Near Skolem Difference Mean Labeling of Special Types of Trees

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Abstract: *Let G be a (p,q)graph and $f: V(G) \to \{1,2,\ldots,p+q-1,p+q+2\}$ be an injection. For each edge $e=uv$, the induced edge labeling f^* is defined as follows:*

$$f^*(e)=\begin{cases}\dfrac{|f(u)-f(v)|}{2} & if\ |f(u)-f(v)|\ is\ even\\ \dfrac{|f(u)-f(v)|+1}{2} & if\ |f(u)-f(v)|\ is\ odd\end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and from $\{1,2,3,\ldots.q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, we investigate near Skolem difference mean labelling of some special types of trees like the banana tree, the coconut tree, the $H-$ graph, the lily graph, the jelly fish graph and the graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ\circ\circ K_{1,n_m}\rangle$.

Key words: *Near Skolem difference mean labeling, Near Skolem difference mean graphs.*

I. INTRODUCTION

All graphs in this paper represent finite, undirected and simple one. The vertex set and the edge set of a graph G are denoted by $V(G)$and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary [1].

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. There are several types of graph labeling and a detailed survey is found in [2]. The notion of kolem difference mean labeling was due to Murugan and Subramanian [3]. It motivates us to define near skolem difference mean labelling.

In this paper, we define near skolem difference mean labeling and show that the banana tree $Bt(n_1, n_2, \ldots\ldots\ldots\ldots.n_m)$, the coconut tree$T(n,m)$, the $H-$ graph of a path P_n, the graph$T\langle K_{1,n_1} \circ K_{1,n_2} \circ\circ\circ K_{1,n_m}\rangle$, the lily graph I_n and the Jelly fish graph J (m,n) are Near skolem difference mean graphs. We use the following definitions in the subsequent section.

Definition 1.1:A lily graph $I_n, n\geq 2$ can be constructed by two-star graphs $2K_{1,n}, n\geq 2$ joining two path graphs $2P_n$,
$n\geq 2$ with sharing a common vertex, that is, $I_n = 2K_{1,n} + 2P_n$.

Definition 1.2:The Jelly fish graph$J(m,n)$ is obtained from a 4-cycle (v_1, v_2, v_3, v_4)together with an edge v_1v_3 and appending m pendent edges to $v_2$and n pendent edges to v_4.

Definition 1.3: The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

Definition1.4:The Banana tree denoted by $Bt(n_1, n_2, \ldots, n_m)(m\ times\ n)$isa graph obtained by connecting a vertex v_0 to one leaf of each of m number of stars.

Definition1.5: The graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ\circ\circ K_{1,n_m}\rangle$ is obtained from the stars $K_{1,n_1}, K_{1n_2}, \ldots, K_{1n_m}$ by joining a leaf of K_{1,n_j} and a leaf of $K_{1,n_{j+1}}$ to a new vertex w_j $(1\leq j\leq m-1)$ by an edge.

Definition 1.6:The coconut tree graph $T(n,m)$is obtained by identifying the central vertex of $K_{1,n}$ with a pendant vertex of the path P_m.

II. MAIN RESULT

Definition 2.1:A graph $G=(V,E)$ with p vertices and q edges is said to have Near skolem difference mean labeling if it is possible to label the vertices x∈V with distinct elements $f(x)$ from $\{1,2,\ldots,p+q-1,p+q+2\}$ in such a way that each edge e = uv , is labeled as$f^*(e)=\frac{|f(u)-f(v)|}{2}$if $|f(u)-f(v)|$ is even and $f^*(e)=\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and from $\{1,2,\ldots\ldots,q\}$. A graph that admits a Near skolem difference mean labeling is called a Near skolem difference mean graph.

Theorem 2.2: *The Jelly fish $J(m,n)$is Near skolem difference mean.*

Proof: Let G be the graph $J(m,n)$.

