A Finite Single Integral Representation for the Polynomial Set $T_n(x_1,x_2,x_3,x_4)$

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Abstract:*-In the present paper an attempt has been made to express a Finite Single Integral Representation for the Polynomial set $T_n(x_1,x_2,x_3,x_4)$.Many interesting new results may be obtained as particular caseson specializing the parameters.*

Key words: -*Appell function, Lauricella form, Generalized hypergeometric polynomial,* Integral Representation

AMS Subject Classification*: Special function-33*

1. INTRODUCTION

The generalized polynomial set $T_n(x_1,x_2,x_3,x_4)$ is defined by means of generating relation.

$$e^{\lambda_1 t^{m_1}+d_\mu} F\left[\begin{matrix}(A_r);(C_u);(Eg_i) \\ \\ (B_s);(D_v);(Fq_i)\end{matrix} \quad \lambda x_1 t,\ \lambda_2 x_2^{m_2} t^{m_2},\ \lambda_3 x_3^{m_3} t^{m_3},\ \lambda_4 x_4^{m_4} t^{m_4}\right]$$

$$=\sum_{n=0}^{\infty} T_{n,m:m_1:m_2:m_3:m_4:(B_s):(D_v):(Fq_i)}^{\lambda:\lambda_1:\lambda_2:\lambda_3:\lambda_4:(A_r):(C_u):(E_{g_i})}(x_1,x_2,x_3,x_4)t^n \quad \text{...}\ \mathbf{(1.1)}$$

where (i = 1, 2, 3, 4) and $\lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ are real and m, m_1, m_2, m_3, m_4 are positive integer. The left hand side of (1.1) contains the product of two generalized hypergeometric function which contains Appell function of four variables in the notation of Burchanall and Chaundy [1] associated with Lauricella function. The polynomial set contains a number of parameters for simplicity, itis denoted by $T_n(x_1,x_2,x_3,x_4)$, where n is the order of the polynomial set.

After little, simplification (1.1) gives,

$$T_n(x_1,x_2,x_3,x_4)=\sum_{P_1=0}^{\left[\frac{n}{m_1}\right]}\ \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]}\ \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]}\ \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times\frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times\frac{[(C_u)]n-m_1P_1-m_2P_2-m_3P_3-m_4P_4\left[(E_{g_1})\right]_{P_1}\left[(E_{g_2})\right]_{P_2}\left[(E_{g_3})\right]_{P_3}}{[(D_v)]n-m_1P_1-m_2P_2-m_3P_3-m_4P_4\left[(F_{q_1})\right]_{P_1}\left[(F_{q_2})\right]_{P_2}\left[(F_{q_3})\right]_{P_3}}$$

$$\frac{\left[(E_{g_4})\right]_{P_4}\left(\lambda x_1^m\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\lambda_1^{P_1}\left(\alpha_\mu\right)^{P_1}\lambda_2^{P_2}\left(\lambda_3 x_3^{m_3}\right)^{P_3}\left(\lambda_4 x_4^{m_4}\right)^{P_4}}{\left[(F_{q_4})\right]_{P_4}\quad P_1!\quad P_2!\quad x_2^{m_2P_2}\quad P_3!\quad P_4!} \quad \text{...}\mathbf{(1.2)}$$

The polynomial Set $T_n(x_1,x_2,x_3,x_4)$ happens to the generalization of as many thirty four orthogonal and non-orthogonal polynomials.

Notations:

a)

I. (m)=1.2.3…….m

II. $(A_p)=A_1.A_2.A_3\ldots\ldots.A_p$

III. $[(A_p)]=A_1,A_2,A_3\ldots\ldots.A_p$

IV. $[(A_p)]_n=(A_1)_n,(A_2)_n,(A_3)_n\ldots\ldots.(A_p)_n$

V. $\Delta(a,b)=\frac{b}{a},\ \frac{b+1}{a},\ \ldots\ldots\frac{b+a-1}{a}$

b)

