Anti Fuzzy Meet Semi L-Ideal

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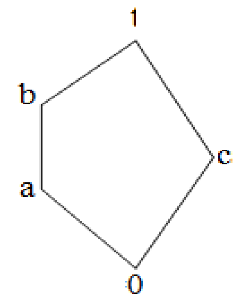
Abstract: *In this paper we made an attempt to define and study some properties of Anti Fuzzy meet semi L-ideal and we introduce some definitions and theorems on the union and intersection of Anti Fuzzy meet semi L-ideal.*

Key words: *Fuzzy meet semilattice, Anti Fuzzy meet subsemilattice,Anti fuzzy meet semi L-ideal,Anti Fuzzy level meet semi L-ideal.*

Introduction: The notion of Fuzzy sets was introduced by Zadeh,L.A.[38] in 1965.He has initiated fuzzy set theory as a modification of ordinary set theory. In this paper we define Anti fuzzy meet semi L-ideal, Anti Fuzzy level meet semi L-ideal and some related theorems.

Definition 1: Let A be a Fuzzy meet semilattice. An Anti fuzzy meet subsemilattice $T_\mu : A \to [0,1]$ is called an anti Fuzzy meet semi L-ideal of A if $\forall x, y \in A,\ T_\mu(x \wedge y) \le \min\{T_\mu(x), T_\mu(y)\}$

Example 1: Let A={0,a,b,c,1}. Let $T_\mu : A \to [0,1]$ be a Fuzzy subset in A defined by $T_\mu(0) = 0.4, T_\mu(a) = 0.7, T_\mu(b) = 0.6,$ $T_\mu(c) = 0.5, T_\mu(1) = 0.9$



Thus, T_μ is an anti fuzzy meet semi L-ideal.

Remark: $T_{\mu t} \subseteq T_{\mu S}$ whenever t>s

Definition 2: Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet semi L-ideals of a fuzzy meet semi lattice A. Then $T_{\mu 1}$ is said to be contained in $T_{\mu 2}$ if $T_{\mu 1}(x) \le T_{\mu 2}(x), \forall x \in A$ and is denoted by $T_{\mu 1} \subseteq T_{\mu 2.}$

Definition 3: Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet semi L-ideals of a fuzzy meet semi lattice A. If $T_{\mu 1}(x) = T_{\mu 2}(x), \forall x \in A$, then $T_{\mu 1}$ and $T_{\mu 2}$ are said to be equal and it is written as $T_{\mu 1} = T_{\mu 2}$.

Definition 4: The complement of a anti fuzzy meet semi L-ideal T_μ of a fuzzy meet semilattice A is symbolized by $\sim T_\mu(x) = 1 - T_\mu(x), \forall x \in A$.

Properties: Let A be a fuzzy meet semilattice and B be a fuzzy meet subset of A. Define $\chi_B : A \to [0,1]$ as $\chi_B(x) = \begin{cases} 1 & if \quad x \in B \\ 0 & if \quad x \notin B \end{cases}$

Then the following properties hold:

1. χ_ϕ and χ_A denote constant functions from A to 0 and 1 respeactively.
2. $T_\mu \cap \chi_\phi = \chi_\phi$.
3. $T_\mu \cup \chi_\phi = T_\mu$.
4. $T_\mu \cap \chi_A = T_\mu$.

5. $T_\mu \cup \chi_A = \chi_A$.

Definition 5: The intersection of two anti fuzzy meet semi L-ideal $T_{\mu 1}$ and $T_{\mu 2}$ of a fuzzy meet semi lattice A is defined as
$[T_{\mu 1} \cap T_{\mu 2}](x) = \min\{T_{\mu 1}(x), T_{\mu 2}(x)\}, \forall x \in A$

Definition 6: The union of two anti fuzzy meet semi L-ideal $T_{\mu 1}$ and $T_{\mu 2}$ of a fuzzy meet semilattice A is defined as
$[T_{\mu 1} \cup T_{\mu 2}](x) = \max\{T_{\mu 1}(x), T_{\mu 2}(x)\}, \forall x \in A$

