

A Description of the Coupled Schrödinger-KDV Equation of Dusty Plasma

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Abstract:

In this article, we offer a reliable blend of Homotopy analysis method (HAM) and integral transform (Laplace method) to solve nonlinear wave equations. This method has with less computation and all so proposes nonlinear wave- travelling, soliton and shock solution. This method is called homotopy analysis transform method (HATM). This study represents significant character of HATM.

Keywords:

Homotopy analysis method; integral transform (Laplace method); homotopy analysis transform method

INTRODUCTION:

Nonlinear partial differential equations (NPDEs) are broadly used to describe variety of complex phenomena in the field of mathematical physics and engineering, such as fluids dynamics, solid mechanics, propagation of shallow water waves, long wave and chemical reaction-diffusion models, biophysics, quantum field theory and plasma physics etc. However, to solve the NPDEs this method is difficult and time consuming. Powerful and direct methods are being searched by many researchers by that NPDEs can solve easily. In the recent years, to find analytic solutions for NPDEs some methods have proposed, such as the homotopy analysis method [1, 2], three-wave method [3], extended homoclinic test approach [4], the improved F-expansion method [5], the projective Riccati equation method [6], the Weierstrass elliptic function method [7], the tanh-function method [8, 9, 10] and the Jacobi elliptic function expansion method [8, 9]. For integral nonlinear differential equations, the inverse scattering transform method [11], the Hirota method [12], the Exp-function method [13, 14, 15, 16], the truncated Painleve expansion method [17], the extended tanh-method [18, 19], the homogeneous balance method [20, 21, 22] and other various methods [23, 24, 25] are used for searching the exact solutions. Recently, many researchers [26, 27] investigated the exact solutions of the coupled NPDEs, that is, the Maccari system, Higgs field equation etc using Exp-function method and truncated expansion method. Moreover, in different literatures [28, 29, 30], The aim of this paper is to find exact travelling solutions of the coupled Schrödinger-KdV equation by using the homotopy analysis transform method (HATM) which is more useful than the other existing method.

In this task we use the homotopy analysis method combined with the Laplace transform for solving nonlinear wave equations. It is notable that the proposed method is a simple combination of the homotopy analysis method and Laplace transform. The advantage of this proposed method is its capability of combining two powerful methods for obtaining rapid convergent series partial differential equations

BASIC IDEA OF HAM BASIC IDEA OF HAM

The homotopy analysis method (HAM) is an analytical technique for solving nonlinear differential equations. HAM proposed by Liao (Liao 1992) [13], this technique is superior to the traditional perturbation methods in that it leads to convergent series solutions of strongly nonlinear problems, independent of any small or large physical parameter associated with the problem (Liao 2009)[23]. The HAM provides a more viable alternative to non-perturbation techniques such as the Adomian decomposition method (ADM) (Adomian 1976; 1991) [24, 25] and other techniques that cannot guarantee the convergence of the solution series and may be only valid for weakly nonlinear problems (Liao 2009) [23]

In HAM, a system can be written as:

$$M[E(x,t)] = 0 \quad (1)$$

Where M is a nonlinear operator, $E(x,t)$ is unknown function of x and t , $E_0(x,t)$ is the initial guess, $\hbar \neq 0$ an auxiliary parameter and \mathfrak{R} is a auxiliary linear operator. Also $q \in [0,1]$ is an embedding parameter. We can construct a Homotopy as follows

$$(1-q)\mathfrak{R}[\phi(x,t;q) - E_0(x,t)] = q\hbar N[\phi(x,t;q)] \quad (2)$$

when $q = 0$, the zero-order deformation become

$$\phi(x,t;0) = E_0(x,t)$$

when $q = 1$, since $\hbar \neq 0$, we get solution expression as follows

$$\phi(x,t;1) = E(x,t)$$

The embedding parameter q increases from 0 to 1. Using Taylor's theorem, $\phi(x,t;q)$ can be expanded in a power series of q as follows

$$\phi(x,t;q) = E_0(x,t) + \sum_{n=1}^{\infty} E_n(x,t)q^n \quad (3)$$

Where

$$E_n(x,t) = \frac{1}{n!} \left. \frac{\partial^n \phi(x,t;q)}{\partial q^n} \right|_{q=0} \quad (4)$$