I. $M=\frac{[(A_r)]_n[(C_u)]_n\left(\lambda x^m\right)^n}{[(B_s)]_n\,[(D_v)]_n\ n!}$

2. Theorem : For $m_2>1,\ m_3>1$ and $m_4>1$.

$$T_n(x_1,x_2,x_3,x_4)=\frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\,\Gamma(1+a-b-d)\,\Gamma(1+a-c-d)}{\Gamma(d)\,\Gamma(1+a-b)\Gamma(1+a-c)\,\Gamma(1+a-2d)\,\Gamma(1+a-b-c-d)}$$

$$\times M_1\int_0^1 x^{d-1}(1-x)^{a-2d}\,{}_4F_3\left[\begin{matrix}a,1+\frac{a}{2},b,c;\\ \frac{a}{2},1+a-b,1+a-c;\end{matrix}\ x\right]$$

$$\times F^{1+s+v\,:\,g_1\,:\,g_2\,:\,g_3\,:\,g_4\,:1:1}_{r+u\,:\,q_1\,:\,q_2\,:\,q_3\,:\,q_4\,:1:1:1}\left[\begin{matrix}[(-n):m_1,m_2,m_3,m_4];\\ \rule{4cm}{0.4pt};\end{matrix}\right.$$

$$\left[(1-(B_s)-n):m_1,m_2-1,m_3-1,m_4-1\right],\ \left[(1-(D_u)-n):m_1,m_2,m_3,m_4\right];$$
$$\left[(1-(A_r)-n):m_1,m_2-1,m_3-1,m_4-1\right],\ \left[(1-(C_u)-n):m_1,m_2,m_3,m_4\right];$$

$$\left[(E_{g_1}):1\right],\left[(E_{g_2}):1\right],\left[(E_{g_3}):1\right],\left[(E_{g_4}):1\right],\ [(1+a-2d):2],[(1+a-b-c-d):1];$$
$$\left[(F_{q_1}):1\right],\left[(F_{q_2}):1\right],\left[(F_{q_3}):1\right],\left[(F_{q_4}):1\right],[(d):1],\ [(1+a-b-d):],[(1+a-c-d):1];$$

$$\frac{(-1)^{m_1(r+s+u+v+1)}(\lambda_1\alpha_\mu)}{(\lambda x_1^m)^{m_1}},\ \frac{\lambda_2(-1)^{m_2(r+s+u+v+1)+r+s}}{\left(\lambda x_1^m x_2\right)^{m_2}}$$

$$\left.\frac{\lambda_3 x_3^{m_3}(-1)^{m_3(r+s+u+v+1)+r+s}}{\left(\lambda x_1^m\right)^{m_3}},\ \frac{\lambda_4 x_4^{m_4}(-1)^{m_4(r+s+u+v+1)+r+s}}{\left(\lambda x_1^m\right)^{m_4}}\right]dx \cdots \textbf{(2.1)}$$

Proof: We have

$$I_1 = \int_0^1 x^{d-1}(1-x)^{a-2d} \; {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix} \; x\right]$$

$$\times \sum_{P_1=0}^{\left[\frac{n}{m_1}\right]} \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]} \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]} \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times \frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times \frac{[(C_u)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4} \left[(E_{g_1})\right]_{P_1} \left[(E_{g_2})\right]_{P_2} \left[(E_{g_3})\right]_{P_3}}{[(D_v)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4} \left[(F_{q_1})\right]_{P_1} \left[(F_{q_2})\right]_{P_2} \left[(F_{q_3})\right]_{P_3}}$$

$$\times \frac{\left[(E_{g_4})\right]_{P_4} \left(\lambda x_1^m\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4} \lambda_1^{P_1}(\alpha_\mu)^{P_1} \; \lambda_2^{P_2} \left(\lambda_3 x_3^{m_3}\right)^{P_3} \left(\lambda_4 x_4^{m_4}\right)^{P_4}}{\left[(F_{q_4})\right]_{P_4} (n-m_1P_1-m_2P_2-m_3P_3-m_4P_4)! \;\; P_1! \;\; x_2^{m_2P_2} \;\; P_2! \;\; P_3! \;\; P_4!}$$

$$\times \frac{(1+a-2d)_{2P_1} \; (1+a-b-c-d)_{P_1}}{(d)_{P_1} \; (1+a-b-d)_{P_1} \; (1+a-c-d)_{P_1}} dx$$

