On Fuzzy γ^* Generalized Closed Sets in Fuzzy Topological Spaces

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Abstract: *In this paper, we have introduced a new class of fuzzy set called fuzzy γ^* generalized closed set, and investigated some of their properties. Some characterizations of the fuzzy γ^* generalized closed sets are also studied.*

Keywords: *Fuzzy sets, fuzzy topology, fuzzy point, fuzzy γ closed sets, fuzzy γ^* generalized closed sets.*

I. INTRODUCTION

The concept of fuzzy set and fuzzy set operations was introduced by L.A.Zadeh [11]. A fuzzy topological space using the concept of fuzzy sets was introduced by C.L.Chang [2]. Thakur S.S [9] introduced the concept of fuzzy generalized closed sets. In this paper we have introduced a new type of fuzzy closed set called fuzzy γ^* generalized closed set and investigated some of their properties.

II.PRELIMINARIES

Definition 2.1: [11] Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \rightarrow [0,1]$ and $\mu_A(x)$ is interpreted as the degree of member of element x in a fuzzy set A, for each $x \in X$. It is clear that A is determined by the set of tuples of $A = \{(x, \mu_A(x)) : x \in X\}$.

Definition 2.2: [11] Let A and B be two fuzzy sets $A = \{(x, \mu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x)) : x \in X\}$.Then, their union $A \vee B$, intersection $A \wedge B$ and complement A^c are also fuzzy sets with membership functions defined as follows :

(a) $\mu_A{}^c(x) = 1-\mu_A(x), \forall x \in X$,
(b) $\mu_{A\vee B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$,
(c) $\mu_{A\wedge B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X$.

Further,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x), \forall x \in X$,
(b) $A = B$ if and only if $\mu_A(x) = \mu_B(x), \forall x \in X$.

Definition 2.3: [4] A family τ of fuzzy sets is called fuzzy topology (FT in short) for X if it satisfy the three axioms:

(a) $\overline{0}, \overline{1} \in \tau$
(b) $\forall A, B \in \tau \Rightarrow A \wedge B \in \tau$
(c) $\forall (A_j)_{j\in J} \in \tau \Rightarrow \vee_{j\in J} A_j \in \tau$

The pair (X, τ) is called a fuzzy topological space (FTS for short). The elements of τ are called fuzzy open sets in X and their respective complements are called fuzzy closed sets of (X, τ).

Definition 2.4: [3] A fuzzy set A in a FTS (X, τ) is said to be a

(a) fuzzy γ closed set (FγCS) if $cl(int(A)) \wedge int(cl(A)) \leq A$
(b) fuzzy γ open set (FγOS) if $A \leq int(cl(A)) \vee cl(int(A))$

Remark 2.5: [3]

(i) Any union of fuzzy γ open sets in a FTS X is a fuzzy γ open set in a FTS X.
(ii) Any intersection of fuzzy γ closed sets is a fuzzy γ closed set in a FTS X.

Definition 2.6: [3] Let A be a fuzzy set in a FTS X. Then we define γ interior and γ closure as

$\gamma cl(A) = \wedge\{B : B \geq A, B$ is a fuzzy γ closed set in X$\}$
$\gamma int(A) = \vee\{B : B \leq A, B$ is a fuzzy γ open set in X$\}$.

Properties 2.7: [3] Let A be a fuzzy set in a FTS X. Then

$\gamma cl(A^c) = (\gamma int(A))^c$
$\gamma int(A^c) = (\gamma cl(A))^c$

Properties 2.8: [3] Let A and B be any two fuzzy sets in a FTS X. Then

1) $\gamma int(\overline{0}) = \overline{0}, \gamma int(\overline{1}) = \overline{1}$,
2) $\gamma int(A)$ is a fuzzy γ open set in X,
3) $\gamma int(\gamma int(A)) = \gamma int(A)$,
4) If $A \leq B$ then $\gamma int(A) \leq \gamma int(B)$,

5) γint(A∧B) = γint(A) ∧ γint(B),
6) γint(A∨B) ≥ γint(A) ∨ γint(B).