If auxiliary linear operators, the initial guesses, the auxiliary parameters, are so properly chosen, then the series (3) converges at $q = 1$ and

$$\phi(x,t;q) = E_0(x,t) + \sum_{n=1}^{\infty} E_n(x,t) \quad (5)$$

Differentiating (2) n times with respect to the embedding parameter q and then setting $q = 0$, we have the so-called n^{th} order deformation equation

$$\mathfrak{R}[\phi(x,t;q) - \lambda_n E_0(x,t)] = \hbar R_n [\vec{E}_{n-1}(x,t)] \quad (6)$$

Using the last equation the series solution is given by

$$E_n(x,t) = \lambda_n E_0(x,t) + \hbar L^{-1} \{ R_n [\vec{E}_{n-1}(x,t)] \} \quad (7)$$

Where

$$R_n [\vec{E}_{n-1}(x,t)] = \frac{1}{(n-1)!} \left. \frac{\partial^{n-1} N[\phi(x,t;q)]}{\partial q^{n-1}} \right|_{q=0} \quad (8)$$

And

$$\lambda_n = \begin{cases} 1 & n > 1 \\ 0 & n \leq 1 \end{cases} \quad (9)$$

HOMOTOPY ANALYSIS TRANSFORM METHOD

We consider a general nonlinear partial differential equation

$$K_i \{E(x,t)\} + \mu_j \{E(x,t)\} + N\{E(x,t)\} = 0 \quad (10)$$

Where K_i is a linear operator $\frac{\partial^i}{\partial t^i}$ ($i=1, 2\dots$), μ_i is a linear operator $\frac{\partial^j}{\partial x^j}$ ($j=0, 1, 2\dots$), and N is a nonlinear operator. The initial conditions are also as

$$E(x,0) = g(x), E_t(x,t) = h(x)$$

Applying the Laplace transforms and we obtain ($i=2$)

$$L\{E(x,t)\} = \frac{g(x)}{p} + \frac{h(x)}{p^2} - \frac{1}{p^2} \left\{ L[N\{E(x,t)\}] + \mu_j \{E(x,t)\} \right\} \quad (11)$$

Now we embed the HAM in Laplace transform method. Hence we may write non linear equation in the form

$$\begin{aligned} N\{E(x,t)\} &= 0 \\ N[\{\phi(x,t;q)\}] &= L\{\phi(x,t;q)\} - \frac{g(x)}{p} - \frac{h(x)}{p^2} + \frac{1}{p^2} \left\{ L[N\{\phi(x,t;q)\}] + \mu_j \{\phi(x,t;q)\} \right\} \end{aligned} \quad (12)$$

Where N is a nonlinear operator, $E(x,t)$ is unknown function of x and t , $\hbar \neq 0$ an auxiliary parameter and \mathfrak{R} is an auxiliary linear operator. Also $q \in [0,1]$ is an embedding parameter

We can construct a Homotopy as follows

$$(1-q)L[\phi(x,t;q) - E_0(x,t)] = q\hbar N[\phi(x,t;q)] \quad (13)$$

when $q = 0$, the zero-order deformation become

$$\phi(x,t;0) = E_0(x,t)$$

since $\hbar \neq 0$, we get solution expression as follows

$$\phi(x,t;1) = E(x,t)$$

The embedding parameter q increases from 0 to 1. Using Taylor's theorem, $\phi(x,t;q)$ can be expanded in a power series of q as follows

$$\phi(x,t;q) = E_0(x,t) + \sum_{n=1}^{\infty} E_n(x,t)q^n \quad (14)$$

Where

$$E_n(x,t) = \left. \frac{1}{n!} \frac{\partial^n \phi(x,t;q)}{\partial q^n} \right|_{q=0} \quad (15)$$

If auxiliary linear operators, the initial guesses, the auxiliary parameters, are so properly chosen, then the series (14) converges at $q = 1$ and

$$\phi(x,t;1) = E_0(x,t) + \sum_{n=1}^{\infty} E_n(x,t) \quad (16)$$

Differentiating (13) n times with respect to the embedding parameter q and then setting $q = 1$ we have the so-called n^{th} order deformation equation

$$L[\phi(x,t;q) - \lambda_n E_0(x,t)] = \hbar R_n [\vec{E}_{n-1}(x,t)] \quad (17)$$