$$= \int_0^1 x^{d+P_1-1}(1-x)^{a-2d-2P_1} \; {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix} \; x\right]$$

$$\times \sum_{P_1=0}^{\left[\frac{n}{m_1}\right]} \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]} \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]} \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times \frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times\frac{[(C_u)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\left[(E_{g_1})\right]_{P_1}\left[(E_{g_2})\right]_{P_2}\left[(E_{g_3})\right]_{P_3}}{[(D_v)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\left[(F_{q_1})\right]_{P_1}\left[(F_{q_2})\right]_{P_2}\left[(F_{q_3})\right]_{P_3}}$$

$$\times\frac{\left[(E_{g_4})\right]_{P_4}\left(\lambda x_1^{m}\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\lambda^{P_1}\left(\alpha_\mu\right)^{P_1}\ \lambda_2^{P_2}}{\left[(F_{q_4})\right]_{P_4}(n-m_1P_1-m_2P_2-m_3P_3-m_4P_4)!\ \ P_1!\ \ P_2!\ \ x_2^{m_2P_2}}$$

$$\times\frac{\left(\lambda_3x_3^{m_3}\right)^{P_3}\left(\lambda_4x_4^{m_4}\right)^{P_4}\ (1+a-2d)_{2P_1}\ \ (1+a-b-c-d)_{P_1}}{P_3!\ \ P_4!\ \ (d)_{P_1}\ \ (1+a-b-d)_{P_1}\ \ (1+a-c-d)_{P_1}}dx$$

$$=\sum_{P_1=0}^{\left[\frac{n}{m_1}\right]}\ \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]}\ \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]}\ \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times\frac{[(A_r)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{[(B_s)]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times\frac{[(C_u)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\left[(E_{g_1})\right]_{P_1}\left[(E_{g_2})\right]_{P_2}\left[(E_{g_3})\right]_{P_3}}{[(D_v)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\left[(F_{q_1})\right]_{P_1}\left[(F_{q_2})\right]_{P_2}\left[(F_{q_3})\right]_{P_3}}$$

$$\times\frac{\left[(E_{g_4})\right]_{P_4}\left(\lambda x_1^{m}\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4}\left(\lambda_1\alpha_\mu\right)^{P_1}\ \lambda_2^{P_2}}{\left[(F_{q_4})\right]_{P_4}\ (n-m_1P_1-m_2P_2-m_3P_3-m_4P_4)!\ \ P_1!\ \ (x_2^{m_2})^{P_2}\ \ P_2!}$$

$$\times\frac{\left(\lambda_3x_3^{,m_3}\right)^{P_3}\left(\lambda_4x_4^{m_4}\right)^{P_4}\ (1+a-2d)_{2P_1}\ \ (1+a-b-c-d)_{P_1}}{P_3!\ \ P_4!\ \ (d)_{P_1}\ (1+a-b-d)_{P_1}\ \ (1+a-c-d)_{P_1}}$$

$$\times\frac{\Gamma(d+P_1)\ \Gamma(1+a-2d-2P_1)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma(1+a-b-c-d-P_1)}{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \ \Gamma(1+a-b-d-P_1)\ \ \Gamma(1+a-c-d-P_1)}dx$$

$$=\sum_{P_1=0}^{\left[\frac{n}{m_1}\right]}\ \sum_{P_2=0}^{\left[\frac{n-m_1P_1}{m_2}\right]}\ \sum_{P_3=0}^{\left[\frac{n-m_1P_1-m_2P_2}{m_3}\right]}\ \sum_{P_4=0}^{\left[\frac{n-m_1P_1-m_2P_2-m_3P_3}{m_4}\right]}$$

$$\times \frac{\left[(A_r)\right]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}{\left[(B_s)\right]_{n-m_1P_1-(m_2-1)P_2-(m_3-1)P_3-(m_4-1)P_4}}$$

$$\times \frac{[(C_u)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4} \left[(E_{g_1})\right]_{P_1} \left[(E_{g_2})\right]_{P_2} \left[(E_{g_3})\right]_{P_3}}{[(D_v)]_{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4} \left[(F_{q_1})\right]_{P_1} \left[(F_{q_2})\right]_{P_2} \left[(F_{q_3})\right]_{P_3}}$$