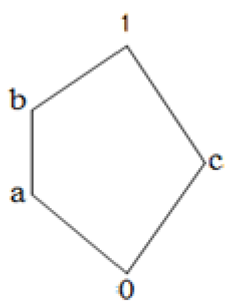
FUZZY LEVEL MEET SEMI L- IDEAL

Definition: 7

Let T_μ be any anti fuzzy meet semi L- ideal of a fuzzy meet semi lattice A and Let $t \in [0,1]$. Then $T_{\mu t} = \{x \in A / T_\mu (x) \geq t\}$ is called anti fuzzy level meet semi L- ideal of T_μ.

Example 2:

Let A = {0, a, b, 1}. Let $T_\mu : A \to [0, 1]$ is a fuzzy meet set in A defined by $T_\mu (0) = 0.7$; $T_\mu (a) = 0.6$; $T_\mu (b) = 0.5$; $T_\mu (1) = 0.4$.



Then T_μ is a anti fuzzy meet semi L-ideal of A.

In this example if t = 0.5, then $T_{\mu t} = T_\mu = \{0,a,b\}$.

Definition 8:

Let T_μ be a anti fuzzy meet semi L-ideal of a fuzzy meet semi lattice A. Then the fuzzy level meet semi L-ideals are defined by

$T_{\mu t} = \{x \in A / T_\mu (x) \geq t\}$

$T_{\mu s} = \{x \in A / T_\mu (x) \geq s\}$

Clearly, $T_{\mu t} \subseteq T_{\mu s}$, whenever t > s.

Lemma 1: Let T_μ be an anti fuzzy meet semil L-ideal of a fuzzy meet semilattice A and let $t,s \in \operatorname{Im} T_\mu$. Then $T_{\mu t} = T_{\mu s}$ if t=s.

Proof:

If t=s, then $T_{\mu t} = T_{\mu s}$.

Conversely,

Let $T_{\mu t} = T_{\mu s}$.

Since $t \in \operatorname{Im} T_\mu$, $\exists$ $x \in A$ such that $T_\mu (x) = t$,

$\Rightarrow t \in T_{\mu s}$

Hence $t = T_\mu (x) \geq s$ -------------- (1)

Similarly,

it can be proved that $s \geq t$----------- (2)

Then from (1) and (2), t=s.

Proposition 1: Let T_μ, T_σ and T_θ be anti fuzzy meet semi L-ideals of a fuzzy meet semilattice A.Then the following are true.

(i) Commutativity:

$$T_\mu \cup T_\sigma = T_\sigma \cup T_\mu$$
$$T_\mu \cap T_\sigma = T_\sigma \cap T_\mu$$

(ii) Associativity:

$$T_\mu \cup [T_\sigma \cup T_\theta] = [T_\mu \cup T_\sigma] \cup T_\theta$$
$$T_\mu \cap [T_\sigma \cap T_\theta] = [T_\mu \cap T_\sigma] \cap T_\theta$$

(iii) Idempotency:

$$T_\mu \cup T_\mu = T_\mu$$
$$T_\mu \cap T_\mu = T_\mu$$

(iv) Distributivity:

$$T_\mu \cup [T_\sigma \cap T_\theta] = [T_\mu \cup T_\sigma] \cap [T_\mu \cup T_\theta]$$
$$T_\mu \cap [T_\sigma \cup T_\theta] = [T_\mu \cap T_\sigma] \cup [T_\mu \cap T_\theta]$$

(v) Identity:

$$T_\mu \cup \chi_\phi = T_\mu \text{ and } T_\mu \cap \chi_\phi = \chi_\phi$$

$$T_\mu \cup \chi_A = \chi_A \text{ and } T_\mu \cap \chi_A = T_\mu$$

(vi) Absorption:

$$T_\mu \cup [T_\mu \cap T_\sigma] = T_\mu$$
$$T_\mu \cap [T_\mu \cup T_\sigma] = T_\mu$$

(vii) Demorgan's Law:

(1) $\sim [T_\mu \cap T_\sigma] = [\sim T_\mu] \cup [\sim T_\sigma]$

(2) $\sim [T_\mu \cup T_\sigma] = [\sim T_\mu] \cap [\sim T_\sigma]$

(viii) Involution:

$$\sim (\sim T_\mu) = T_\mu$$

Remark: Let C be any fuzzy subset of a fuzzy meet semilattice A and T_μ be any anti fuzzy meet semi L-ideal of A. The the following properties hold.

