Domination Number in Sierpriński 4-Cycle Graph

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Abstract**—***In this paper I found the edge domination number and vertex domination number of Sierpriński 4- cycle graph.*

Keywords: *Domination Number, Sierpriński Graph, Cycle Graph*

1. Introduction

In the past some years intensely those Graphsstudied whose drawings can be viewed as approximations to the famous Sierprińskitriangle. The curiosity for thesegraphs comes from many diverse sources such as games like the Chinese ringsor the Hanoi Tower, physics, topology the study of interconnection systems,and elsewhere.In 1997 Klavžar and Milutinovic were defined generalized Sierpriński graphs, $S(n,k)$. $S(1,k)$ graphis the simply complete graph K_k and $S(n,3)$ are the graphsof problem of Hanoi Tower.Hinzand Schief[1, Section 2], found the isomorphism betweenHanoi graphs and a sequence of graphs obtained from approximations to theSierprińskitriangle was constructed. Hanoi graphs H_p^n are the state graphs of theTower of Hanoi (TH) game with $p \in N_3$ pegs and $n \in N_0$ discs. Hinz, Klavzar, Milutinovic and Petr, gave the definition and properties of these graphs. [2, Sections 2.3 and 5.5].

In Section 2 we give a definition of generalized Sierpriński and domination number and cycle graphs.In Section 3, we study Sierpriński 4-cycle graph. In Section 4, we study domination number of Sierpriński 4-cycle graph and generalizing some results.

2. Preliminaries

Generalized Sierpriński Graph

From Gravier, Kovše and Aline [2011] let k be an integer and G be a finite undirected graph on a vertex set $\{1,2,.....k\}$. In the following, vertices of graphs will be identified with wordson integers. We denote by $\{1,2,.....k\}^n$ the set of words of size n on alphabet $\{1,2,.....k\}$ The letters of a word u of $\{1,2,.....k\}^n$ are denoted by $u = u_1u_2u_3....u_n$. The concatenation of two words u and v is denoted by uv.

The generalized Sierpriński graph of G of dimension n denoted by $S(n,G)$ is the graph with vertex set $\{1,2,.....k\}^n$ and edge set defined by: $\{u,v\}$ is anedge if and only if there exists $i \in \{1,2....,n\}$ such that:

i.) $u_i = v_j$ if $j < i$,

ii.) $u_i \neq v_i$ and $(u_iv_i) \in E(G)$,

iii.) $u_i = v_i$ and $v_j = u_i$ if $j > i$.

In other words, if $\{u,v\}$ is an edge of $S(n,G)$ there is an edge $\{x,y\}$ of G and a wordsuch that $u = wxy....y$ and $v = wyx....x$. We says that edge $\{u,v\}$ is using edge $\{x,y\}$ of G .Graphs $S(n,G)$ can be constructed recursively from G with the followingprocess: $S(1,G)$ is isomorphic to G. To construct $S(n,G)$ for $n>1$, copy k times $S(n-1,G)$ and add to labels of vertices in copy x of $S(n-1,G)$ theletter x at the beginning. Then for any edge $\{x,y\}$ of G, add an edge betweenvertex $xy....y$ and vertex $yx....x$. Forany word u of length d , with $1 \le d < n$, the subgraph of $S(n,G)$ induced byvertices with label beginning by u, is isomorphic to $S(n-d,G)$. For a vertex x of G , we call extremevertex x of $S(n,G)$ the vertex with label $x......x.$

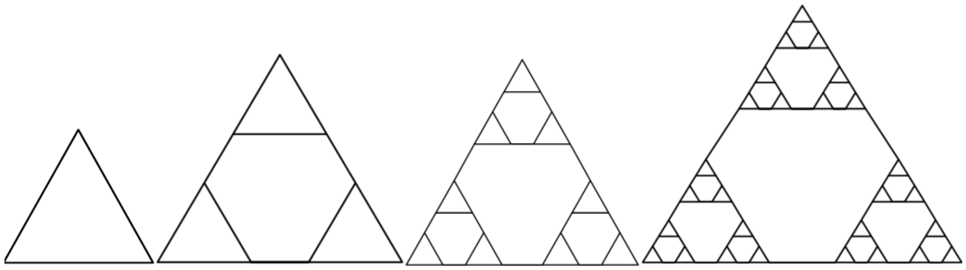
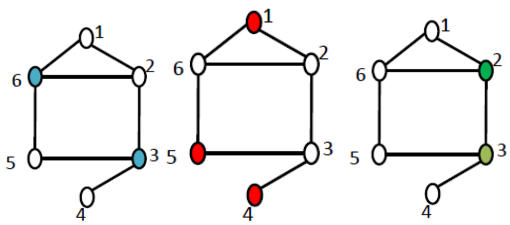