Let $V(G) = \{u,v,x,y,u_i,v_j \,/\, 1 \le i \le m, 1 \le j \le n\}$ and

$E(G) = \{xu,xv,yu,yv,xy\} \cup \{uu_i/1 \le i \le m\} \cup \{vv_j/1 \le j \le n\}$.

Then $|V(G)| = m+n+4$ and $|E(G)| = m+n+5$

Define:$f: V(G) \to \{1,2,\ldots\ldots,2m+2n+8, 2m+2n+11\}$

as follows:

$f(u) = 2m+2n+11$

$f(v) = 2m+2n+7$

$f(x) = 2m+1$

$f(y) = 2m+3$

$f(u_i) = 2i-1, \quad 1 \le i \le m$

$f(v_j) = 2m+3+2j, \quad 1 \le j \le n.$

Let f^* be the induced edge labeling of f. Then,

$f^*(uu_i) = m+n+6-i, \quad 1 \le i \le m$

$f^*(vv_j) = n+2-j, \quad 1 \le j \le n$

$f^*(xu) = n+5$

$f^*(xv) = n+3$

$f^*(yu) = n+4$

$f^*(yv) = n+2$

$f^*(xy) = 1$

The induced edge labels are all distinct and are $f^*(E(G)) = \{1,2,\ldots,m+n+5\}$.

Hence, from the above labeling pattern, the Jelly fish graph $J(m,n)$ admits Near skolem difference mean labeling.

Example 2.3:*The labeling patterns of $J(4,5)$ and $J(5,6)$ are shown in fig 1 and fig 2 respectively*

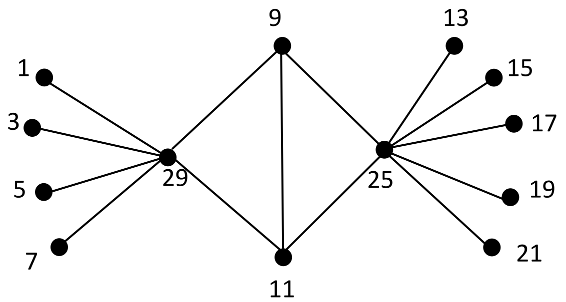


Fig 1

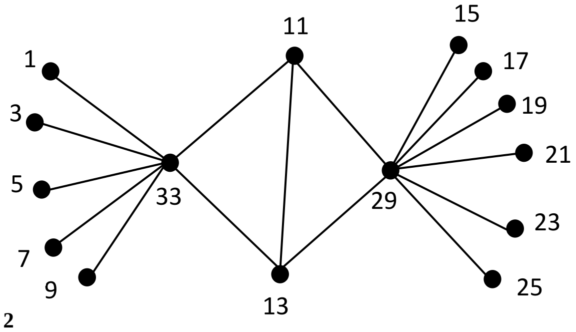


Fig 2

Theorem 2.4:*The Banana tree $Bt(n,n,\ldots\ldots\ldots\ldots.n), (m\ times\ n)$ is a Near skolem difference mean graph.*

Proof: Let the graph be denoted by G

Let $V(G) = \{v_0, v_j, u_{ij} / 1 \le i \le n \ and\ 1 \le j \le m\}$ and

$E(G) = \{v_0u_{1j}, v_ju_{ij} / 1 \le i \le n, 1 \le j \le m\}$.

