) = \emptyset$.Thus A_1 and B_1 are spZc- separated.
(iii) Let A and B are spZc-open.So, $(X-A)$ and $(X-B)$ are spZc-closed. Since $U\subseteq(X-B)$, spZccl(U) $\subseteq$spZccl$(X-B) = X-B$ and so spZccl(U) $\cap$ B = $\emptyset$.Thus $V\cap$ spZccl(U) = $\emptyset$.Similarly $U\cap$spZccl(V) $=\emptyset$.Thus U and V are spZc- separated.

Lemma 3.15: (1). Let $f : (X,\tau) \to (Y, \sigma)$ be a function. The following are equivalent
(i) f is spZc continuous.
(ii)The inverse image of each closed(respectively open) set in Y is spZc-closed(spZc-open) in X.
(2). $f : X\to Y$ is spZc-irresolute if and only if the inverse image of every spZc-open set in Y is spZc-open in X.

Theorem 3.16: If A is spZc-connected, then spZccl(A) is spZc-connected.
Proof: Suppose, spZccl(A) is spZc-disconnected,then there exists two non-empty spZc separated sets U and V in X such that spZccl(A) = $U\cup V$.Since $A= (U \cap A)\cup (V \cap A)$ and spZccl$(U \cap A) \subseteq$spZccl(U) and spZccl$(V \cap A) \subseteq$ spZccl(V) and $U\cap V = \emptyset$, then (spZccl$(U \cap A))\cap V= \emptyset$.Hence (spZccl$(U \cap A)) \cap (V \cap A) = \emptyset$.Similarly, (spZccl$(V \cap A)) \cap (U \cap A) = \emptyset$.So A is spZc-disconnected.

Theorem 3.17: Let $f : X\to Y$ be a function.
(i) If X is spZc-connected and if f is spZc-continuous, surjective, then Y is connected.
(ii) If X is spZc-connected and if f is spZc-irresolute, surjective ,then Y is spZc-connected.
Proof: (i) Let X be spZc-connected and f is a spZc-continuous, surjective function. Suppose if Y is disconnected, then there exists disjoint,non-empty open subsets A,B of Y such that $Y = A \cup B$. Given f is spZc-continuous, surjective by lemma 3.15, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint, nonempty, spZc-open subsets of X which is a contradiction to X is spZc-connected, thus Y is connected.
(ii) Let X be spZc-connected and f be spZc-irresolute,surjective. Assume Y is not spZc-connected then $Y=A \cup B$ where A and B are disjoint, non-empty, spZc open subsets of Y. Since f is spZc-irresolute, surjective then $X = f^{-1}(A)\cup f^{-1}(B)$, by lemma 3.15 where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint,non-empty spZc-open subsets of X. Thus we get a contradiction that X is not spZc-connected. Therefore Y is spZc-connected.

Theorem 3.18: Let $f : X\to Y$ be a spZc-continuous function of X into a discrete space Y with atleast two points which is a constant map then empty set and X are the only subsets of X that are both spZc-open and spZc-closed.
Proof: Let A be both spZc-open and spZc-closed in X and $A\neq \emptyset$.Let $f : X\to Y$ be a spZc-continuous function defined by $f(A) =\{y\}$ and $f(X -A) =\{w\}$ for some distinct points y and w in Y.Since f is a constant function we get A=X and hence the proof follows.

Theorem 3.19: A contra- spZc-continuous image of an spZc-connected space is connected.

Proof: Let f : X→Y be contra- spZc-continuous function where X is spZc-connected. To prove: Y is connected. Suppose if Y is disconnected, then Y =A ∪B where A and B are non-empty clopen sets in Y such that A ∩B= ∅.Since f is contra spZc-continuous, we have f^{-1}(A) and f^{-1}(B) are non-empty spZc-open sets in X with f^{-1}(A) ∪ f^{-1}(B) = f^{-1}(A ∪B) = f^{-1}(Y) =X and f^{-1}(A) ∩ f^{-1}(B) = f^{-1}(A∩B) =f^{-1}(∅) = ∅ which implies X is not spZc-connected, which is a contradiction. Hence Y is connected.

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