(1) $C \cap (\sim C) = \phi$ and $C \cup (\sim C) = A$

(2) $T_\mu \cap (\sim T_\mu) \neq \chi_\phi$ and $T_\mu \cup (\sim T_\mu) \neq \chi_A$

Theorem 1: Two anti fuzzy meet semi L-ideals T_μ and T_θ of a fuzzy meet semilattice A such that the card Im $T_\mu < \infty$ and card Im $T_\theta < \infty$ are equal iff Im T_μ = Im T_θ and $F_{S\mu} = F_{S\theta}$.

Proof: Let T_μ and T_θ be two anti fuzzy meet semi L-ideals of a fuzzy meet semilattice A such that the card Im $T_\mu < \infty$ and card $T_\theta < \infty$.

Assume that T_μ and T_θ are equal

(ie) $T_\mu(x) = T_\theta(x)$ ------------------------ (1)

$$\Rightarrow T_\mu(x) \in \mathrm{Im}\, T_\mu$$
$$\Rightarrow T_\theta(x) \in \mathrm{Im}\, T_\mu$$
$$But T_\theta(x) \in \mathrm{Im}\, T_\theta$$
$$\Rightarrow \mathrm{Im}\, T_\mu \subseteq \mathrm{Im}\, T_\theta ---------- (2)$$

Similarly ,

it can be proved that $\mathrm{Im}\, T_\theta \subseteq \mathrm{Im}\, T_\mu$ ------- (3)

(2) and (3) $\Rightarrow \mathrm{Im}\, T_\mu = \mathrm{Im}\, T_\theta$ ------- (4)

$$Let T_{\mu t} \in F_{S\mu} and, x \in T_{\mu t}$$
$$\Rightarrow T_\mu(x) \geq t, t \in \mathrm{Im}\, T_\mu$$
$$\Rightarrow T_\theta(x) \geq t, t \in \mathrm{Im}\, T_\theta, by (1) and (4).$$
$$\Rightarrow x \in T_{\theta t}$$

$$\Rightarrow T_{\mu t} \subseteq T_{\theta t}$$

Similarly, it can be proved that $T_{\theta t} \subseteq T_{\mu t}$

$$Hence, T_{\mu t} = T_{\theta t}$$
$$\Rightarrow T_{\theta t} \in F_{S\mu}$$
$$But, T_{\theta t} \in F_{S\theta}$$
$$\Rightarrow F_{S\theta} \subseteq F_{S\mu} ------------- (5)$$
$$Similarly, F_{S\mu} \subseteq F_{S\theta} ---------- (6)$$
$$(5) \,\&\, (6) \Rightarrow F_{S\mu} = F_{S\theta} --------- (7)$$

Equation (4) and (7) completes the proof.

Conversely, assume that $\operatorname{Im} T_{\mu} = \operatorname{Im} T_{\theta}$ and $F_{S\mu} = F_{S\theta}$

To prove: T_{μ} and T_{θ} are equal.

Suppose $T_{\mu}(x) \neq T_{\theta}(x)$ for $x \in A$

Then either $\operatorname{Im} T_{\mu} \neq \operatorname{Im} T_{\theta}$ or $F_{S\mu} \neq F_{S\theta}$.

This is a contradiction

Hence $T_{\mu}(x) = T_{\theta}(x), \forall x \in A.$

Therefore $T_{\mu} and, T_{\theta}$ are equal.