Properties 2.9: [3] Let A and B be any two fuzzy sets in a fuzzy topological spaces X. Then

1) γcl($\bar{0}$) = $\bar{0}$; γcl($\bar{1}$) = $\bar{1}$,
2) γcl(A) is a fuzzy γ closed set in X,
3) γcl(γcl(A)) = γcl(A),
4) If A ≤ B then γcl(A) ≤ γcl(B),
5) γcl(A∨B) = γcl(A) ∨ γcl(B),
6) γcl(A∧B) ≤ γcl(A) ∧ γcl(B).

Definition 2.10: [5] A fuzzy set A is a quasi-coincident with a fuzzy set B, denoted by A_qB, if there exists x ∈ X such that A(x)+B(x) > 1.

Definition 2.11: [5] If A and B are not quasi-coincident then we write $A_{\bar{q}}B$. A ≤ B ⟺ $A_{\bar{q}}(1-B)$.

Definition 2.12: [8] A fuzzy Set A in a FTS (X, τ) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set B in (X, τ) such that B < cl(A) that is intcl(A) = $\bar{0}$.

Definition 2.13: [6] The intersection of all fuzzy open subsets of a topological space (X, τ) containing A is called the Kernel of A, this means ker(A) = ∧ {G ∈ τ, A ≤ G}.

Definition 2.14: [7] A fuzzy point $\tilde{p}$ in a set X is also a fuzzy set with membership function:

$$\mu_{\tilde{p}}(x) = \begin{cases} r, & \text{for } x = y \\ 0, & \text{for } x \neq y \end{cases}$$

where x ∈ X and 0 < r ≤ 1, y is called the support of $\tilde{p}$ and r the value of $\tilde{p}$. We denote this fuzzy point by x_r or $\tilde{p}$. A fuzzy point x_r is said to be belonged to a fuzzy subset $\tilde{A}$ in X, denoted by $x_r \in \tilde{A}$ if and only if $r \leq \mu_{\tilde{A}}(x)$.

III. FUZZY γ* GENERALIZED CLOSED SETS

In this section we have introduced a new type of fuzzy closed set called fuzzy γ* generalized closed set and studied some of the properties.

Definition 3.1: An fuzzy set A of a FTS (X, τ) is said to be a fuzzy γ* generalized closed set (Fγ*GCS for short) if cl(int(A)) ∧ int(cl(A)) ≤ U, whenever A ≤ U and U is a fuzzy open set in X.

Example 3.2: Let X = {a, b} and τ ={$\bar{0}$, $\bar{1}$, G_1, G_2} be a FT on X, where G_1 = ⟨x, $(0.5_a, 0.6_b)$⟩, G_2 = ⟨x, $(0.4_a, 0.5_b)$⟩. Then (X, τ) is a FTS. Let A = ⟨x, $(0.5_a, 0.5_b)$⟩ be a fuzzy set in (X, τ). We have A ≤ G_1. Now cl(int(A)) ∧ int(cl(A)) = G_2^c ∧ G_2 = G_2 ≤ G_1, where G_1 is a FOS in X. This implies A is a Fγ*GCS in X.

Theorem 3.3: Every FCS is a Fγ*GCS in (X, τ) but not conversely in general.

Proof: Let A be a FCS in (X, τ), then cl(A) = A. Let A ≤ U and U be a FOS in (X, τ). Now cl(int(A)) ∧ int(cl(A)) ≤ cl(A) = A ≤ U, by hypothesis. Hence A is a Fγ*GCS in (X, τ).

Example 3.4: In Example 3.2, the FS A = ⟨x, $(0.5_a, 0.5_b)$⟩ is a Fγ*GCS but not a FCS in (X, τ), as cl(A) = G_2^c ≠ A.