$$\times \frac{\left[(E_{g_4})\right]_{P_4} \left(\lambda_1\alpha_\mu\right)^{P_1} \ \lambda_2^{P_2} \left(\lambda_3 x_3^{m_3}\right)^{P_3} \left(\lambda_4 x_4^{m_4}\right)^{P_4}}{\left[(F_{q_4})\right]_{P_4} \ P_1! \ x_2^{m_2P_2} P_2! \ P_3! \ P_4!}$$

$$\times \frac{\left(\lambda x_1^m\right)^{n-m_1P_1-m_2P_2-m_3P_3-m_4P_4} \ (1+a-2d)_{2P_1} (1+a-b-c-d)_{P_1}}{(n-m_1P_1-m_2P_2-m_3P_3-m_4P_4)! \ (d)_{P_1} \ (1+a-b-d)_{P_1}}$$

$$\times \frac{(1+a-b-d)_{P_1} \ \Gamma(d) \ \Gamma(1+a-b) \ \Gamma(1+a-c) \ \Gamma(1+a-2d)}{(1+a-c-d)_{P_1} \ \Gamma(1+a) \ \Gamma(1+a-b-c) \ \Gamma(1+a-b-d)}$$

$$\times \frac{\Gamma(1+a-b-c-d) \ (d)_{P_1} \ (1+a-b-d)_{P_1} \ (1+a-c-d)_{P_1}}{\Gamma(1+a-c-d) \ (1+a-2d)_{2P_1} \ 1+a-b-c-d)_{P_1}}$$

$$= \frac{\mathrm{M}_1 \ \Gamma(\mathrm{d}) \ \Gamma(1+\mathrm{a}-\mathrm{b}) \Gamma(1+\mathrm{a}-\mathrm{c}) \Gamma(1+\mathrm{a}-2\mathrm{d}) \Gamma(1+\mathrm{a}-\mathrm{b}-\mathrm{c}-\mathrm{d})}{\Gamma(1+\mathrm{a}) \ \Gamma(1+\mathrm{a}-\mathrm{b}-\mathrm{c}) \ \Gamma(1+\mathrm{a}-\mathrm{b}-\mathrm{d}) \ \Gamma(1+\mathrm{a}-\mathrm{c}-\mathrm{d})}$$

$$\times \sum_{P_1,\, P_2,\, P_3,\, P_4=0}^{\infty} \frac{\left[1-(B_s)-n\right]_{m_1P_1+(m_2-1)P_2+(m_3-1)P_3+(m_4-1)P_4}}{\left[1-(A_r)-n\right]_{m_1P_1+(m_2-1)P_2+(m_3-1)P_3+(m_4-1)P_4}}$$

$$\times \frac{\left[1-(D_v)-n\right]_{m_1P_1+m_2P_2+m_3P_3+m_4P_4} \left[(E_{g_1})\right]_{P_1} \left[(E_{g_2})\right]_{P_2}}{\left[1-(C_u)-n\right]_{m_1P_1+m_2P_2+m_3P_3+m_4P_4} \left[(F_{q_1})\right]_{P_1} \left[(F_{q_2})\right]_{P_2}}$$

$$\times \frac{\left[(E_{g_3})\right]_{P_3} \left[(E_{g_4})\right]_{P_4} (-n)_{m_1P_1+m_2P_2+m_3P_3+m_4P_4} \left(\lambda_1\alpha_\mu\right)^{P_1}}{\left[(F_{q_3})\right]_{P_2} \left[(F_{q_4})\right]_{P_4} \left(\lambda x_1^m\right)^{m_1P_1} \ P_1!}$$

$$\times \frac{(-1)^{m_1(r+s+u+v+1)P_1} \lambda_2 P_2 (-1)^{\{m_2(r+s+u+v+1)+r+s\}P_2} \left(\lambda_3 x_3^{m_3}\right)^{P_3}}{\left(\lambda x_1^m\right)^{m_2P_2} \ x_2^{m_2P_2} \ P_2! \ P_3!}$$

$$\times \frac{(-1)^{\{m_3(r++s+u+v+1)+r+s\}P_3} \left(\lambda_4 x_4^{m_3}\right)^{P_4} (-1)^{\{m_4(r+s+u+v+1)+r+s\}P_4}}{\left(\lambda x_1^m\right)^{m_3 P_3} \left(\lambda x_1^m\right)^{m_4 P_4} \; P_4!} \quad \ldots (2.2)$$