Theorem 2

If J is an anti fuzzy meet semi L- ideal of a fuzzy meet semilattice A, J≠A ,Then T_{μ} is the anti fuzzy meet semi L-ideal of A is defined by

$$T_{\mu}(x) = \begin{Bmatrix} s & if & x \in I \\ t & if & x \in A \sim I \end{Bmatrix}$$

where s,t $\in$[0,1] and s > t.

Proof:

Let x,y$\in$A

To prove: T_{μ} is an anti fuzzy meet semi L-ideal of A.

(ie)To Prove: .$T_{\mu}(x \wedge y) \leq \min\{T_{\mu}(x), T_{\mu}(y)\}$

It is proved by considering exhaustive three cases.

Case(i)

Let x,y$\in$J. $T_{\mu}(x) = s, T_{\mu}(y) = s.$

As x,y$\in J$, $x \wedge y \in J$,since J is a fuzzy meet semi L-ideal of A.

Then x,y $\in$ J $\Rightarrow T_{\mu}(x \wedge y) = s$

$$T_{\mu}(x \wedge y) \leq \min\{T_{\mu}(x), T_{\mu}(y)\}$$

= min{s.s}

=s

Case(ii):

Let x$\in$J,y$\in$A~J. $T_{\mu}(x) = s, T_{\mu}(y) = t$

As x$\in$J,y$\in$A~J, $x \wedge y \in A \sim J$

Now $T_{\mu}(x) = s > t = T_{\mu}(y)$

(ie) $T_{\mu}(x) > T_{\mu}(y)$

$$x \wedge y \in J \Rightarrow T_{\mu}(x \wedge y) = t$$

=min{ s, t}

=t since s>t

$$T_{\mu}(x \wedge y) \leq \min\{T_{\mu}(x), T_{\mu}(y)\}$$

Hence T_{μ} is an anti fuzzy meet semi L ideal.

Case(iii)

Let x, y$\in$A~J, $T_{\mu}(x) = t, T_{\mu}(y) = t$

As x,y$\in$A ~ J, $x \wedge y \in A \sim J$ or J

If x$\wedge y \in A$~J, then

$$T_{\mu}(x \wedge y) = t$$

$\min\{T_{\mu}(x), T_{\mu}(y)\} = \min\{t, t\}$ =t

$$T_{\mu}(x \wedge y) \leq \min\{T_{\mu}(x), T_{\mu}(y)\}$$

Therefore T_{μ} is an anti fuzzy meet semi L ideal of A.

Hence T_{μ} is an anti fuzzy meet semi L ideal of A in all the three cases.

Proposition 2:

A non empty fuzzy meet subset C of A is a fuzzy meet semilattice of A iff χ_C is an anti fuzzy meet semi L ideal of A.

Proof:

χ_C is nothing but characteristic function of the anti fuzzy meet semi L ideal of C.

(ie) $\chi_C(x) = \begin{Bmatrix} 1 & x \in C \\ 0 & x \in A \sim C \end{Bmatrix}$

Where s,t $\in [0,1]$

Then by previous theorem,the proof is complete.

Theorem 3:

The intersection of two anti fuzzy meet semi L ideal of a fuzzy meet semilattice A is also an anti fuzzy meet semi L ideal of A.

Proof:

Let A be a fuzzy meet semilattice.

Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet semi L ideal of A.

T.P: $T_{\mu 1} \cap T_{\mu 2}$ is an anti fuzzy meet semi L ideal of A.

Let a,b $\in T_{\mu 1} \cap T_{\mu 2}$

Then a,b $\in T_{\mu 1}$ and a,b $\in T_{\mu 2}$.

$\Rightarrow a \wedge b \in T_{\mu 1} and, a \wedge b \in T_{\mu 2}$

Hence $a \wedge b \in T_{\mu 1} \cap T_{\mu 2}$.

Therefore $T_{\mu 1} \cap T_{\mu 2}$ is an anti fuzzy meet semi L ideal of A.

Theorem 4: The union of anti fuzzy meet semi L ideal of a fuzzy meet semilattice A is also an anti fuzzy meet semi L ideal of A iff one contained in the other.