Theorem 3.5: Every FSCS [1] in (X, τ) is a Fγ*GCS but not conversely in general.

Proof: Let A be a FSCS in X, then int(cl(A)) ≤ A. Let A ≤ U and U be a FOS in (X, τ). Now cl(int(A)) ∧ int(cl(A)) ≤ cl(int(A)) ∧ A ≤ cl(A) ∧ A = A ≤ U. Hence A is a Fγ*GCS in (X, τ).

Example 3.6: Let X = {a, b} and τ = {$\bar{0}$, $\bar{1}$, G_1, G_2} be a FT on X, where G_1 = ⟨x, $(0.5_a, 0.4_b)$⟩, G_2 = ⟨x, $(0.6_a, 0.5_b)$⟩. Then (X, τ) is a FTS. Let A = ⟨x, $(0.5_a, 0.3_b)$⟩ be a fuzzy set in (X, τ). Now cl(int(A)) ∧ int(cl(A)) = $\bar{0}$ ∧ G_1 = $\bar{0}$ ≤ U, then A is a Fγ*GCS but not a FSCS in (X, τ), as int(cl(A)) = G_1 ≰ A.

Theorem 3.7: Every FPCS [10] is a Fγ*GCS in (X, τ) but not conversely in general.

Proof: Let A be a FPCS in X, then cl(int(A) ≤ A. Let A ≤ U and U be a FOS in (X, τ). Now cl(int(A)) ∧ int(cl(A)) ≤ A ∧ int(cl(A)) ≤ A ∧ cl(A) = A ≤ U. Hence A is a Fγ*GCS in (X, τ).

Example 3.8: Let X = {a, b} and τ = {$\bar{0}$, $\bar{1}$, G_1, G_2} be a FT on X, where G_1 = ⟨x, $(0.3_a, 0.3_b)$⟩, G_2 = ⟨x, $(0.5_a, 0.6_b)$⟩. Then (X, τ) is a FTS. Let A = ⟨x, $(0.4_a, 0.4_b)$⟩ be a fuzzy set in (X, τ). Now cl(int(A)) ∧ int(cl(A)) = G_2^c ∧ G_1 = G_1 ≤ G_2, where A ≤ G_2. Then A is a Fγ*GCS in X, but not a FPCS in (X, τ), as cl(int(A)) = G_2^c ≰ A.

Theorem 3.9: Every FRCS [10] is a Fγ*GCS in (X, τ) but not conversely in general.

Proof: Let A be a FRCS in X, then cl(int(A) = A. Let A $\leq$ U and U be a FOS in (X, τ). Now cl(int(A)) $\wedge$ int(cl(A)) = A $\wedge$ int(cl(A)) $\leq$ A $\wedge$ cl(A) = A $\leq$ U. Hence A is a Fγ*GCS in (X, τ).

Example 3.10: In Example 3.8, A is a Fγ*GCS in (X, τ) but not a FRCS as cl(int(A) = $G_2^c \neq$ A.
Theorem 3.11: Every FαCS [10] is a Fγ*GCS in (X, τ) but not conversely in general.
Proof: Let A be a FαCS in X, then cl(int(cl(A)) $\leq$ A. Let A $\leq$ U and U be a FOS in (X, τ). Now cl(int(A)) $\wedge$ int(cl(A)) $\leq$ cl(int(cl(A))) $\wedge$ int(cl(A)) $\leq$ A $\wedge$ cl(A) = A $\leq$ U. Hence A is a Fγ*GCS in (X, τ).

Example 3.12: In Example 3.8, A is a Fγ*GCS but not a FαCS as cl(int(cl(A)) = $G_2^c \nleq$ A.

