Anti-homomorphism of an intuitionistic fuzzy ℓ-ring ideals

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Abstract

In this paper the notion of Intuitionistic fuzzy ℓ-ring ideals(left, right) is introduced and some anti-homomorphism of an intuitionstic fuzzy ℓ-ring ideals(left, right) properties of image have been derived.

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1 Introduction

Zadeh in [23], who was the first reacher initiated the theory of fuzzy sets in 1965. In fuzzy set theory the membership of an element to a fuzzy set is a single value between zero and one. Therefore, a generalization of fuzzy sets was considered by Atanassov 1986 as intuitionistic fuzzy sets which incorporate the degree of membership and non-membership degrees respectively [3]. The concept of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. [1] N. Ajmal and K. V. Thomas discussed the lattice of fuzzy ideals of a ring. [4] K. T. Atanassov proved new operations defined over the intuitionstic fuzzy sets. [5] B. Banerjee and D. K. Basnet introduced the concept of intuitionistic fuzzy subring and ideals. [12] K. Hur, H. W. Kang and H. K. song developed the concept of intuitionistic fuzzy subgroups and subrings. Also K. Hur, SY. Jang and HW. Kang Intuitionistic fuzzy ideals of a ring [11].

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Further, K. Hur, Y. S. Ahn and D. S. Kim [10] extented the lattice of intuitionistic fuzzy ideals of rings. [14] Marashdeh and salleh they have also introduced and studied the notion of intuitionistic fuzzy rings based on the notion of fuzzy space. K. Meena and K. V. thomas [13] discussed the idea about intuitionistic ℓ-fuzzy subring.

Lattice theory has been applied to all kinds of fields. [17] R. Natarajan and S. Moganavalli applied the concept of fuzzy sets to lattice theory. [15] M. Mullai B. Chellappa have introduced fuzzy ℓ-fliter. [20] Also Double representation for an intuitionistic fuzzy ℓ-filter was introduced by KR. sasireka, KE. Sathappan and B. Chellappa. We also established by the properties of intuitionistic fuzzy sub ℓ-ring. [23] G. J. Wang generalized order-homomorphism on fuzzy. [18] D. M. Olson proved on the anti-homomorphism for hemirings. [19] N. Palaniappan K. Arjunan, introduced and studied the anti-homomorphism, anti-homomorphism of a fuzzy and anti-fuzzy ideals. [7] K. Chandrasekhara rao and V. Swaminathan studied in detail anti-homomorphism in near-rings.

In this paper, we define the notion of the intuitionistic fuzzy ℓ-ring ideals(left, right) and have studied Anti-Homomorphism of an intuitionistic fuzzy ℓ-ring ideals.

This paper has been organized as follows: In section 2, some preliminary definitions and examples have been outlined. In section 3, the definition of intuitionistic fuzzy ℓ-ring ideals(left, right) is given and few anti-homomorphism images have been analysed.

2 Preliminaries

In this section, we give some basic definitions. There were two definitions for a lattice, one as a poset and the other as an algebraic structure. A poset (L,$\leq$) is said to form a lattice if for any a, b $\in$ L, Sup{a,b} and inf{ a,b} exist in L. In this case we write Sup{a,b} = a$\vee$b and inf{a,b} = a$\wedge$b. Throughout this paper we denote a lattice with join $'\vee'$ and meet $'\wedge'$ by simply L.