Proof:

Let A be a fuzzy meet semilattice.

Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet semi L ideal of A such that one contained in the other.

$T_{\mu 1} \subseteq T_{\mu 2}$ or $T_{\mu 2} \subseteq T_{\mu 1}$

$\Rightarrow T_{\mu 1} \cup T_{\mu 2} = T_{\mu 1}$ or $T_{\mu 1} \cup T_{\mu 2} = T_{\mu 2}$

Then $T_{\mu 1} \cup T_{\mu 2}$ is an anti fuzzy meet semi L ideal of A.

Conversely, suppose $T_{\mu 1} \cup T_{\mu 2}$ is an anti fuzzy meet semi L ideal of A.

T.P: $T_{\mu 1} \subseteq T_{\mu 2}$ or $T_{\mu 2} \subseteq T_{\mu 1}$

Suppose $T_{\mu 1}$ is not contained in $T_{\mu 2}$

Then there exist an element a,b such that

$$a \in T_{\mu 1} and, a \notin T_{\mu 2} ------------(1)$$
$$b \in T_{\mu 2} and, b \notin T_{\mu 1} ------------(2)$$

Clearly, a,b $\in T_{\mu 1} \cup T_{\mu 2}$

Since $T_{\mu 1} \cup T_{\mu 2}$ is an antifuzzy meet semi L ideal of A, $a \wedge b \in T_{\mu 1}$ or $T_{\mu 2}$

Case(i)

Let $a \wedge b \in T_{\mu 1}$

Since a $\in T_{\mu 1}$ and $a' \in T_{\mu 1}$

$$a' \vee (a \wedge b) = (a' \vee a) \wedge b = 1 \wedge b = b \in T_{\mu 1}$$

Which is a contradiction to b $\notin T_{\mu 1,} by(2)$

Case(ii)

Let $a \wedge b \in T_{\mu 2}$

Since $b \in T_{\mu 2} and, b' \in T_{\mu 2}$

Hence
$(a \wedge b) \vee b' = a \wedge (b \vee b') = a \wedge 1 = a \in T_{\mu 2}$

Which is a contradiction to the assumption $a \notin T_{\mu 2} by (1)$

Hence the assumption that $T_{\mu 1}$ is not contained in $T_{\mu 1}$ and $T_{\mu 2}$ is not contained in $T_{\mu 1}$ is false.

Therefore either $T_{\mu 1} \subseteq T_{\mu 2}$ or $T_{\mu 2} \subseteq T_{\mu 1}$.

Theorem 5 :

Let T_θ be any anti fuzzy meet semi L ideal of a fuzzy meet semilattice A Such that Im T_θ = {0, t}, where t$\in$ [0,1]. If $T_\theta = T_\mu \cup T_\sigma$, where T_μ, and T_σ are anti fuzzy meet semi L ideal of A, then either $T_\mu \subseteq T_\sigma$, or $T_\sigma \subseteq T_\mu$.

Proof

Suppose $T\mu \not\subset T_\sigma$ or $T_\sigma \not\subset T\mu$ then there exist some x, y $\in$A such that

$T_\mu(x) > T_\sigma (x)$ and $T_\sigma (y) > T_\mu (y)$

Then $t = T_\theta(x) = [T_\mu \cup T_\sigma] (x)$.

$= \max \{T_\mu, (x), T_\sigma(x)\}$

$= T_\mu (x) \geq 0$, Since $T_\mu (x) > T_\sigma(x)$

and $t = T_\theta (y) = [T_\mu \cup T_\sigma] (y)$

$= \max [T_\mu (y), T_\sigma (y)\}$

$= T_\sigma (y) \geq 0$, Since $T_\sigma (y) > T_\mu (y)$

Therefore $T_\theta(x) = t = T_\sigma (y)$

$\Rightarrow T_\mu (x) = t = T_\sigma (y)$

$\Rightarrow T_\sigma (y) < T_\sigma (x)$ and $T_\mu (x) < T_\mu (y)$

Then $T_\mu (x \wedge y) \leq \min \{T_\mu(x), T_\mu (y)\}$

$= T_\mu (y) < t$ …………………..(1)