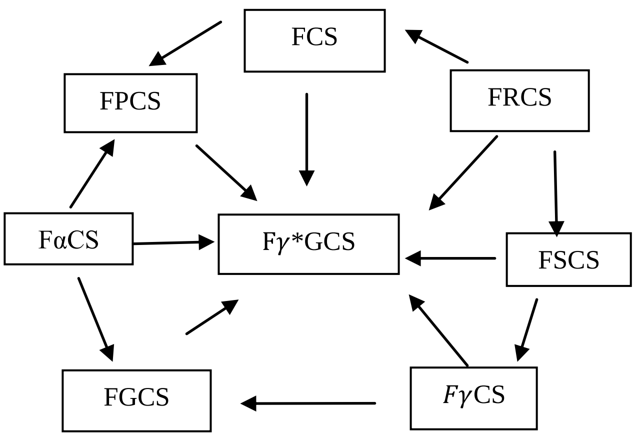
Theorem 3.13: Every FγCS [3] is a F γ*GCS in (X, τ) but not conversely in general.
Proof: Let A be a FγCS in X, then cl(int(A) $\wedge$ (int(cl(A)) $\leq$ A. Let A $\leq$ U and U be a FOS in (X, τ). Now cl(int(A)) $\wedge$ int(cl(A)) $\leq$ A $\leq$ U. Hence A is a Fγ*GCS in (X, τ).

Example 3.14: Let X = {a, b} and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a FT on X, where $G_1 = \langle x, (0.3_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.5_b)\rangle$. Then (X, τ) is a FTS. Let A = $\langle x, (0.4_a, 0.4_b)\rangle$ be a fuzzy set in (X, τ). Now cl(int(A)) $\wedge$ int(cl(A)) = $G_2^c \wedge G_2 = G_2 \leq G_2$ where A $\leq$ G_2. Then A is a Fγ*GCS but not a FγCS as cl(int(A)) $\wedge$ int(cl(A)) = $G_2 \nleq$ A.

Theorem 3.15: Every FGCS [9] is a Fγ*GCS in (X, τ) but not conversely in general.
Proof: Let A be a FGCS in X. Let A $\leq$ U and U be a FOS in (X, τ). Now cl(int(A)) $\wedge$ int(cl(A)) $\leq$ cl(A) $\wedge$ cl(A) = cl(A) $\leq$ U, by hypothesis. Hence A is a Fγ*GCS in (X, τ).

Example 3.16: Let X = {a, b} and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a FT on X, where $G_1 = \langle x, (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.5_b)\rangle$. Then (X, τ) is a FTS. Let A = $\langle x, (0.4_a, 0.5_b)\rangle$ be a fuzzy set in (X, τ). Now cl(int(A)) $\wedge$ int(cl(A)) = $G_2^c \wedge G_2 = G_2 \leq G_1, G_2$ where A $\leq G_1, G_2$. Then A is a Fγ*GCS but not a FGCS as cl(A) =$G_2^c \nleq G_1, G_2$ but A $\leq G_1, G_2$.

In the following diagram we have provided relation between various types of fuzzy closedness.



Theorem 3.17: Let (X, τ) be a FTS. Then for every A $\in$ Fγ*GC(X) and for every B $\in$ FS(X), A $\leq$ B $\leq$ cl(int(A) $\Rightarrow$ B $\in$ F γ*GC(X).

Proof: Let B $\leq$ U and U be a FOS in X. Let A $\leq$ B, A $\leq$ U, by hypothesis. Since B $\leq$ cl(int(A), cl(int(B)) $\leq$ cl(int(A)). Also int(cl(B)) $\leq$ int(cl(cl(int(A)))) $\leq$ int(cl(int(A))) $\leq$ int(cl(A)). Therefore cl(int(B)) $\wedge$ int(cl(B)) $\leq$ cl(int(A)) $\wedge$ int(cl(A)) $\leq$ U, by hypothesis. Hence B $\in$ Fγ*GC(X).

Theorem 3.18: A fuzzy set A of a FTS (X, τ) is a Fγ*GCS if and only if $A_{\bar{q}}F \Rightarrow (\mathrm{int}(\mathrm{cl}(A) \wedge \mathrm{cl}(\mathrm{int}(A)))_{\bar{q}}F$ for every FCS F of X.