Definition 2.1 *[13] Let X be a non-empty. An intuitionistic fuzzy set A of X is an object of the following from* $A = \{\langle X, \mu_A(x), \nu_A(x)\rangle / x \in X\}$*, where* $\mu_A : X \to [0, 1]$ *and* ν_A*:* $X \to [0, 1]$*, define the degree of membership and the degree of non-membership of the element* $x \in X$*, respectively and* $\forall\ x \in X$*,* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2 *[13] Let L be a lattice and* $A = \{\langle X, \mu_A(x), V_A(x)\rangle / x \in X\}$ *be an intuitionistic fuzzy set of L. Then A is called an intuitionistic fuzzy sublattice(intuitionistic fuzzy lattice) of L if the following conditions are satisfied*

(i) $\mu_A(x \vee y) \geq min\ \{\ \mu_A(x)\ ,\ \mu_A(y)\}$

(ii) $\mu_A(x \wedge y) \geq min\ \{\ \mu_A(x), \mu_A(y)\}$

(iii)$\nu_A(x \vee y) \leq max\ \{\ \nu_A(x), \nu_A(y)\}$

(iv) $\nu_A(x \wedge y) \leq max\ \{\ \nu_A(x), \nu_A(y)\}\quad \forall\ x, y \in L$.

Definition 2.3 *[13] An Intuitionistic fuzzy sublattice A of L is called an intuitionistic fuzzy ideal of L (Intuitionistic fuzzy ℓ-ideal) if the following conditions are satisfied.*

(i) $\mu(x \vee y) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$ *(iii)* $\nu(x \vee y) \leq max\ \{\ \nu(x)\ ,\ \nu(y)\}$

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(ii) $\mu(x \wedge y) \geq max\ \{\ \mu(x)\ ,\ \mu(y)\}$ *(iv)* $\nu(x \wedge y) \leq min\ \{\ \nu(x)\ ,\ \nu(y)\}$ $\forall$ *x*, *y* $\in$ *L*.

Definition 2.4 *[17] A fuzzy subset* μ *of a lattice ordered ring(or* ℓ*-ring in short) R, is called fuzzy sub* ℓ*-ring of R, if the following conditions are satisfied.*
(i) $\mu(x \vee y) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$ *(iii)* $\mu(x - y) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$
(ii) $\mu(x \wedge y) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$ *(iv)* $\mu(xy) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$
for all x, y $\in$ *L.*

Definition 2.5 *[17] A fuzzy subset* μ *of an* ℓ*-ring R, is called a fuzzy* ℓ*-ring ideal (or) fuzzy* ℓ*-ideals of R, if for all x, y* $\in$ *R the following conditions are satisfied.*
(i) $\mu(x \vee y) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$ *(iii)* $\mu(x - y) \geq min\ \{\ \mu(x)\ ,\ \mu(y)\}$
(ii) $\mu(x \wedge y) \geq max\ \{\ \mu(x)\ ,\ \mu(y)\}$ *(iv)* $\mu(xy) \geq max\ \{\ \mu(x)\ ,\ \mu(y)\}$
for all x, y $\in$ *L.*

Definition 2.6 *[16] A mapping f from R to a ring S is called an anti-homomorphism,* $\forall$ *x,y* $\in$ *R*
(i) $f(x+y) = f(x) + f(y)$
(ii) $f(xy) = f(x)\ .\ f(y)$.

Definition 2.7 *[17] Let R and S be two l-rings. A function* $f : R \rightarrow S$ *is called an anti-*ℓ*-homomorphism if for all x, y* $\in$ *R.*
(i) $f(x \vee y) = f(x) \vee f(y)$
(ii) $f(x \wedge y) = f(x) \wedge f(y)$
(iii) $f(x + y) = f(x) + f(y)$
(iv) $f(xy) = f(x).\ f(y)$

Definition 2.8 *[16] Let f be a mapping from a set R to a set S and let A be a fuzzy subset in R. Then A is called f-invarient if f(x) = f(y) implies A(x) = A(y) for all x, y* $\in$ *R. Clearly, if A is f-invarient, then* $f^{-1}(f(A)) = A$.