and $T_\sigma (x \wedge y) \leq \min \{T_\sigma (x), T_\sigma (y)\}$

$=T_\sigma(x) < t$……………………..…(2)

hence $t = T_\theta(x \wedge y) = [T_\mu \cup T_\sigma] (x \wedge y)$

$= \max \{T_\mu (x \wedge y), T_\sigma (x \wedge y)\}$

$=\max\{T_\mu(y),T_\sigma(x)\}$

$< t$ by (1) and (2) which is a contradiction.

Therefore, if $T_\theta = T\mu \cup T_\sigma$, then either $T_\mu \subseteq T_\sigma$ or $T_\sigma \subseteq T_\mu$.

Proposition : 3

Let A be any fuzzy meet semi lattice. Let I ={T_μ / T_μ is a anti fuzzy meet semi L-ideal of A}. Then for any x $\in$ A, $T_\mu^*(x) = \sup \{k /x \in T_{\mu k} ;T_\mu \in I\}$ is a anti fuzzy meet semi L-ideal.

Proof

Let $T_\alpha = \sup \{k /x \in T_{\mu k}, T_\mu \in I \}$ and let $\in > 0$ be arbitrary.

Then $T_\alpha - \in < \sup \{k / x \in T_{\mu k}, T_\mu \in I\} = k_1$ (say)

$\Rightarrow T_\alpha - \in < k_1$, for some k such that $x \in T_{\mu k1}$

$\Rightarrow T_\alpha - \in < T_\mu(x)$, since $T_\mu (x) \geq k_1$,

$\Rightarrow T_\alpha \geq T_\mu (x)$---------------------------(1)
since $\in > 0$ is arbitrary.

If $T_\mu (x) = t$, then $x \in T_{\mu t}$,and also $t = \sup\{k/x \in T_{\mu k}\}$

$t - \in \leq \sup \{k / x \in T_{\mu k}\}$

Hence $t \leq \sup \{k / x \in T_{\mu k}\}$, since $\in$ is arbitrary.

$\Rightarrow t \leq T_\alpha$ .... (2)

Therefore (1) and (2) $\Rightarrow T_\mu (x) = T_\alpha$

Hence $T_\mu{}^*(x) = T_\mu (x)$, for some $T_\mu \in I$.

Therefore $T_\mu{}^* (x \wedge y) \geq \max \{T_\mu * (x), T_\mu * (y)\}$ for some $T_\mu \in I$.

Theorem : 6

If T_μ and T_α are any two anti fuzzy meet semi L- ideals of a fuzzy meet semi lattice A, then $[T_\mu \wedge T_\alpha](x \wedge y) \leq \min \{ T_\mu (x), T_\alpha (y)\}$

Proof:

Let $T_\theta = T_\mu \wedge T_\alpha$

$T_\theta (x \wedge y) \leq \min \{T_\theta (x), T_\theta (y)\}$

$= \min \{[T_\mu \wedge T_\alpha] (x), [T_\mu \wedge T_\alpha] (y)\}$

$= \min\{\min\{\max\{T_\mu(a), T_\alpha(b)\}\},$
$x=a\wedge b$
$\min\{\max\{T_\mu(c), T_\alpha(d)\}\}\}$
$x=c\wedge b$

$= \min\{\{\max \{T_\mu (a), T_\alpha(b)\}, \{\max \{T_\mu (c), T_\alpha (d)\}\}$

$\leq \min\{ T_\mu (x), T_\alpha (y)\}$, where $x=a\wedge b$, $y = c\wedge b$

Therefore, $[T_\mu \wedge T_\sigma] (x \wedge y) \leq \min \{ T_\mu (x), T_\alpha (y)\}$

Clearly, $T_{\mu t} \subseteq T_{\mu s}$, ,whenever $t > s$.