Then $|V(G)| = mn+m+1$ and$|E(G)| = mn+m$

Define$f: V(G) \to \{1,2,\ldots.,2mn+2m, 2mn+2m+3\}$ as follows:

$f(v_0) = 2mn+2m+3$

$f(v_j) = 2mn-2m+4j, \quad 1 \le j \le m$

$f(u_{1j}) = 2j+1, \quad 1 \le j \le m$

$f(u_{ij}) = (2i-2)m+2j+1, \quad 2 \le i \le n-1\ and\ 1 \le j \le m$

$f(u_{nj}) = 2mn+1+2j, \quad 1 \le j \le m-1$

$f(u_{nm}) = 2mn+2m-2$

Let f^* be the induced edge labeling of f.Then,

$f^*(v_0u_{1j}) = m(n+1)+1-j, \quad 1 \le j \le m$

$f^*(v_ju_{ij}) = m(n-i)+j, \quad 1 \le i \le n-1\ and\ 1 \le j \le m$

$f^*(v_ju_{nj}) = m+1-j, \quad 1 \le j \le m$

Thus, the induced edge labels are all distinct and are $f^*(E(G)) = \{1,2,\ldots\ldots,mn+m\}$.

Hence, the banana tree $Bt(n,n,\ldots\ldots\ldots\ldots.n)$is a Near skolem difference mean graph.

Example 2.5: *The labelling pattern of $Bt(4,4,4,4,4,4,4,4)$ is shown in fig 3*

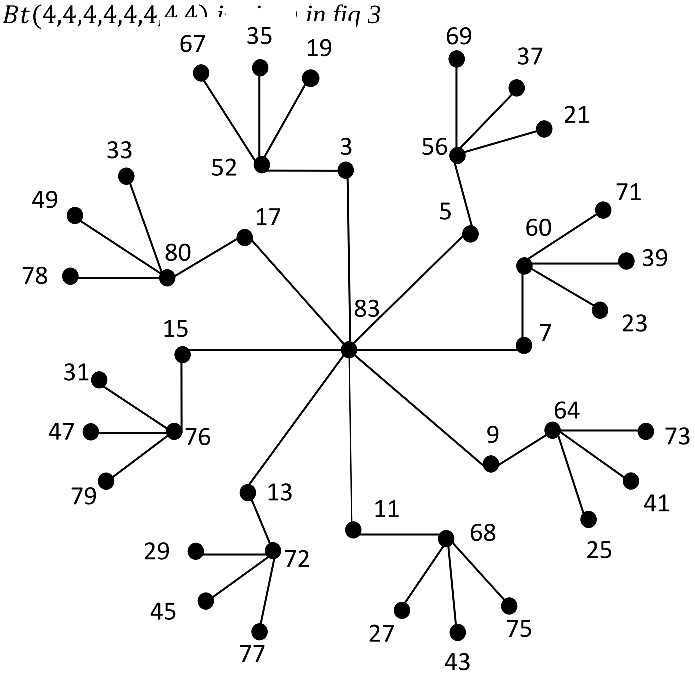


Fig 3

Theorem 2.6:*The lily graph I_n $(n \ge 2)$ admits Near skolem difference mean labeling.*

Proof: Let G be the lily graph I_n with $n \ge 2$.

Let $V(G) = \{u_i, v_j \,/\, 1 \le i \le 2n,\ 1 \le j \le 2n-1\}$ and

$E(G) = \{v_nu_i, v_jv_{j+1} / 1 \le i \le 2n, 1 \le j \le 2n-2\}$

Then $|V(G)| = 4n-1$ and$|E(G)| = 4n-2$.

Define: $f: V(G) \to \{1,2,..,8n-4, 8n-1\}$ as follows:

Case (i)When n is odd:

$f(v_1) = 8n - 1$
$f(v_{2i+1}) = 8n - 2i - 2, \quad 1 \le i \le n - 1$
$f(v_{2i}) = 2i + 1, \quad 1 \le i \le n - 1$
$f(u_i) = 7n - 2i, \quad 1 \le i \le 2n.$

Case(ii) When n is even:

$f(v_{2i+1}) = 3 + 2i, \quad 0 \le i \le n - 1$
$f(v_2) = 8n - 1$
$f(v_{2i}) = 8n - 2i, \quad 1 \le i \le n - 1$
$f(u_i) = 7n + 1 - 2i, \quad 1 \le i \le 2n$

Let f^* be the induced edge labeling of f. Then,

$f^*(v_j v_{j+1}) = 4n - 1 - i, \quad 1 \le i \le 2n - 2$
$f^*(v_n u_i) = i, \quad 1 \le i \le 2n.$

Thus, the induced edge labels are all distinct and are $f^*(E(G)) = \{1,2,\ldots,4n - 2\}$.