Proof: Necessity: Let F be a FCS and $A_{\bar{q}}F$, then A$\leq F^c$, where F^c is a FOS in X. Then int(cl(A)) $\wedge$ cl(int(A)) $\leq$ F^c, by hypothesis. Hence by Definition 2.11, $(\mathrm{int}(\mathrm{cl}(A) \wedge \mathrm{cl}(\mathrm{int}(A)))_{\bar{q}}$ F.

Sufficiency: Let U be a FOS in X such that A ≤ U.Then U^c is a FCS and A ≤ $(U^c)^c$. Therefore $A_{\bar{q}}\, U^c$. By hypothesis, $A_{\bar{q}} U^c \Rightarrow (\mathrm{int}(\mathrm{cl}(A) \wedge \mathrm{cl}(\mathrm{int}(A)))_{\bar{q}} U^c$. Hence int(cl(A)) ∧ cl(int(A)) ≤ $(U^c)^c$ = U. Therefore int(cl(A)) ∧ cl(int(A)) ≤ U. Hence A is a Fγ*GCS.

Theorem 3.19: If A is both a FOS and a Fγ*GCS then A is a FγCS in (X,τ).

Proof: Let A be a FOS and a Fγ*GCS in (X, τ). Then as A ≤ A, cl(int(A) ∧ int(cl(A)) ≤ A. Hence A is a FγCS in (X, τ).

Theorem 3.20: For a fuzzy set A in (X, τ) the following are equivalent:

i. A is both a FOS and a Fγ*GCS
ii. A is a FROS

Proof: (i) ⟹ (ii) Let A be a FOS and a Fγ*GCS in X. Then by Theorem 3.19, A is a FγCS. So cl(int(A)) ∧ int(cl(A)) ≤ A. We have int(cl(A)) = int(cl(A)) ∧ cl(A) = int(cl(A)) ∧ cl(int(A)) ≤ A. Hence int(cl(A)) ≤ A⟶ (1). Since A is a FOS, it is a FPOS. Hence A ≤ int(cl(A)) ⟶ (2). Therefore from (1) and (2) A= int(cl(A)) and A is a FROS in X.

(ii)⟹(i) Let A be a FROS in X then A = int(cl(A)). Since every FROS is a FOS, As is a FOS in X and A ≤ A, Therefore cl(int(A)) ∧ int(cl(A)) = cl(int(A)) ∧ A = A ∧ cl(A) ≤ A. Hence A is a Fγ*GCS.

Theorem 3.21: Let F ≤ A ≤ X where A is a FOS and a Fγ*GCS in X. Then F is a Fγ*GCS in A if and only if F is a Fγ*GCS in X.

Proof: Necessity: Let F be a Fγ*GCS in A. Let U be a FOS in X and F ≤ U. Then F ≤ A ∧ U and A ∧ U is a FOS in A. Hence $\mathrm{int}_A(\mathrm{cl}_A(F)) \wedge \mathrm{cl}_A(\mathrm{int}_A(F))$ ≤ A ∧ U and by Theorem 3.19, A is a FγCS. Therefore int(cl(A)) ∧ cl(int(A)) ≤ A. Now int(cl(F)) ∧ cl(int(F)) ≤ [int(cl(F)) ∧ cl(int(F))] ∧ [int(cl(A)) ∧ cl(int(A))] ≤ (int(cl(F)) ∧ cl(int(F))) ∧ A = $\mathrm{int}_A(\mathrm{cl}_A(F)) \wedge \mathrm{cl}_A(\mathrm{int}_A(F))$ ≤ A ∧ U ≤ U. That is int(cl(F)) ∧ cl(int(F)) ≤ U, whenever F ≤ U. Hence F is a Fγ*GCS in X.