Using the last equation the series solution is given by

$$E_n(x,t) = \lambda_n E_{n-1}(x,t) + \hbar L^{-1} \{ R_n [\vec{E}_{n-1}(x,t)] \} \quad (18)$$

Where

$$R_n[\vec{E}_{n-1}(x, t)] = \frac{1}{(n-1)!} \left. \frac{\partial^{n-1} N\{\phi(x, t; q)\}}{\partial q^{n-1}} \right|_{q=0} \quad (19)$$

and

$$\lambda_n = \begin{cases} 1 & n > 1 \\ 0 & n \leq 1 \end{cases} \quad (20)$$

APPLICATION

In order to elucidate the solution procedure of the homotopy Analysis transform method (HATM), we solve The Coupled Schrödinger-Kdv Equation which shows the effectiveness and generalizations of our proposed method.

$$\begin{aligned} iE_t &= E_{xx} + \eta E \\ \eta_t + 6E\eta_x + \eta_{xxx} &= (|E|^2)_x \end{aligned} \quad (21)$$

with initial condition

$$\begin{aligned} E_0(x, t) &= 6\sqrt{2}k^2 \sec h^2(kx)e^{i\alpha x} \\ \eta_0(x, t) &= \frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(kx) \end{aligned} \quad (22)$$

Where α and k are arbitrary constants.

Applying Laplace transformation on (21) we have,

$$\begin{aligned} L[E(x, t)] + \frac{i}{p} \{L[E_{xx} + \eta E]\} - \frac{1}{p} [6\sqrt{2}k^2 \sec h^2(kx)e^{i\alpha x}] &= 0 \\ L[\eta(x, t)] - \frac{1}{p} \{L[|E|^2]_x - 6E\eta_x - \eta_{xxx}\} - \frac{1}{p} \left[\frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(kx) \right] &= 0 \end{aligned} \quad (23)$$

We define a nonlinear operator according equation (12)

$$\begin{aligned} N_1[\phi(x, t; q)] &= L[\phi(x, t; q)] - \frac{1}{p} [6\sqrt{2}k^2 \sec h^2(kx)e^{i\alpha x}] + \frac{i}{p} \left\{ L \left[\frac{\partial^2 \phi(x, t; q)}{\partial x^2} + \eta \phi(x, t; q) \right] \right\} \\ N_2[\phi_1(x, t; q)] &= L[\phi_1(x, t; q)] - \frac{1}{p} \left[\frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(kx) \right] - \frac{1}{p} \left\{ L \left[\frac{\partial |E|^2}{\partial x} - 6E \frac{\partial \phi_1(x, t; q)}{\partial x} - \frac{\partial^3 \phi_1(x, t; q)}{\partial x^3} \right] \right\} \end{aligned} \quad (24)$$

(24)

Using above definitions, we can construct a Homotopy as follows

$$\begin{aligned} q\hbar N_1[\phi(x, t; q)] &= L[\phi(x, t; q) - E_0(x, t)] \\ q\hbar N_2[\phi_1(x, t; q)] &= L[\phi_1(x, t; q) - \eta_0(x, t)] \end{aligned} \quad (25)$$

Where $q \in [0,1]$, $E_0(x,t), \eta_0(x,t)$ is an initial guess of $E(x,t), \eta(x,t)$ and $\Phi(x,t;q)$ is unknown function. When $q = 0$ and $q = 1$ we have

$$\phi(x,t;0) = E_0(x,t), \phi(x,t;1) = E(x,t), \phi(x,t;0) = \eta_0(x,t), \phi(x,t;1) = \eta(x,t)$$

The n^{th} order deformation equation is

$$E_n(x,t) = \lambda_n E_{n-1}(x,t) + \hbar L^{-1} \{R_n[E_{n-1}(x,t)]\}$$

$$\eta_n(x,t) = \lambda_n \eta_{n-1}(x,t) + \hbar L^{-1} \{R_n[\eta_{n-1}(x,t)]\} \quad (26)$$