Fig 1: **Sierpiński 3-cycle graphs** $S(1,C_3), S(2,C_3), S(3,C_3)$ and $S(4,C_3)$

Domination Number

Let $G=(V,E)$ be a graph, a subset D of $V(G)$ is said to dominating set for a graph G if every vertex not in D is adjacent to at least one member of D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for $G.$



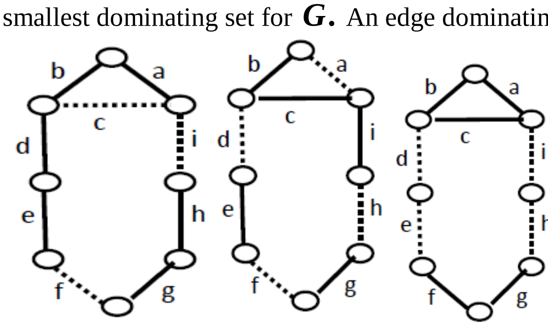
(a) (b) (c)

Fig (2)

In this example $V(G)=\{1,2,3,4,5,6\}$ Subset D for Graph (a), (b) and(c) are $\{6,3\},\{1,5,6\}$ and $\{2,3\}$. So its vertex dominating number $\gamma(G)=2.$

Edge Dominating Set

Let $G=(V,E)$ be a graph, a subset D of $E(G)$ is said to dominating set for a graph G if every edge not in D is adjacent to at least one member of D. The domination number $\gamma'(G)$ is the number of edges in a smallest dominating set for $G.$ An edge dominating set is also known as line dominating set.



(a) (b) (c)

Fig (3)

In this example $E(G)=\{a,b,c,d,e,f,g,h,i\}$ Subset D for Graph (a), (b) and (c) are $\{c,f,i\},\{a,d,f,h\}$ and $\{d,e,h,i\}$. So edge dominating number $\gamma'(G)=3.$

Cycle Graph

A graph on n vertices $\{v_1, v_2, v_3, v_4, \ldots\ldots v_{n-1}, v_n\}$ where $n \geq 3$, is said to be cycle graph if its edge set is given as $\{(v_1,v_2),(v_2,v_3),(v_3,v_4),\ldots\ldots(v_{n-1},v_n),(v_n,v_1)\}$. Every Cycle graph is two regular graphs. It is also known as circular graphs. It is denoted by C_n.

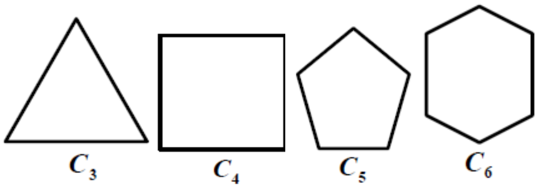


Fig (4)

Sierpriński 4-cycle graph

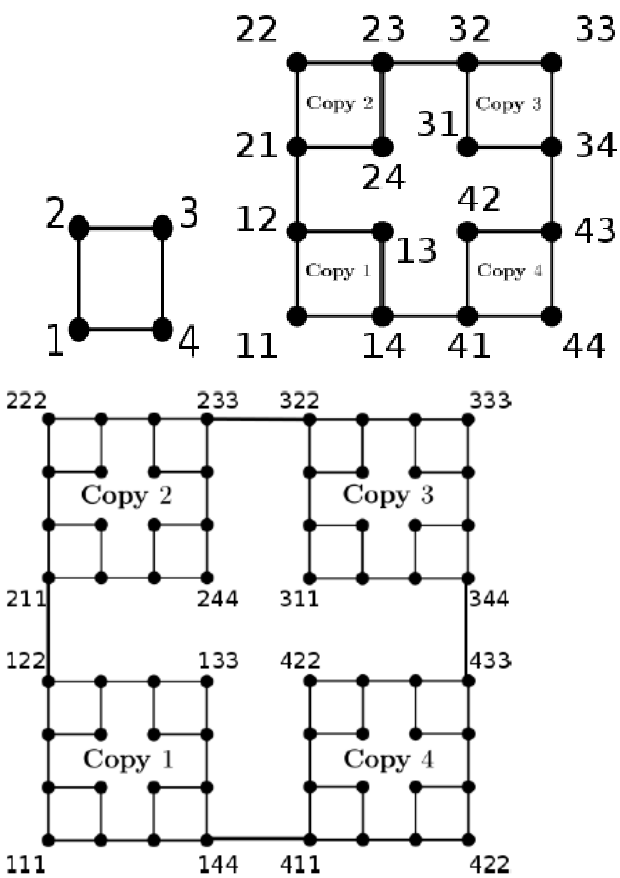
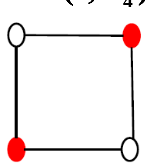