Sufficiency: Let V be a FOS in A such that F ≤ V. Since A is a FOS in X, V is a FOS in X. Therefore int(cl(F)) ∧ cl(int(F)) ≤ V as F is a Fγ*GCS in X. Thus, $\mathrm{int}_A(\mathrm{cl}_A(F)) \wedge \mathrm{cl}_A(\mathrm{int}_A(F))$ = int(cl(F)) ∧ cl(int(F)) ∧ A ≤ V ∧ A ≤ V. Hence F is a Fγ*GCS in A.

Theorem 3.22: For a Fγ*GCS A in a FTS (X, τ), the following condition hold:

i. A is a FROS then scl(A) is a Fγ*GCS
ii. A is a FRCS then sint(A) is a Fγ*GCS

Proof: (i) Let A be a FROS in (X, τ). Then int(cl(A)) = A. By the definition of semi closure we have scl(A) = A∨ int(cl(A)) = A. Since A is a Fγ*GCS in X, scl(A) is a Fγ*GCS in X.

(ii) Let A be a FRCS in (X, τ). Then cl(int(A)) = A. By the definition of semi interior we have sint(A) = A ∧ cl(int(A)) = A. Since A is a Fγ*GCS in X, sint(A) is a Fγ*GCS in X.

Theorem 3.23: If every fuzzy set in (X, τ) is a Fγ*GCS then FO(X) ≤ FγC(X).

Proof: Suppose that every fuzzy set is a Fγ*GCS in (X, τ). Let U ∈ FO(X) then as U ≤ U and by hypothesis, int(cl(U)) ∧ cl(int(U)) ≤ U. Therefore U ∈ FγC(X). Hence FO(X) ≤ FγC(X).

Theorem 3.24: A fuzzy set A of X is a Fγ*GCS if int(cl(A)) ∧ cl(int(A)) ≤ ker(A).

Proof: Let A be any fuzzy set and let U be any FOS in X such that A ≤ U. By hypothesis int(cl(A)) ∧ cl(int(A)) ≤ ker(A). Since A ≤ U, ker(A) ≤ U. Therefore int(cl(A)) ∧ cl(int(A)) ≤ U and hence A is a Fγ*GCS in X.

Theorem 3.25: If a fuzzy set A of a FTS X is nowhere dense, then A is a Fγ*GCS in X.

Proof: If A is a fuzzy nowhere dense subset, then by Definition 2.12, int(cl(A)) = $\bar{0}$. Let A ≤ U where U is a FOS in X. Then cl(int(A)) ∧ int(cl(A)) = $\bar{0}$ ≤ U and hence A is a Fγ*GCS in X.

Theorem 3.26: Let A be a Fγ*GCS in (X, τ) and $\mu_{\tilde{p}}(x)$ be a fuzzy point such that $\mu_{\tilde{p}}(x)_q(cl(int(A)) \wedge int(cl(A))$. Then $cl(\mu_{\tilde{p}}(x))_q A$.

Proof: Assume that A is a Fγ*GCS in (X, τ) and $\mu_{\tilde{p}}(x)_q(cl(int(A)) \wedge int(cl(A))$.Suppose that $cl(\mu_{\tilde{p}}(x))_{\bar{q}}A$, then $A \le (cl(\mu_{\tilde{p}}(x)))^c$ where $(cl(\mu_{\tilde{p}}(x)))^c$ is a FOS in (X, τ). Then by hypothesis , $cl(int(A)) \wedge int(cl(A)) \le (cl(\mu_{\tilde{p}}(x)))^c = int(\mu_{\tilde{p}}(x))^c \le (\mu_{\tilde{p}}(x))^c$. Therefore$(cl(int(A)) \wedge int(cl(A))_{\bar{q}}(\mu_{\tilde{p}}(x))$, which is a contradiction to the hypothesis. Hence $cl(\mu_{\tilde{p}}(x))_q A$.

Theorem 3.27: If A is a FOS and a Fγ*GCS in (X, τ), then int(A) is a FROS in X.