Hence, the lily graph admits Near skolem difference mean labeling for $n \ge 2$.

Example 2.7: *The labeling patterns of I_5 and I_6 are given in fig 4 and fig 5 respectively.*

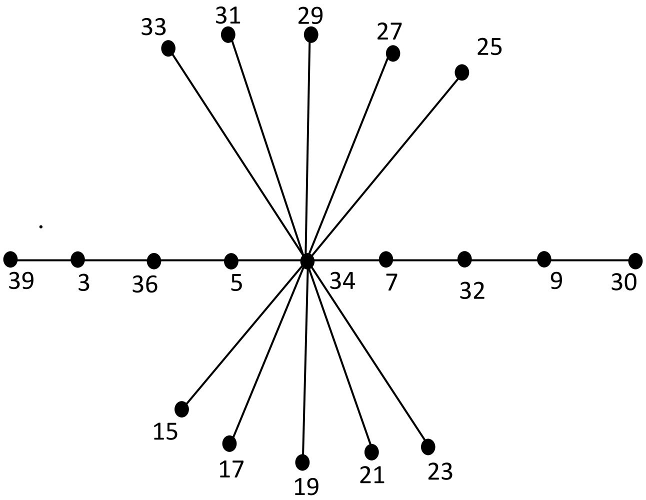


Fig 4

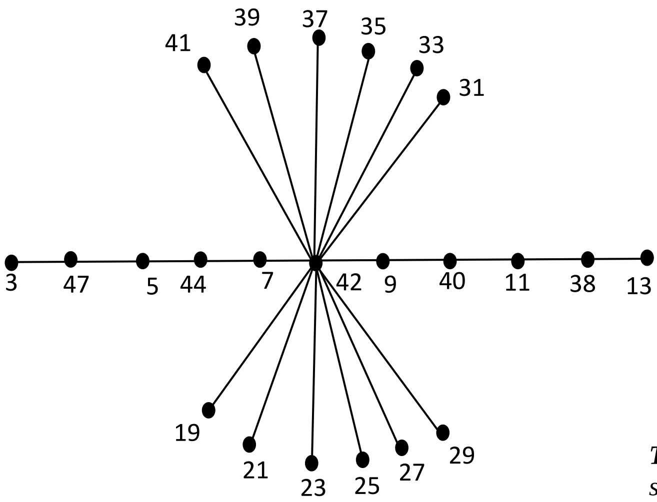


Fig 5

Theorem 2.8: *The H-graph G is Near skolem difference mean graph.*

Proof: Let $V(G) = \{v_i, u_i / 1 \le i \le n\}$ and

$E(G) = \{v_i v_{i+1}, u_i u_{i+1} / 1 \le i \le n - 1\} \cup \{v_{\frac{n+1}{2}} u_{\frac{n+1}{2}} (\text{when } n \text{ is odd}) \text{ and } v_{\frac{n}{2}+1} u_{\frac{n}{2}} (\text{when } n \text{ is even})\}$

Then $|V(G)| = 2n$ and $|E(G)| = 2n - 1$

Define $f: V(G) \to \{1,2,\ldots,4n - 2, 4n + 1\}$ as follows:

Case(i): When n is odd

$f(v_{2i+1}) = 2i + 3, \quad 0 \le i \le \frac{n-1}{2}.$
$f(v_2) = 4n + 1$
$f(v_{2i}) = 4n + 2 - 2i, \quad 2 \le i \le \frac{n-1}{2}.$
$f(u_{2i+1}) = 3n + 1 - 2i, \quad 0 \le i \le \frac{n-1}{2}.$
$f(u_{2i}) = n + 2 + 2i, \quad 1 \le i \le \frac{n-1}{2}.$

