Effect of Human Behaviour in Spreading of Disease

Pravitha V. R.*

* *Research Scholar, Manonmaniam Sundaranar University,*
Thirunalveli, Tamil Nadu, India

Abstract — *In this paper we discuss the variation in the spreading of disease that can be caused by behavioural variations in human being. We suggest some ways to incorporate the behavioural variations in the models. Using some examples we explain how the new models can be used to control the spreading of diseases that spread by human contact.*

Keywords — *Mathematical epidemiology, Contagious diseases, Fuzzy Sets, Fuzzy graphs, SIS models, Fuzzy SIS models.*

I. Introduction

Kermack - McKendrick models are very famous in the field of mathematical epidemiology [2]. It mainly divides the population under threat into different compartment. Susceptible (S), Infected (I) and Recovered (R) are the main compartments. So these models are also called compartmental models. Compartmental models are mainly used to study the dynamics of diseases that spread by human contact. SIS models and SIR models are the compartmental models which are very basic. SIS models are used to represent the diseases which do not provide immunity to the infected. Some diseases that provide immunity to disease can be modeled by SIR models.

These models are developed based on some simplifying assumptions. One of the assumptions is that the contacts between the members of the population are uniform. This assumption may lead to wrong conclusions when the disease strictly follows the path determined by the human contacts in the network of members in a society. When the disease spreads in a society, the members of the society responds to it in different ways. Some members may break all contacts with the diseased parsons to avoid the chance of getting affected by the disease. Some people may continue indifferent. It is possible to find people who come forward to help the diseased persons. This may increase the rate of contact of such people and causing the infection of the person. In short the response of the members of the population is of varying nature, which may add complexity to the situation. In order to study the spreading process of the disease in the above mentioned situation we need some modified models in which the variations in the human behaviour also is given due consideration.

Network of human contacts is the matter of serious concerns in the mathematical study of epidemics [1]. Each network has its own topology. The topology is decided by the connections that exist in a network. There are different types of topologies. Tree structure, cyclic structure, star structure, multi partite structure, small world topology, scale free topology random graph structure etc some important network structures that need be considered in the study of epidemics [3]. In the next section we initiate a study to incorporate human behaviour in the study of disease.

II. Modeling Human Behaviour

Mathematical theory of fuzzy sets was introduced by L. A. Zadeh [4] in 1960's. This concept is perved best to handle vagueness and imprecise concepts in mathematics. Fuzzy set expresses every subset of a universal set together with a membership function, which is defined on the universal set.

We denote the network model representing the population and their inter connections by the graph $G_t = (V_t, E_t)$, where V_t represent the members of the population and E_t represent personal contacts at the time point ***t***. A membership function $f_t : V_t \to [0,1]$. is defined to represent the behaviour of the person $v \in V_t$. In the beginning of each time period we calculate the function values. It is assumed that the function values of f is not a constant. It can vary depending on time and the member to which it belong. Each time we calculate a new function value ψ_t for each pair of vertices $(u,v)\in E$ so that these function values also belong to $[0,1]$. This ensures that the second function defines a fuzzy subset of the edge set E. Thus f_t, ψ_t defines a fuzzy graph on G. Clearly $G_t = (f_t, \psi_t)$ defines a fuzzy graph for all t. Additionally we assume that that $\psi_t(u,v) = \min(f(u), f(v))$. In the following section we analyze the possible dynamics of a network taking the Karate club as an example (Figure 2.1.).

In the social network of Karate club, there are 34 nodes. These nodes are connected with edges. The network is famous for clashes between two of its members, labeled 34 and 1. Whole network is divided into two groups. The groups were under control of the members 1 and 34. Latter the group broke and separated forever. This is the end of the story. But we develop our analysis by assuming that a disease is introduced into the group.

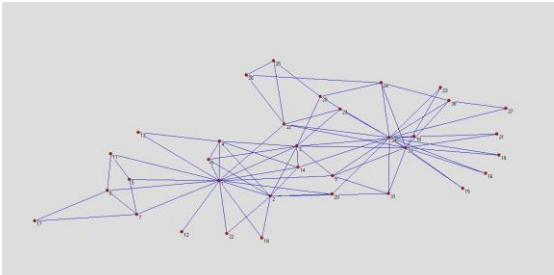

Figure: 2.1. Karate Club Network.