The single terminating factor $(-n)_{m_1P_1+m_2P_2+m_3P_3+m_4P_4}$ makes all summation in (2.2) runs up to ∞.

$$= \frac{\Gamma(d)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma(1+a-2d)\ \Gamma(1+a-b-d)}{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \Gamma(1+a-b-d)\ \Gamma(1+a-c-d)} T_n\left(x_1, x_2, x_3, x_4\right)$$

The above result is obtained on using the result [5].

$$\int_0^1 x^{d-1}(1-x)^{a-2d} {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix} \; x\right] dx$$

$$= \frac{\Gamma(d)\ \ \Gamma(1+a-2d)\ \Gamma(1+a-b)\ \ \Gamma(1+a-c)\ \ \Gamma(1+a-b-c-d)}{\Gamma(1+a)\ \ \Gamma(1+a-b-c)\ \ \Gamma(1+a-b-d)\ \ \Gamma(1+a-c-d)}$$

provided that $R_e(d),\ R_c(a-2d) > -1$ and $R(b+c+d-a) > -1$.

Particular Cases of (2.1)

(i) If we take $r=0=s=v=g_1=q_1;\ u=1=m=m_1=\alpha_\mu=\lambda=\lambda_1;\ c_1=-\lambda$ **and** $\frac{1}{x}$ **for** x**, in (2.1),we get**

$$G_n(\alpha, \beta, x) = \frac{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \ \Gamma(1+a-b-d)\ \ \Gamma(1+a-c-d)}{\Gamma(a)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma(1+a-2d)\ \Gamma(1+a-b-c-d)}$$

$$\times \frac{(\alpha)_n\ (\beta)_n\ (2x)^n}{(\alpha+\beta)_n\ \ n!} \int_0^1 x^{d-1}\ (1-x)^{a-2d}\ \ {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix} \; x\right]$$

$$F\left[\begin{matrix} -\frac{n}{2}, \frac{-n}{2}+\frac{1}{2}, 1-\alpha-\beta-n,\ \Delta(2; 1+a-2d),\ 1+a-b-c-d; \\ 1-\alpha-n,\ 1-\beta-n,\ 1+a-b-d,\ 1+a-c-d; \end{matrix} \; \frac{-1}{x^2}\right] dx$$

(ii) On making the substitution $r=0=s=u=v=g_1;\ q_1=1 \text{ or } 2;\ \lambda=1=m=m_1=\alpha_\mu;$ $\lambda_1=-1, F_1=u+1,\ F_2=v+1+\frac{u}{2};\ \frac{1}{z}$ for x_1, in (2.1), we get

$$C_n^\nu(x) = \frac{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \Gamma(1+a-b-d)\ \Gamma(1+a-c-d)\ (\nu)_n\ (2x)^n}{\Gamma(d)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \ \Gamma(1+a-2d)\ \Gamma(1+a-b-c-d)}$$

$$\times\frac{1}{n}\int_0^1 x^{d-1}(1-x)^{1-2d}\ {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\ x\right]$$

$$\times F\left[\begin{matrix} -\frac{n}{2}, \frac{-n}{2}+\frac{1}{2}, \Delta(2;1+a-2d), 1+a-b-c-d; \\ 1-\nu-n, 1+a-b-d, 1+a-c-d; \end{matrix}\ \frac{1}{x^2}\right]dx$$