Case(ii): When n is even

$f(v_{2i+1}) = 2i + 3, \quad 0 \le i \le \frac{n-2}{2}.$
$f(v_2) = 4n + 1$
$f(v_{2i}) = 4n + 2 - 2i, \quad 2 \le i \le \frac{n}{2}.$
$f(u_{2i+1}) = n + 3 + 2i, \quad 0 \le i \le \frac{n-2}{2}.$
$f(u_{2i}) = 3n + 2 - 2i, \quad 1 \le i \le \frac{n}{2}.$

This f^* be the induced edge labelling of f. Then,

$f^*(v_i v_{i+1}) = 2n - i, \quad 1 \le i \le n - 1$
$f^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) = n$, when n is odd
$f^*\left(v_{\frac{n+1}{2}} u_{\frac{n}{2}}\right) = n$, when n is even
$f^*(u_i u_{i+1}) = n - i, \quad 1 \le i \le n - 1.$

In both cases, the induced edge labels are all distinct and $f^*(E(G)) = \{1,2,\ldots,2n - 1\}$.

Hence, the $H - graph$ G is a Near skolem difference mean graph.

Example 2.9: *The Near skolem difference mean labeling of the graph for $n = 7$ and $n = 6$ are given fig 6 andfig 7 respectively.*

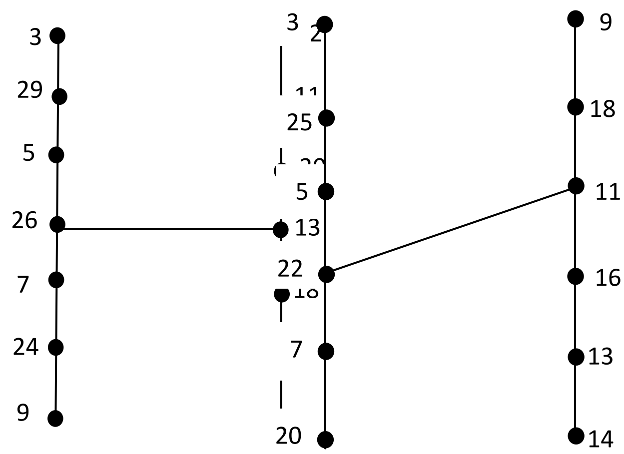


Fig 6 **Fig 7**

Theorem 2.10: *The coconut tree $T(n,m)$ is Near skolem difference mean for every $n, m \ge 1$.*

Proof: Let G be the coconut tree $T(n,m)$.

Let $V(G) = \{v_i, u_j,\ 1 \le i \le n, 1 \le j \le m\}$ and
$E(G) = \{v_i v_{i+1}, v_1 u_j, 1 \le i \le n - 1, 1 \le j \le m\}$
Then $|V(G)| = n + m$ and $|E(G)| = n + m - 1$

Define $f: V(G) \to \{1,2,\dots,2n+2m-2, 2n+2m+1\}$ as follows:

***Case (i)* When n is odd:**

$f(v_{2i+1}) = 3 + 2i, \qquad 0 \le i \le \frac{n-1}{2}$

$f(v_2) = 2n + 2m + 1$

$f(v_{2i}) = 2m + 2n + 2 - 2i, \qquad 1 \le i \le \frac{n-1}{2}$

$f(u_i) = 2 + 2i, \qquad 1 \le i \le m$

***Case (ii)* When n is even:**

$f(v_{2i+1}) = 3 + 2i, \qquad 0 \le i \le \frac{n-2}{2}$

$f(v_2) = 2n + 2m + 1$

$f(v_{2i}) = 2m + 2n + 2 - 2i, \qquad 1 \le i \le \frac{n}{2}$

$f(u_i) = 2 + 2i, \qquad 1 \le i \le m$

In both cases let f^*be the induced edge labeling of f. Then,

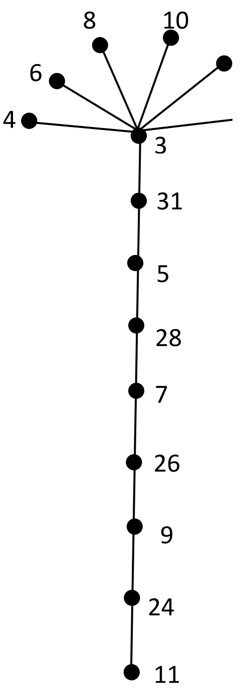
$f^*(v_i, v_{i+1}) = n + m - i, \qquad 1 \le i \le n-1$

$f^*(v_1 u_j) = j, \qquad 1 \le j \le m$

The induced edge labeling are all distinct and are $f^*(E(G)) = \{1,2,\dots,n+m-1\}$.

Hence, the coconut tree is Near skolem difference mean for every $n, m \ge 1$.

Example 2.11: *The labeling pattern of the coconut trees T(9,6) and T(8,8) are given in fig 8 and fig 9 respectively*



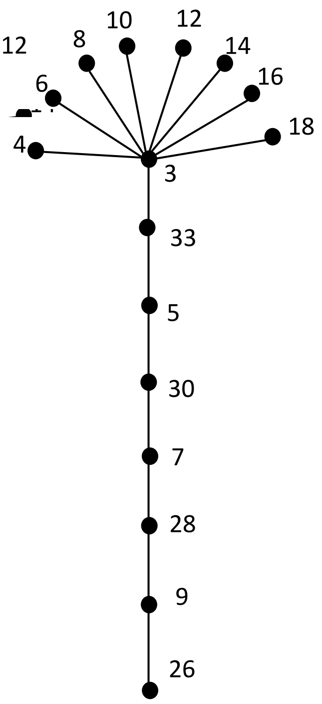


Fig 8Fig 9

Theorem 2.12: *The graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ\circ\circ K_{1,n_m}\rangle$is a Near skolem difference mean graph for every $m > 1$.*

Proof: Let G be the graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ\circ\circ K_{1,n_m}\rangle$.

Let $u_i^j \left(1 \le j \le n_j\right)$ be the pendant vertices andv_j $(1 \le j \le m)$ be the central vertex of the star$K_{1,n_j}$$(1 \le j \le m)$.

Then G is a graph obtained by joining $u_{n_j}^j$ and u_1^{j+1} to a new vertex $w_k (1 \le k \le m-1)$ by an edge.

Then $V(G) = \{u_i^j, v_j, w_k / 1 \le i \le n_j, 1 \le j \le m, 1 \le k \le m-1\}$

and $E(G) = \{v_j u_i^j, w_k u_{n_j}^j, w_k u_1^{j+1} / 1 \le i \le n_j, 1 \le j \le m, 1 \le k \le m-1\}$.

Then $|V(G)| = \sum_{k=1}^m n_k + 2m - 1$ and

$$|E(G)| = \sum_{k=1}^{m} n_k + 2m - 2$$

Define$f: V(G) \to \{1,2,\dots\dots,2\sum_{k=1}^m n_k + 4m - 4,$ *2k=1mnk+4m−1}* as follows:

$f(v_j) = 2\sum_{k=1}^m n_k + 4m + 3 - 4j, \qquad 1 \le j \le m.$

$f(w_j) = 2\sum_{k=1}^m n_k + 4m - 4j, \qquad 1 \le j \le m - 1.$

$f(u_i^1) = 2i + 1, \qquad 1 \le i \le n_1.$

$f(u_i^j) = u_{n_j}^{j-1} + 2i, \qquad 1 \le i \le n_m$ *and* $2 \le j \le m.$

Let f^* be the induced edge labeling. Then,

$f^*(v_1 u_i^1) = \sum_{k=1}^m n_k + 2m - 1 - i, \qquad 1 \le i \le n_1.$

$f^*(v_j u_i^j) = n_j + n_{j+1} + \dots + n_m + 2m - 2j + 1 - i, \qquad 2 \le j \le m$ *and* $1 \le i \le n_j$