We need more information about the network in order to make the analysis meaningful. In the network there are 34 nodes and 78 edges (ties). Average degree is 4.5882. Maximum degree in the graph is 17 and minimum degree is 1. Total number of triangles is 45 and total number of squares is 154. Diameter is 5 edges [5].

There are two big communities in the network. One is centered around the node 34 and the other is centered around the node 1. Some smaller communities also exist in the network. There exists communities with center at the nodes 2, 3, 8, 33 etc. The nodes 24, 25, 28 and 32 form a community. Nodes 6, 7 and 17 form a small community. The nodes 3, 9, 14, 20, 28, 29, 31, 32, 33 etc are particularly important. Because they have a key role in the network by connecting to the members of the two major communities in the network. They can play the role of mediators at the time of clashes. In the spreading of diseases such nodes become very crucial. Epidemic that is spreading in one community can pass on to the other community only through these mediators.

First suppose either the node 1 is infected. If the contacts in the community continue for a long time the disease will be transmitted to all nodes in the community. At this level the nodes having common connections with both communities become very critical. Disease will reach the other community through theses nodes. Breaking these contacts is very important for the healthy existence of the whole network.

We define, $N^r(v)$ the set of all nodes in the network, which are exactly at a distance of r from the fixed node v. It can be called the rth neighbourhood set of the node v. Starting from the vertex 1, the neighbourhood sets in the network are

$N^1(v) = \{2,3,4,5,6,7,8,9,11,12,13,14,18,20,22,32\}$. $N^2(v) = \{10,15,17,25,26,28,29,31,33,34\}$.

$N^3(v) = \{16,19,21,23,24,27,30\}$.

Since the diameter of the graph is 5, every node is within 5 steps from any other node. Thus a disease can reach all nodes in 5 steps in the network. Most important nodes in the network are 1, 34, 33, 2 and 3. Almost all nodes in the network can be reached in maximum two steps from these nodes. Thus in the process of disease spreading these nodes are most important.

Another set of nodes which contains 3, 9, 14, 20 and 32 is also important. The removal of the nodes can break the network into two components having almost same size. There are two main communities. The most important node in one community is 1 and the important node in the second community is 34. The nodes 2 and 3 are also prominent in the first community and the nodes 32 and 33 are two prominent nodes in the second community. The nodes 3, 9, 14, 20 and 32 have common contacts with both communities. By removing all contacts with the above nodes in the network we can hinder the transfer of disease from one community to the other. Furthermore, these nodes have idea about both communities simultaneously.

III. FUZZY SOCIAL NETWORK AND EPIDEMIOLOGY

The graph $G_t = (f_t, \psi_t)$ is a fuzzy sub-graph or fuzzy network of the original network, G for all t. In the fuzzy network, there exists a fuzzy path from ***u*** to ***v***, if there exists a sequence of nodes $u = v_0, v_1, \ldots, v_r = v$ such that $f(v_i) > 0$ for i = 1, 2, r. There exists no fuzzy path in the network through the node *v* if *f(v) = 0*. Fuzzy paths play an important role in the spread of disease in a fuzzy network. So we state and prove the following results regarding fuzzy paths.

Theorem 3.1. Let $P = v_0, v_1, \ldots, v_r$ be a path in the network G_t at time t. According to the assumption $\psi_t(u,v) =$ $\min(f(u), f(v))$ for all $(u,v) \in E(G)$, the strength of the path is given by $\min\{f(v_0), f(v_1), \ldots, f(v_r)\}$.

Proof. We know that $\psi_t(u,v) =$ $\min(f(u), f(v))$ for al $(u,v) \in E(G)$, strength of the path P is given by = $\min_i\{\min(f(v_{i-1}), f(v_i))\}$ for all *i*. This quantity is equal to $\min\{f(v_0), f(v_1), \ldots, f(v_r)\}$.

Theorem 3.2. Let v be a cut-vertex in $G = (V, E)$. The vertex can become cut-vertex in the fuzzy graph and then no disease can be transmitted through v.

Proof. Since we can make the membership function value $f(v)$ arbitrarily small, the strength of all paths passing through the vertex v can be made zero.