Definition 2.9 *[13] An intuitionistic fuzzy set A is said to have sup-property[inf-property], if for each subset* $T \subseteq A$ *there exist* $t_0 \in T$ *such that,*

$$\sup_{t \in T}; \{\mu(t)\} = \mu(t_o)$$
$$\inf_{t \in T} \{\nu(t)\} = \nu(t_o)$$

Definition 2.10 *[13] Let f be a mapping from a set X to a set Y and let* μ_A *and* ν_A *be intuitionistic fuzzy subset in X and Y respectively.*
(i) f(A), the image of A under f, is a intuitionistic fuzzy subset in Y for all y$\in$ *Y. We define,*

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & if\ f^{-1}(y) \neq \phi \\ 0 & if\ f^{-1}(y) = \phi \end{cases}$$

and

$$f(\nu_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x), & if\ f^{-1}(y) \neq \phi \\ 1 & if\ f^{-1}(y) = \phi \end{cases}$$

$f^{-1}(B)$ *is the pre-image of B under f, is a intuitionistic fuzzy set in X.*

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ie) $f^{-1}(\mu_A)(x)=\mu_A(f(x))$ *and*

$f^{-1}(\nu_A)(x)=\nu_A(f(x)), \forall\ x \in R$

Definition 2.11 *[20] Let R be a ring. An Intuitionistic fuzzy set* $A = \{\langle\ x, \mu(x), \nu(x)\ \rangle : x \in R\}$ *of R is said to be intuitionistic fuzzy sub* ℓ*-ring on R if for all x, y* $\in R$.

(i) $\mu(x - y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$ (ii) $\mu(xy) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$
(iii) $\mu(x \vee y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$ (iv) $\mu(x \wedge y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$
(v) $\nu(x - y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$ (vi) $\nu(xy) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$
(vii) $\nu(x \vee y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$ (viii) $\nu(x \wedge y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$

Example 2.12 *[17] Now* $(R = a, b, c, d, +, \cdot, \vee, \wedge)$ *is an* ℓ*-ring under the operations* $+, \cdot, \vee,$ *and* $\wedge$ *defined by the following tables,*

3 Theorems on Intuitionistic fuzzy ℓ-ring ideals.

Definition 3.1 *An intuitionistic fuzzy sub* ℓ*-ring A on R is said to be a intuitionistic fuzzy left* ℓ*-ideal if for all x, y* $\in R$.

(i) $\mu(x - y) \geq min\ \{\mu(x), \mu(y)\}$ (ii) $\mu(xy) \geq \mu(y)$
(iii) $\mu(x \vee y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$ (iv) $\mu(x \wedge y) \geq max\ \{\mu(x)\ ,\ \mu(y)\}$
(v) $\nu(x - y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$ (vi) $\nu(xy) \leq \nu(y)$
(vii) $\nu(x \vee y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$ (viii) $\nu(x \wedge y) \leq min\ \{\nu(x)\ ,\ \nu(y)\}$

Definition 3.2 *An intuitionistic fuzzy sub* ℓ*-ring A on R is said to be a intuitionistic fuzzy right* ℓ*-ideal if for all x, y* $\in R$

(i) $\mu(x - y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$ (ii) $\mu(xy) \geq \mu(x)$
(iii) $\mu(x \vee y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$ (iv) $\mu(x \wedge y) \geq max\ \{\mu(x)\ ,\ \mu(y)\}$
(v) $\nu(x - y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$ (vi) $\nu(xy) \leq \nu(x)$
(vii) $\nu(x \vee y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$ (viii) $\nu(x \wedge y) \leq min\ \{\nu(x)\ ,\ \nu(y)\}$

Definition 3.3 *An intuitionistic fuzzy sub* ℓ*-ring A on R is said to be an intuitionistic fuzzy* ℓ*- ring ideal if it is both an intuitionistic fuzzy left* ℓ*- ideal and an intuitionistic fuzzy right* ℓ*- ideal of R.*

Example 3.4 *Consider the Intuitionistic fuzzy* ℓ*-ring ideal R defined in example 2.10*

$$\mu(x) = \begin{cases} \cdot 9 & if\ x = a \\ \cdot 6 & if\ x = b \\ \cdot 4 & if\ x = c, d. \end{cases}$$

$$\nu(x) = \begin{cases} \cdot 1 & if\ x = a \\ \cdot 3 & if\ x = b \\ \cdot 5 & if\ x = c, d. \end{cases}$$