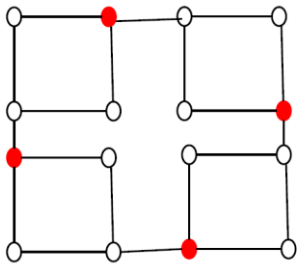
Fig (5) $S(1, C_4), S(2, C_4)$ and $S(3, C_4)$

Domination Number of $S(1, C_4)$



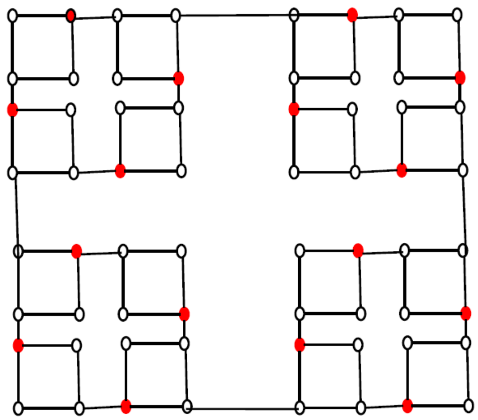
$$\gamma(S(1, C_4)) = 2$$

Domination Number of $S(2,C_4)$



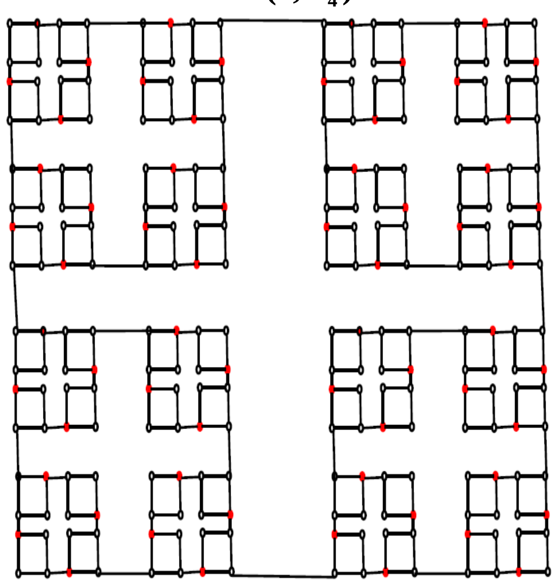
$\gamma(S(2,C_4))=4$

Domination Number of $S(3,C_4)$



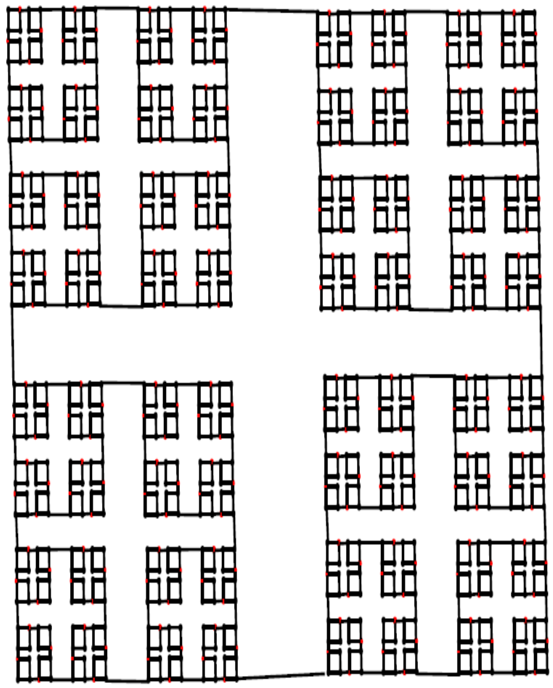
$\gamma(S(3,C_4))=16$

Domination Number of $S(4,C_4)$



$\gamma(S(4,C_4))=64$

Domination Number of $S(5,C_4)$



$\gamma(S(5,C_4)) = 256$

Table No. I

Vertex Domination Number of Sierpriński 4-Cycle Graph

S. No.	Sierpriński 4-cycle graph	Vertices	Domination Number ($\gamma(G)$)
1	$S(1,C_4)$	4	2
2	$S(2,C_4)$	16	4
3	$S(3,C_4)$	64	16
4	$S(4,C_4)$	256	64
5	$S(5,C_4)$	1024	256
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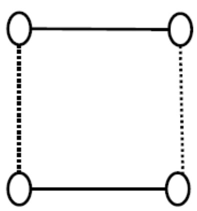
n	$S(n,C_4)$	2^{2n}	2^{2n-2}

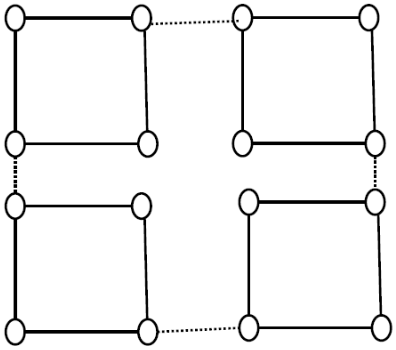
Edge Domination number in Sierpriński 4-cycle graph