Theorem 3.3. Let (v_1, v_2), (v_3, v_4), …, (v_r, v_{r+1}) be a collection of edges which form an edge cut in the network $G = (V, E)$. The end vertices of the edges are not necessarily distinct. Then there exists a fuzzy graph $G = (V, \sigma, \mu)$ such that the strength of all paths on which at least one of the edges of the above edges is present, is arbitrarily small.

Proof. The given edges form an edge cut in the original network. We can chose one vertex from each edge and define a fuzzy graph with same vertex set as that of *G* and the membership values of at least one end vertex of each edge arbitrarily small. Since the removal of all edges from the original network can increase the number of components in it, there exists two components in the network such that all paths from one component to another contains at least one of the edges.

Theorem 3.4. $v_0, v_1, \ldots, v_r$ be a collection of nodes which form a vertex-cut in the network $G = (V, E)$. Then there exists a fuzzy graph $G = (V, \sigma, \mu)$ such that the strength of all paths on which at least one of the vertices is present is arbitrarily small.

Proof. The given vertices form a vertex cut in the original network. We can chose the membership values of these vertices arbitrarily small. Since the removal of all the vertices from the original network can increase the number of components in it, there exists two components in the network such that all paths from one component to another contains at least one of these vertices.

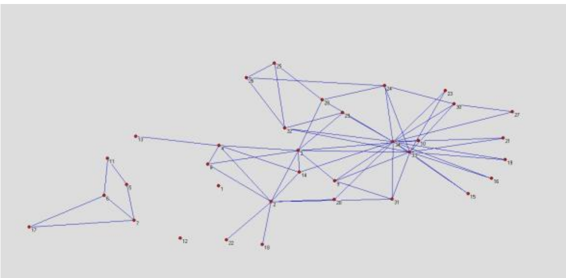

Figure: 3.1. Karate Club Network with *f(1) = 0*

Imagine that the node 1 is infected in the beginning. We consider two possibilities. *f(1) = 0* immediately after the infection. This leads to the disconnection of all contacts that 1 has with others in the community. Consequently $\psi_t(1,v)=0$ for all $v \in V$. This results in a modified network G_1 as shown in the Figure 3.1. From the figure it is clear that the disease can be controlled effectively since majority in the network lose contact with disease.

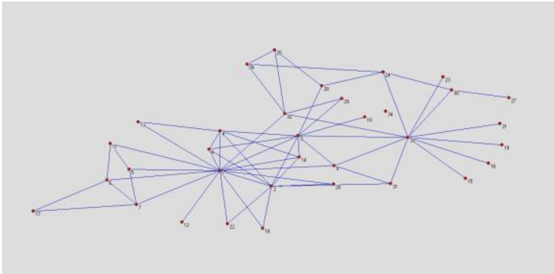

Figure: 3.2. Karate Club Network *f(34) = 0*

A similar situation can be assumed with the node 34. Corresponding network when we assume that the *f(34) = 0* is given in the next Figure 3.2. It is interesting to note that after the removal all edges from 34 the network is still connected unlike the removal of edges from 1. Next we imagine that *f(1)* does not change. This may lead to spreading the disease in the neighborhood of the node, if the nodes in the network are unaware of the infectious situation. Even if some nodes has function value below the threshold level some other nodes may have high values. This increases the chance of continuation of disease in the network.

IV. CONCLUSION

In the paper we initiate a study on the effect of behavioural change in the dynamics of disease spread in a human population. Human contacts among the members of the population are assumed to build a network. Based on the network we analyse the changers in the topology of the network, which is caused by the information about the spread of disease in the network. We use concepts in fuzzy mathematics to incorporate the vagueness caused by variations is the human behaviour. The study presented here can be extended to a much broader setting and can be strengthened by the support of analysis of data collected from real situations.

REFERENCES

[1] Andersson H, 1999, Epidemic models and social networks, Math. Sci., 24, 128 - 147.

[2] Brauer F, Pauline van den Driessche and Jianhong Wu (editors), 2008, Mathematical Epidemiology, Springer - Verlag, Berlin, Heidelberg.

[3] Pravitha V. R. and Kumar K. R., 2015, Social Network - A new Perspective in Mathematical Epidemiology, International Journal of New Technology and Research, Vol: 1, 4, 09 - 12.

[4] Zadeh L. A, Fuzzy sets, Inf. Control 8 (1965) 378-352.

[5] http://konect.uni-koblenz.de/networks/ucidata-zachary