Remark 3.5 *Every Intuitionistic fuzzy* ℓ*-ring ideal R is a Intuitionistic fuzzy sub* ℓ*-ring of R. But converse need not be true.*

Proof: *Consider the intuitionistic fuzzy subset* μ, V *of* ℓ*-ring* $(Z, +, \cdot, \vee, \wedge)$

$$\mu_1(x) = \begin{cases} \cdot 6 & if\ x \in < 2 > \\ \cdot 3 & otherwise \end{cases}$$

$$\nu_1(x) = \begin{cases} \cdot 4 & if\ x \in < 2 > \\ \cdot 7 & otherwise \end{cases}$$

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

.	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

∨	a	b	c	d
a	a	b	c	d
b	b	b	d	d
c	c	d	c	d
d	d	d	d	d

∧	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

Theorem 3.6 *Let f be a anti-homomorphism from a* ℓ*-ring R into* ℓ*-ring S and let* μ_A, ν_A *be an Intuitionistic fuzzy left* ℓ*-ideal of S. Then the pre-image* $f^{-1}(\mu_A)$ *and* $f^{-1}(\nu_A)$ *is a intuitionistic fuzzy right* ℓ*-ideal of R.*

*Proof:*Consider a ℓ-ring anti-homomorphism f : R → S
Let μ_A,ν_A be a Intuitionistic fuzzy left ℓ-ideal of S. ∀ x, y ∈ R.

$$\begin{aligned}
(i) f^{-1}(\mu_A)(x-y) &= \mu_A f(x-y) \\
&\geq min\{\mu_A f(y), \mu_A f(x)\} \\
&\geq min\{f^{-1}(\mu_A)(y), f^{-1}(\mu_A)(x))\} \\
f^{-1}(\mu_A)(x-y) &\geq min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y))\}. \\
(ii) f^{-1}(\mu_A)(xy) &= \mu_A f(xy) \\
&\geq \mu_A f(x) \\
&\geq f^{-1}(\mu_A(x)) \\
f^{-1}(\mu_A)(xy) &\geq f^{-1}(\mu_A(x)) \\
(iii) f^{-1}(\mu_A)(x \vee y) &= \mu_A f(x \vee y) \\
&\geq min\{\mu_A f(y), \mu_A f(x)\} \\
&\geq min\{f^{-1}(\mu_A)(y), f^{-1}(\mu_A)(x))\} \\
&\geq min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y))\} \\
(iv) f^{-1}(\mu_A)(x \wedge y) &= \mu_A f(x \wedge y) \\
&\geq max\{\mu_A f(y), \mu_A f(x)\} \\
&\geq max\{f^{-1}(\mu_A)(y), f^{-1}(\mu_A)(x))\} \\
&\geq max\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y))\}
\end{aligned}$$

$$\begin{aligned}
(v)f^{-1}(\nu_A)(x-y) &= \nu_A f(x-y) \\
&\leq max\{\nu_A f(y), \nu_A f(x)\} \\
&\leq max\{f^{-1}(\nu_A)(y), f^{-1}(\nu_A)(x))\} \\
&\leq max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y))\}. \\
(vi)f^{-1}(\nu_A)(xy) &= \nu_A(f(xy)) \\
&\leq \nu_A f(x) \\
&\leq f^{-1}(\nu_A(x))\} \\
(vii)f^{-1}(\nu_A)(x \vee y) &= \nu_A f(x \vee y) \\
&\leq max\{\nu_A f(y), \nu_A f(x)\} \\
&\leq max\{f^{-1}(\nu_A)(y), f^{-1}(\nu_A)(x))\} \\
&\leq max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y))\}. \\
(viii)f^{-1}(\nu_A)(x \wedge y) &= \nu_A f(x \wedge y) \\
&\leq min\{\nu_A f(y), \nu_A f(x)\} \\
&\leq min\{f^{-1}(\nu_A)(y), f^{-1}(\nu_A)(x))\} \\
&\leq min\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y))\}.
\end{aligned}$$

$\therefore$ $\mathrm{f}^{-1}(\mu_A)$, $\mathrm{f}^{-1}(\nu_A)$ is a intuitionistic fuzzy right ℓ-ideal of R.