Where

$$R_n[E_{n-1}(x,t)] = L[E_{n-1}(x,t)] + \frac{i}{p} \left\{ L \left[\frac{\partial^2 E_{n-1}(x,t)}{\partial x^2} + \eta_{n-1} E_{n-1}(x,t) \right] \right\} - \frac{(1-\lambda_n)}{p} \left[6\sqrt{2} e^{i\alpha x} k^2 \operatorname{sech}^2(kx) \right]$$

$$R_n[\eta_{n-1}(x,t)] = L[\eta_{n-1}(x,t)] - \frac{1}{p} \left\{ L \left[\frac{\partial |E_{n-1}|^2}{\partial x} - 6E_{n-1} \frac{\partial \eta_{n-1}}{\partial x} - \frac{\partial^3 \eta_{n-1}}{\partial x^3} \right] \right\} - \frac{(1-\lambda_n)}{p} \left[\frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(kx) \right]$$

(27)

Obtain the series solution (using Mathematica 5.2 package)

$$E_1(x,t) = -24\sqrt{2}\alpha\hbar k^3 t e^{i\alpha x} \operatorname{sech}^2(kx) \tanh(kx) \\ + 6\sqrt{2}it\hbar e^{i\alpha x} \left[\left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) k^2 \operatorname{sech}^2(kx) + (4 - 6\sqrt{2})k^4 \operatorname{sech}^2(kx) \tanh^2(kx) - 2k^4 \operatorname{sech}^4(kx) \right] \quad (28)$$

$$\eta_1(x,t) = \hbar t k^5 \left[432\sqrt{2}e^{i\alpha x} \operatorname{sech}^4(kx) \tanh(kx) + 240 \operatorname{sech}^4(kx) \tanh(kx) + 48 \operatorname{sech}^2 \alpha x \tanh^2(kx) - 24 \operatorname{sech}^4(kx) \right] \quad (29)$$

$$E_2(x,t) = -3\sqrt{2}t^2\hbar e^{i\alpha x} \left[-\alpha^2 k^2 \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) \operatorname{sech}^2(kx) + \left(-12k^4 \alpha^2 + \frac{4}{3}k^4 \alpha + \frac{64}{3}k^6 \right) \operatorname{sech}^2(kx) \tanh(kx) + i \operatorname{sech}^2(kx) \tanh(kx) \right. \\ \left. \left(\frac{16}{3}\alpha^2 k^3 - \frac{2}{3}\alpha^2 k^2 - \frac{64}{3}\alpha k^5 + 2\alpha^3 k^3 \right) - 2(8 - \sqrt{2})\alpha^2 k^4 \operatorname{sech}^2(kx) \tanh^2(kx) + \left(12k^4 \alpha^2 - \frac{2}{3}\alpha k^4 - \frac{32}{3}k^6 \right) \operatorname{sech}^4(kx) - 8\alpha k^5 (5 - 3\sqrt{2}) \right. \\ \left. \operatorname{sech}^2(kx) \tanh^3(kx) + 24(3 - \sqrt{2})\alpha k^5 \operatorname{sech}^4(kx) \tanh(kx) + 8(2 - 3\sqrt{2})k^6 \operatorname{sech}^2(kx) \tanh^4(kx) + 4(-22 + 21\sqrt{2}) \operatorname{sech}^4(kx) \tanh^2(kx) + \right. \\ \left. 2k^6(8 - \sqrt{2}) \operatorname{sech}^6(kx) \right] + \sqrt{2}t^3 k^5 \hbar \left[80k^2 \alpha - 240\alpha^2 k^2 + 1280k^4 + 432\sqrt{2} \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) k^2 e^{i\alpha x} \right] \operatorname{sech}^6(kx) \tanh(kx) + \\ \left. \left(16k - 48\alpha^2 + 256k^2 \right) k^2 \operatorname{sech}^4(kx) \tanh^2(kx) - (8\alpha - 24\alpha^2 + 128k^2) \operatorname{sech}^6(kx) + 4k^3 (48k + 36\sqrt{2}k) \operatorname{sech}^4(kx) \tanh^2(kx) - 4k^3 i \right. \\ \left. (240\alpha + 432\sqrt{2}\alpha e^{i\alpha x}) \operatorname{sech}^6(kx) \tanh^3(kx) - 96k^4 [10 - 15\sqrt{2} + 9\sqrt{2}e^{i\alpha x}] (2 - 3\sqrt{2}) \operatorname{sech}^6(kx) \tanh^2(kx) + 96k^4 (5 + 9\sqrt{2}e^{i\alpha x}) \operatorname{sech}^8(kx) \tanh(kx) \right]$$