$f^*(w_1 u_{n_1}^1) = \sum_{k=1}^m n_k + 2m - 2 - n_1.$

$f^*\left(w_j u_{n_j}^j\right) = n_{j+1} + n_{j+2} + \dots + n_m + 2(m-j), \qquad 2 \le j \le m - 1$

$f^*(w_1 u_1^2) = \sum_{k=2}^m n_k + 2m - 3.$

$f^*(w_j u_1^{j+1}) = n_{j+1} + n_{j+2} + \dots + n_m + 2m - 2j - 1, \qquad 2 \le j \le m - 1.$

The induced edge labels are all distinct and are $f^*(E(G)) = \{1,2,\dots,\sum_{k=1}^m n_k + 2m - 2\}$.

Hence, the graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ\circ\circ K_{1,n_m}\rangle$ is Near skolem difference mean for every $m > 1$.

Example 2.13: *The labeling pattern of $T\langle K_{1,5} \circ K_{1,4} \circ K_{1,6} \circ K_{1,3}\rangle$is shown in fig 10*

III. CONCLUSION

In this paper, we investigated the Near skolem difference meanlabeling of some special types of trees. We have already investigated graphs which are Near Skolem difference mean only for certain cases [4] and have planned to investigate the Near skolem difference labeling of some special cases of cycle related graphs in our next paper.

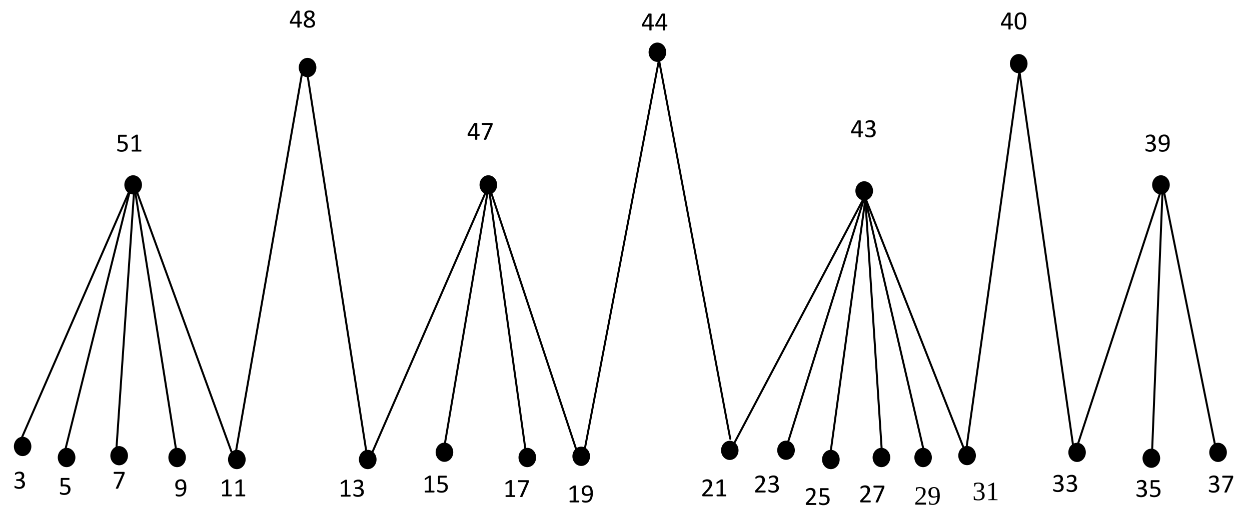


Fig 10

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