Theorem 3.7 *Let $f : R \to S$ be an anti-homomorphism from ℓ-ring R into S. (i) If μ_A, ν_A is a Instuitionistic fuzzy right ℓ-ideal of S , then pre-image $f^{-1}(\mu_A)$, $f^{-1}(\nu_A)$ is a Intuitionistic fuzzy left ℓ-ideal of R.*

Proof: Consider a ℓ-ring homomorphism f : R $\to$ S. Let μ_A, ν_A be a Intuitionistic fuzzy right ℓ-ideal of S, for all x, y $\in$ R.

$$\begin{aligned}
(i)f^{-1}(\mu_A)(x-y) &= \mu_A f(x-y) \\
&\geq min\{\mu_A f(y), \mu_A f(x)\} \\
&\geq min\{f^{-1}(\mu_A)(y), f^{-1}(\mu_A)(x))\} \\
f^{-1}(\mu_A)(x-y) &\geq min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y))\}. \\
(ii)f^{-1}(\mu_A)(xy) &= \mu_A(f(xy)) \\
&\geq \mu_A(f(y)) \\
&= f^{-1}(\mu_A)(y) \\
&\geq f^{-1}((\mu_A)(y)) \\
(iii)f^{-1}(\mu_A)(x \vee y) &= \mu_A f(x \vee y) \\
&\geq min\{\mu_A f(y), \mu_A f(x)\} \\
&\geq min\{f^{-1}(\mu_A)(y), f^{-1}(\mu_A)(x)\} \\
&\geq min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\}. \\
(iv)f^{-1}(\mu_A)(x \wedge y) &= \mu_A(f(x \wedge y)) \\
&\geq max\{\mu_A f(y), \mu_A f(x)\} \\
&\geq max\{f^{-1}\mu_A(y), f^{-1}\mu_A(x))\} \\
&\geq max\{f^{-1}\mu_A(x), f^{-1}\mu_A(y))\}.
\end{aligned}$$

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$$
\begin{aligned}
(v) f^{-1}(\nu_A)(x-y) &= \nu_A f(x-y) \\
&\leq max\{\nu_A f(y), \nu_A f(x)\} \\
&\leq max\{f^{-1}(\nu_A)(y), f^{-1}(\nu_A)(x))\} \\
&\leq max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y))\}. \\
(vi) f^{-1}(\nu_A)(xy) &= \nu_A(f(xy)) \\
&\leq \nu_A f(y) \\
&\leq f^{-1}(\nu_A)(y))\} \\
(vii) f^{-1}(\nu_A)(x \vee y) &= \nu_A f(x \vee y) \\
&\leq max\{\nu_A f(y), \nu_A f(x)\} \\
&\leq max\{f^{-1}(\nu_A(y)), f^{-1}(\nu_A(x))\} \\
&\leq max\{f^{-1}(\nu_A(x)), f^{-1}(\nu_A(y))\}.
\end{aligned}
$$

$$
\begin{aligned}
(viii) f^{-1}(\nu_A)(x \wedge y) &= \nu_A f(x \wedge y) \\
&\leq min\{\nu_A f(y), \nu_A f(x)\} \\
&\leq min\{f^{-1}(\nu_A(y)), f^{-1}(\nu_A(x))\} \\
&\leq min\{f^{-1}(\nu_A(x)), f^{-1}(\nu_A(y))\}.
\end{aligned}
$$

$\therefore$ $f^{-1}(\mu_A)$, $f^{-1}(\nu_A)$ is a Intuitionistic fuzzy left ℓ-ideal of R.