$$\begin{aligned}
 & \eta_2(x, t) = -12t^3\hbar \left\{ -8k^5 \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right)^2 \sec h^4(kx) \tanh(kx) - 128k^7\alpha^2 \sec h^4(kx) \tanh^3(kx) - 96\sqrt{2}k e^{i\alpha x} \right. \\
 & \left. \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) \sec h^4(kx) \tanh^3(kx) + 16[\alpha + 16k^2 + \alpha^2 - 864\alpha e^{2i\alpha x} \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) - 768\sqrt{2}k^2\alpha e^{i\alpha x} \right. \\
 & \left. (5 + 9\sqrt{2}e^{i\alpha x})] \sec h^6(kx) \tanh(kx) + 3072\sqrt{2}k e^{i\alpha x} \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) \sec h^6(kx) \tanh(kx) - 64k^9(11 - 6\sqrt{2}) \sec h^4(kx) \right. \\
 & \left. \tanh^5(kx) + 192\sqrt{2}k^{10}ie^{i\alpha x}(2 - 3\sqrt{2}) \sec h^4(kx) \tanh^5(kx) + 32k^9[(2 - 3\sqrt{2})(5 - 6\sqrt{2}) - 54\alpha e^{2i\alpha x}(2 - 3\sqrt{2})] \sec h^6(kx) \right. \\
 & \left. \tanh^3(kx) + 192\sqrt{2}ik^9e^{i\alpha x}(4 - 6\sqrt{2} + k) - 32k^9[(4 - 3\sqrt{2}) - 6\alpha e^{i\alpha x}\sqrt{2}(5 + 9\sqrt{2}e^{i\alpha x}) - 54\sqrt{2}\alpha e^{2i\alpha x}] \sec h^8(kx) \tanh(kx) \right. \\
 & \left. - 576\sqrt{2}ik^{10}ie^{i\alpha x} \sec h^8(kx) \tanh(kx) - 1728\sqrt{2}k^8\alpha^2 e^{2i\alpha x} \sec h^6(kx) \tanh^2(kx) - 192\sqrt{2}ik^8e^{i\alpha x} \left[\left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) \right. \right. \\
 & \left. \left. (5 + 9\sqrt{2}e^{i\alpha x}) + 4k\alpha \right] \sec h^6(kx) \tanh^2(kx) + 48\sqrt{2}ik^8e^{i\alpha x} \left(\frac{\alpha}{3} - \alpha^2 + \frac{16k^2}{3} \right) (5 + 9\sqrt{2}e^{i\alpha x}) \sec h^8(kx) - 384\sqrt{2}k^9\alpha e^{i\alpha x} \right. \\
 & \left. \sec h^4(kx) \tanh^4(kx) - 384\sqrt{2}ik^{10}e^{i\alpha x}(2 - 3\sqrt{2})(5 + 9\sqrt{2}e^{i\alpha x}) \sec h^6(kx) \tanh^4(kx) + 144\sqrt{2}i\alpha^{10}e^{i\alpha x}(2 - 3\sqrt{2}) \right. \\
 & \left. (5 + 9\sqrt{2}e^{i\alpha x}) \sec h^8(kx) \tanh^2(kx) - 144\sqrt{2}i\alpha^{10}e^{i\alpha x}(5 + 9\sqrt{2}e^{i\alpha x}) \sec h^{10}(kx) \right\} - 6k^5t^5\hbar \left\{ -3072k^3(5 + 9\sqrt{2}e^{i\alpha x}) \right. \\
 & \left. \sec h^4(kx) \tanh^4(kx) + 6336k^3(5 + 9\sqrt{2}e^{i\alpha x}) \sec h^6(kx) \tanh^2(kx) + 4992k^3 \sec h^4(kx) \tanh^3(kx) + 21504i\alpha k^2 e^{i\alpha x} \right. \\
 & \left. - 64k(51k^2 + 58\sqrt{2}k\alpha e^{i\alpha x}) \sec h^6(kx) \tanh(kx) - 864ik(9\sqrt{2}\alpha e^{i\alpha x} + 7ke^{i\alpha x}) \sec h^6(kx) \tanh(kx) - 96k^3 \sec h^8(kx) \right. \\
 & \left. (35 + 63\sqrt{2}e^{i\alpha x}) - 432\sqrt{2}i\alpha^3 e^{i\alpha x} \sec h^4(kx) \tanh(kx) + 10368\sqrt{2}\alpha^3 e^{i\alpha x} \sec h^4(kx) \tanh^2(kx) - 432\sqrt{2}\alpha^2 e^{i\alpha x}(k + 2) \right. \\
 & \left. \sec h^6(kx) - 384k^3 \sec h^2(kx) \tanh^5(kx) \right\}
 \end{aligned} \tag{30}$$