(iii) If we set $r=0=s=g_1;\ u=1=v=q_1=m=m_1=\lambda_1=\alpha_\mu;\ D_1=c, c_1=1,$ $F_1=c;\ \lambda=-1$ and

writing $\frac{1}{y}$ form, in(2.1),we get

$$C_n^{(\lambda)}(\lambda)=\frac{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \Gamma(1+a-b-d)\ \Gamma(1+a-c-d)\ (2\lambda)_n\ x^n}{\Gamma(d)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \ \Gamma(1+a-2d)\ \ n!}$$

$$\int_0^1 x^{d-1}(1-x)^{a-2d}\ \ {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\ x\right]$$

$$F\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2}+\frac{1}{2}, \Delta(2;1+a-2d), 1+a-b-c-d; \\ \alpha+\frac{1}{2}, 1+a-b-d, 1+a-c-d; \end{matrix}\ \frac{x^2-1}{x^2}\right]dx$$

(iv) On taking $r_1=0=s=u=v=g_2=q_2;\ m_2=2=x;\ \lambda_2=-4,\ x_2=1=m=\alpha_\mu$ and writing *x* for x_1 in (2.1), we have

$$g_n^*(x)=\frac{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \Gamma(1+a-b-d)\ \Gamma(1+a-c-d)\ \left(\frac{1}{2}\right)_n\ 3^n\ x^n}{\Gamma(d)\ \ \Gamma(1+a-b)\ \ \Gamma(1+a-c)\ \ \Gamma(1+a-2d)\ \ \Gamma(1+a-b-c-d)\ \ n!}$$

$$\times\int_0^1 x^{d-1}(1-x)^{a-2d}\ \ {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\ x\right]$$

$$\times\ F\left[\begin{matrix} \Delta(3;-n), \Delta(2;\ 2+a-2d), (1+a-b-c-d); \\ \Delta\left(2;\ \frac{1}{2}-n\right), 1+a-b-d,\ 1+a-c-d; \end{matrix}\ \frac{1}{4x^2}\right]dx$$

(v) On putting $r=0=s=u=v-m;\ q_2=1=g_2=x_2=\alpha_\mu,\ E_1=1+\alpha,\ \lambda=2=m_2;$ $\lambda_2=-4$ and x for *x* in (2.1), we achieve.

$$g_n^m(x,h)=\frac{\Gamma(1+a)\ \Gamma(1+a-b-c)\ \Gamma(1+a-b-d)\ \Gamma(1+a-c-d)\ x^n}{\Gamma(d)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma(1+a-2d)\ \Gamma(1+a-b-c-d)}$$

$$\times\int_0^1 x^{d-1}\left(1-x\right)^{a-2d}\ {}_4F_3\left[\begin{matrix} a,1+\frac{a}{2},b,c; \\ \frac{a}{2},1+a-b,\ 1+a-c; \end{matrix}\ x\right]$$

$$\times F\left[\begin{matrix}\Delta(m;-n),\ \Delta(2;1+a-2d),\ (1+a-b-c-d); \\ 1+a-b-d-,\ 1+a-c-d; \end{matrix}\ h\left(\frac{-m}{x}\right)^m\right]dx$$

(vi) On setting $r=0=s=u=v=g_2;\ q_2=1=m=\alpha_\mu=\lambda=\lambda_1=x_2;$

$F_1 = 1;\ m_2 = 2$ and $x_1=\frac{x}{\sqrt{x^2-1}}$ in (2.1), we get

$$P_n\left(m,x,y,\rho,c\right)=\frac{\Gamma(1+a)\ \Gamma(1+a-b-c)\,\Gamma(1+a-b-d)\ \Gamma(1+a-c-d)}{\Gamma(d)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma(1+a-2d)\ \Gamma(1+a-b-c-d)}$$

$$\times\frac{\{-(m-1)\}^{\frac{n(m-1)}{m}}\ (mn)^n C^{P-n}\ (-P)_n}{n!}\int_0^1 x^{d-1}(1-x)^{a-2d}\ {}_4F_3\left[\begin{matrix} a,1+\frac{a}{2},b,c; \\ \frac{a}{2},1+a-b,1+a-c; \end{matrix}\ x\right]$$

$$\times F\left[\begin{matrix}\Delta(m,-n),\ \Delta(2;c+a-2d),\ (1+a-b-c-d); \\ \Delta(m-1;\ 1+p-n);1+a-b-d,\ 1+a-c-d; \end{matrix}\ -\frac{(-1)^m C^{m-1}y}{x^{-n}\{-(m-1)\}^{m-1}}\right]dx$$

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