Theorem 3.8 *Let R and S be ℓ-rings and $f : R \to S$ be an anti-homomorphism from ℓ-ring R into S.*
(i) If μ_A, ν_A is a Intuitionistic fuzzy ℓ-ideal of S, then the pre-image $f^{-1}(\mu_A)$, $f^{-1}(\nu_A)$ is a Intuitionistic fuzzy ℓ-ideal of R.

Proof: It's trivial

Theorem 3.9 *Let R and S be ℓ-ring and $f : R \to S$ be an anti-homomorphism from ℓ-ring R into S.*
If μ_A, ν_A is a Intuitionistic fuzzy left ℓ-ideal of a ℓ-ring R with sup property, then the image $f(\mu_A)$, $f(\nu_A)$ is a Intuitionistic fuzzy right ℓ-ideal of a S.

Proof: Consider a ℓ-ring homomorphism f:R $\to$ S. Let μ_A, ν_A be a Intuitionistic fuzzy left ℓ-ideal of R. For all x, y $\in$ R.

$$
\begin{aligned}
(i) f(\mu_A)(f(x)-f(y)) &= f(\mu_A)f(x-y) \\
&\geq min\{\mu_A(y), \mu_A(x)\} \\
&\geq min\{f(\mu_A)(y), f(\mu_A)(x))\} \\
&\geq min\{f(\mu_A)(x), f(\mu_A)(y))\}. \\
(ii) f(\mu_A)(f(x)f(y)) &= f(\mu_A)(f(xy)) \\
&\geq \mu_A(x) \\
&= f(\mu_A)f(x) \\
&\geq f(\mu_A)(f(x)) \\
(iii) f(\mu_A)(f(x) \vee f(y)) &= f(\mu_A)f(x \vee y) \\
&\geq min\{\mu_A(y), \mu_A(x)\} \\
&\geq min\{f(\mu_A)(y), f(\mu_A)(x)\} \\
&\geq min\{f(\mu_A)(x), f(\mu_A)(y)\}.
\end{aligned}
$$

$$\begin{aligned}
(iv) f(\mu_A)(f(x) \wedge f(y)) &= f(\mu_A)(f(x \wedge y)) \\
&\geq max\{\mu_A(y), \mu_A(x)\} \\
&\geq max\{f(\mu_A)(y), f(\mu_A)(x)\} \\
&\geq max\{f(\mu_A)(x), f(\mu_A)(y)\}. \\
(v) f(\nu_A)(f(x) - f(y)) &= f(\nu_A)f(x - y) \\
&\leq max\{\nu_A(y), \nu_A(x)\} \\
&\leq max\{f(\nu_A)(y), f(\nu_A)(x)\} \\
&\leq max\{f(\nu_A)(x), f(\nu_A)(y)\}. \\
(vi) f(\nu_A)(f(x)f(y)) &= f(\nu_A)f(xy) \\
&\leq \nu_A(x) \\
&\leq f(\nu_A)(f(x))\} \\
(vii) f(\nu_A)(f(x) \vee f(y)) &= f(\nu_A)f(x \vee y) \\
&\leq max\{\nu_A(y), \nu_A(x)\} \\
&\leq max\{f(\nu_A)(y), f(\nu_A)(x)\} \\
&\leq max\{f(\nu_A)(x), f(\nu_A)(y)\} \\
(viii) f(\nu_A)(f(x) \wedge f(y)) &= f(\nu_A)(f(x \wedge y)) \\
&\leq min\{\nu_A(y), \nu_A(x)\} \\
&\leq min\{f(\nu_A)(y), f(\nu_A)(x)\} \\
&\leq min\{f(\nu_A)(x), f(\nu_A)(y)\}.
\end{aligned}$$

$\therefore$ f(μ_A), f(ν_A) is a Intuitionistic fuzzy right ℓ-ideal of S.