(31)

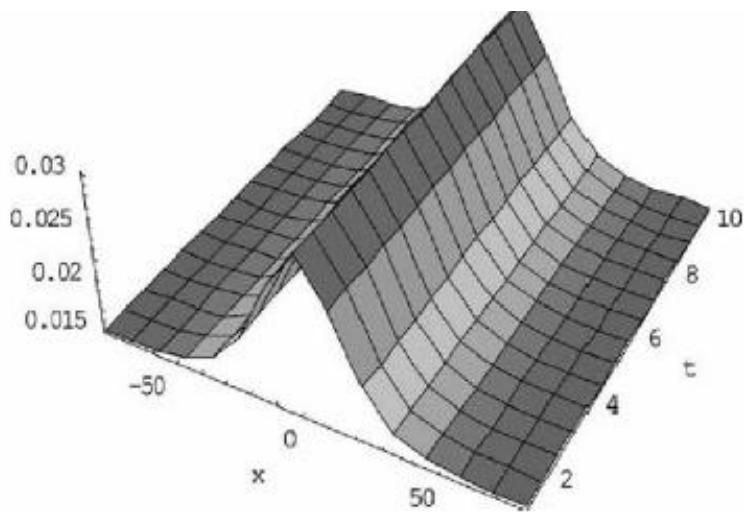
and so on, the other components of the series (15) can be determined in a similar way. Substituting the values in series (15) which is a Taylor series we get the exact solution. In the same manner and for the higher value of n, we obtain the closed form solutions which are same [30].

The solution is

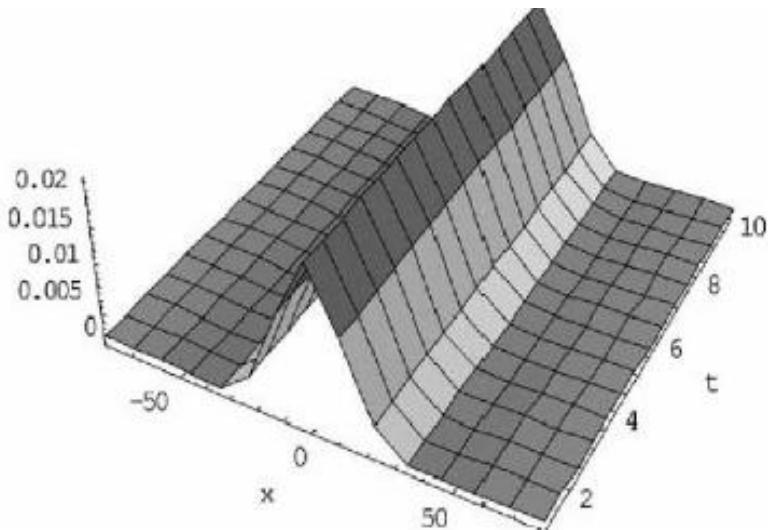
$$E(x, t) = 6\sqrt{6}e^{i\theta}k^2 \sec h(k(x + 2\alpha t))$$

$$\eta(x, t) = \frac{\alpha + 16k^2}{3} - 6k^2 \tanh^2(k(x + 2\alpha t))$$

$$\text{Where } \hbar = -1, \theta = \left(\frac{\alpha t}{3} + t\alpha^2 - \frac{10k^2 t}{3} + x\alpha \right)$$



(a) Numerical results for $E_2(x,t)$ with a fixed values of $\alpha = 0.05$, $k = 0.05$ and for different values of time t .



(b) Numerical results for $\eta_2(x,t)$ with a fixed values of $\alpha = 0.05$, $k = 0.05$ and for different values of time t

CONCLUSIONS

In this paper, the homotopy analysis transform method (HATM) is successfully applied to solve many nonlinear wave problems. It is apparently seen that HATM is very powerful and efficient technique in finding analytical solutions for broadly class of problems. They also do not require large computer memory and discretization of variable x.

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