Proof: Since A is a FOS and a Fγ*GCS in (X, τ), then by Theorem 3.19, A is a FγCS, which implies $int(cl(A)) \wedge cl(int(A)) \le A$. Therefore $int[int(cl(A)) \wedge cl(int(A))] \le int(A)$ which implies $int(cl(int(A)) \le int(A)$. Since A is a FOS, it is a FαOS. Hence $int(A) \le int(cl(int(A))$. Therefore int(A) = int(cl(int(A)). Thus int(A) is a FROS.

IV.FUZZY γ* GENERALIZED OPEN SETS

In this section we have introduced a new type of fuzzy open set called fuzzy γ* generalized open set and studied some of its properties.

Definition 4.1: The complement A^c of a F γ*GCS A in a FTS (X, τ) is called a fuzzy γ* generalized open set (Fγ*GOS in short) in X.

Example 4.2: In Example 3.2, $A=\langle x,(0.5_a, 0.5_b)\rangle$ is a Fγ*GOS in X.

Theorem 4.3: Every FOS, FSOS, FPOS, FROS, FαOS, FγOS, FGOS are Fγ*GOS but not conversely in general.

Proof: Straight forward.

Example 4.4: Obvious from Example 3.4, Example 3.6, Example 3.8, Example 3.10, Example 3.12, Example 3.14, Example 3.16 by taking complement of A in the respective examples.

Theorem 4.5: Let (X, τ) be a FTS. Then for every A ∈ Fγ*GO(X) and for every B ∈ FS(X), $int(cl(A)) \le B \le A \Rightarrow B \in$ Fγ*GO(X) .

Proof: Let A be a Fγ*GOS of X. Let $B \le U$ and U be a FOS in X. As A^c is a Fγ*GCS and $A^c \le B^c \le cl(int(A^c))$ from the hypothesis, B^c is a Fγ*GCS, by Theorem 3.17. This implies B is a Fγ*GOS in X. Hence B ∈ Fγ*GO(X).

Theorem 4.6: If A is a FγCS and a Fγ*GOS in (X, τ), then A is a FγOS in (X, τ).

Proof: Obvious from the Theorem 3.19 by taking complement.

Theorem 4.7: A fuzzy set A of a FTS (X, τ) is a Fγ*GOS if and only if $F \le cl(int(A)) \vee int(cl(A))$ whenever F is a FCS and $F \le A$.

Proof: Necessity: Suppose A is a Fγ*GOS in X. Let F be a FCS, such that $F \le A$.Then F^c is a FOS and $A^c \le F^c$, by hypothesis A^c is a Fγ*GCS. We have $int(cl(A^c)) \wedge cl(int(A^c)) \le F^c$. Therefore $F \le cl(int(A)) \vee int(cl(A))$.

Sufficiency: Let U be a FOS, such that $A^c \le U$. Now $U^c \le A$ and U^c is a FCS in X. Then by hypothesis, $U^c \le cl(int(A)) \vee int(cl(A))$. Therefore $int(cl(A^c)) \wedge cl(int(A^c)) \le U$ and A^c is a Fγ*GCS. Hence A is a Fγ*GOS in X.

Theorem 4.8: A fuzzy set A of a FTS (X,τ) is a Fγ*GOS, then $F \le cl(int(cl(A)))$ whenever F is FCS and $F \le A$.

Proof: Suppose A is a Fγ*GOS in X. Let F be a FCS such that $F \le A$. Then F^c is a FOS and $A^c \le F^c$. By hypothesis A^c is a Fγ*GCS, we have $cl(int(A^c)) \wedge int(cl(A^c)) \le F^c$. Now $int(cl(int(A^c))) = cl(int(A^c)) \wedge int(cl(int(A^c))) \le cl(int(A^c)) \wedge int(cl(A^c)) \le F^c$. Therefore $F \le cl(int(cl(A)))$.

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