Theorem 3.10 *Let R and S be ℓ-ring and $f : R \to S$ be an homomorphism from ℓ-ring R into S.*
If μ_A, ν_A is a Intuitionistic fuzzy right ℓ-ideal of R with sup property, then the image f(μ_A), f(ν_A) is a Intuitionistic fuzzy left ℓ-ideal of S.

Proof: Consider a ℓ-ring homomorphism f: R $\to$ S.
Let μ_A, ν_A a Intuitionistic fuzzy right ℓ-ideal of R. For all x, y $\in$ R.

$$\begin{aligned}
(i) f(\mu_A)(f(x) - f(y)) &= f(\mu_A)f(x - y) \\
&\geq min\{\mu_A(y), \mu_A(x)\} \\
&\geq min\{f(\mu_A)(y), f(\mu_A)(x))\} \\
&\geq min\{f(\mu_A)(x), f(\mu_A)(y))\}. \\
(ii) f(\mu_A)(f(x)f(y)) &= f(\mu_A)(f(xy)) \\
&\geq \mu_A(y) \\
&= f(\mu_A)f(y) \\
&\geq f(\mu_A)(f(y)) \\
(iii) f(\mu_A)(f(x) \vee f(y)) &= f(\mu_A)f(x \vee y) \\
&\geq min\{\mu_A(y), \mu_A(x)\} \\
&\geq min\{f(\mu_A)(y), f(\mu_A)(x)\} \\
&\geq min\{f(\mu_A)(x), f(\mu_A)(y)\}.
\end{aligned}$$

$$\begin{aligned}
(iv) f(\mu_A)(f(x) \wedge f(y)) &= f(\mu_A)(f(x \wedge y)) \\
&\geq max\{\mu_A(y), \mu_A(x)\} \\
&\geq max\{f(\mu_A)(y), f(\mu_A)(x)\} \\
&\geq max\{f(\mu_A)(x), f(\mu_A)(y)\}. \\
(v) f(\nu_A)(f(x) - f(y)) &= f(\nu_A) f(x - y) \\
&\leq max\{\nu_A(y), \nu_A(x)\} \\
&\leq max\{f(\nu_A)(y), f(\nu_A)(x)\} \\
&\leq max\{f(\nu_A)(x), f(\nu_A)(y)\} \\
(vi) f(\nu_A)(f(x) f(y)) &= f(\nu_A) f(xy) \\
&\leq \nu_A(y) \\
&\leq f(\nu_A)(f(y))\} \\
(vii) f(\nu_A)(f(x) \vee f(y)) &= f(\nu_A) f(x \vee y) \\
&\leq max\{\nu_A(y), \nu_A(x)\} \\
&\leq max\{f(\nu_A)(y), f(\nu_A)(x)\} \\
&\leq max\{f(\nu_A)(x), f(\nu_A)(y)\}. \\
(viii) f(\nu_A)(f(x) \wedge f(y)) &= f(\nu_A)(f(x \wedge y)) \\
&\leq min\{\nu_A(y), \nu_A(x)\} \\
&\leq min\{f(\nu_A)(y), f(\nu_A)(x)\} \\
&\leq min\{f(\nu_A)(x), f(\nu_A)(y)\}.
\end{aligned}$$

$\therefore$ f(μ_A), f(ν_A) is a intuitionistic fuzzy left ℓ-ideals of S.

Theorem 3.11 *Let R and S be a ℓ-ring and f: $R \to S$ be an anti-homomorphism from ℓ-ring R into S.*
If μ_A, ν_A is an Intuitionistic fuzzy l-ideal of a ℓ-ring R with sup property, then the image f(μ_A), f(ν_A) is a fuzzy ℓ-ideal of S.

Proof: It's trivial.

4 Conclusion

In this paper,the concept of Homomorphism on Intuitionistic fuzzy l-ring ideal of images are introduced. To extend this work, one can investigate the other anti-homomorphism properties.

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