Theorem: 7

The intersection of two anti fuzzy level meet semi L-ideals of a fuzzy meet semi lattice A is also a anti fuzzy level meet semi L-ideal of A.

Proof :

Let T_μ be a anti fuzzy meet semi L-ideal of a fuzzy meet semilattice A.

Let $T_{\mu t}$, and $T_{\mu s}$ be two anti fuzzy level meet semi L-ideals of a fuzzy meet semilattice A.

Let $x, y \in T_{\mu t} \cap T_{\mu s}$

Then $x, y \in T_{\mu t}$ and $x \in T_{\mu s}$

$\Rightarrow x \wedge y \in T_{\mu t}$

Also $x \wedge y \in T_{\mu s}$

$\Rightarrow x \wedge y \in T_{\mu t} \cap T_{\mu s}$

There $T_{\mu t} \cap T_{\mu s}$ is a anti fuzzy level meet semi L-ideal of A.

Remark :

The intersection of any family of anti fuzzy level meet semi L-ideals of A is also a anti fuzzy level meet semi L-ideal of A.

Theorem : 8

The union of two anti fuzzy level meet semi L-ideals of A is also a anti fuzzy level meet semi L-ideal of A.

Proof:

Let T_μ be a anti fuzzy meet semi L-ideal of A.

Let $T_{\mu t}$ and $T_{\mu s}$ be two anti fuzzy level meet semi L-ideals of A.

Let $x, y \in T_{\mu t} \cap T_{\mu s}$

Then $x, y \in T_{\mu t}$ and $x, y \in T_{\mu s}$

$\Rightarrow x \wedge y \in T_{\mu t} \cap T_{\mu s}$

There $T_{\mu t} \cap T_{\mu s}$ is a anti fuzzy level meet semi L-ideal of A.

Remark :

The union of any family of fuzzy level meet semi L-ideals of A is also a fuzzy level meet semi L-ideal of A.

Theorem : 9

Let A be a fuzzy meet semilattice. Two anti fuzzy level meet semi L-ideals $T_{\mu s}$ and $T_{\mu t}$ (with $t < s$)

of a anti fuzzy meet semi L-ideal T_μ of A are equal iff there if no $x \in A$ such that $s \le T_\mu(x) < t$.

Proof:

Let $T_{\mu s}$ and $T_{\mu t}$ be two fuzzy level meet Semi L- ideals of a fuzzy meet semilattice A, where s<t.

Assume that $T_{\mu s}$ and $T_{\mu t}$ are equal.

To prove : There is no x in A such that $s \le T_\mu(x) < t$.

On the contrary, assume that $s \le T_\mu(x) < t$.for some $x \in A$

$\Rightarrow T_\mu(x) \ge s$ and $T_\mu(x) < t$

$\Rightarrow x \in T_{\mu s}$ and $x \notin T_{\mu t}$

$\Rightarrow T_{\mu s} \neq T_{\mu t}$

This is a contradiction to the assumption.

Hence there is no x in A such that $s \le T_\mu(x) < t$.

Conversely, assume that there is no x in A such that $s \le T_\mu(x) < t$ ……….. (1)

$T_{\mu s} = \{x \in A / T_\mu(x) \ge s\}$ and

$T_{\mu t} = \{x \in A / T_\mu(x) \ge t\}$ and $t > s$

Then $T_{\mu t} \subseteq T_{\mu s}$……………………………………….(2)

It is enough to show that $T_{\mu s} \subseteq T_{\mu t}$

Let $x \in T_{\mu s}$

Then $T_\mu(x) \ge s$

$\Rightarrow T_\mu(x) \ge t$ by (1)

$\Rightarrow x \in T_{\mu t}$

$\Rightarrow T_{\mu s} \subseteq T_{\mu t}$ ……………………………… (3)

Therefore (2) and (3) imply $T_{\mu s} = T_{\mu t}$

Hence the two anti fuzzy level meet semi L- ideals are equal.

Conclution: Thus in this paper, we have defined Anti fuzzy meet semi L ideal,some of its properties and some related theorems.

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