Edge Domination Number of $S(1,C_4)$



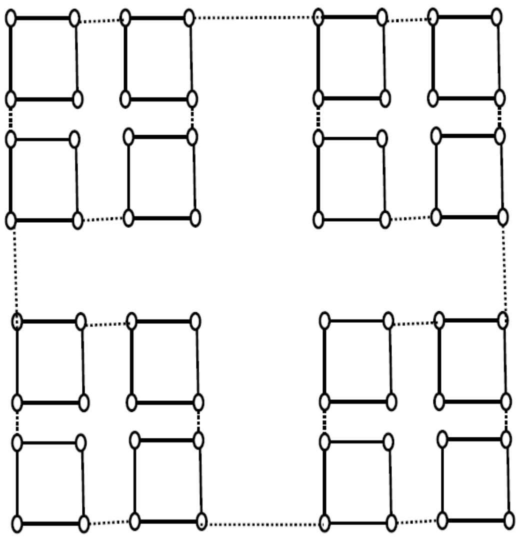
$\gamma'(S(1,C_4)) = 2$

Edge Domination Number of $S(2,C_4)$



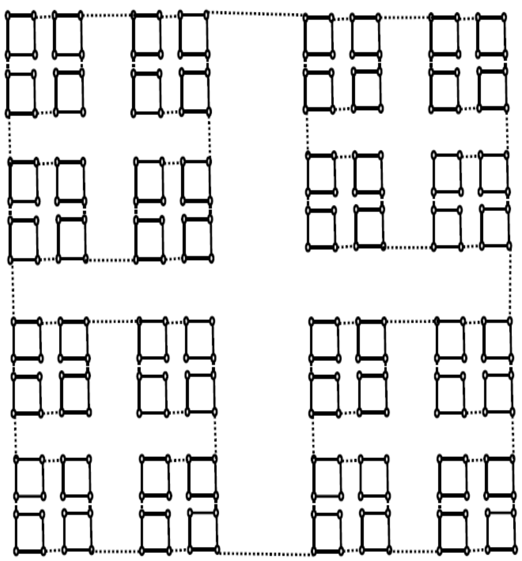
$\gamma'(S(2,C_4))=4$

Edge Domination Number of $S(3,C_4)$



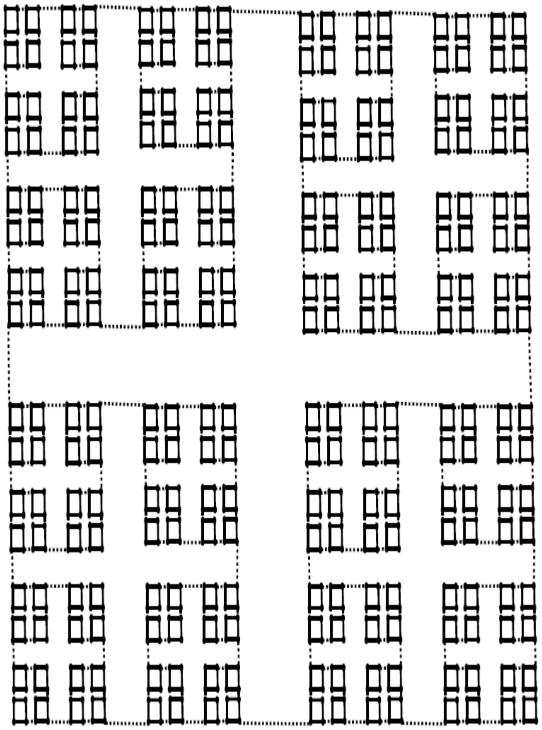
$\gamma'(S(3,C_4))=20$

Edge Domination Number of $S(4,C_4)$



$\gamma'(S(4,C_4)) = 84$

Edge Domination Number of $S(5,C_4)$



$\gamma'(S(5,C_4)) = 340$

Table No. II

Edge Domination Number of Sierpriński 4-Cycle Graph

S. No.	Sierpriński 4-cycle graph	Edges	Domination Number ($\gamma'(G)$)
1.	$S(1,C_4)$	4	2
2.	$S(2,C_4)$	20	4
3.	$S(3,C_4)$	84	20
4.	$S(4,C_4)$	340	84
5.	$S(5,C_4)$	1364	340
6.	--------	-----	------
7.	$\boldsymbol{S(n,C_4)}$		$\lvert E(S(n-1,C_4))\rvert$

There are two types of vertices in Sierpriński 4-cycle graph, one of degree two and others are of degree three.

Table No. III
Types of vertices in Sierpiński 4-Cycle Graph

S. No.	Sierpiński 4-cycle graph	Vertices	Vertices of degree 2	Vertices of degree 3
1	$S(1,C_4)$	4	4	0
2	$S(2,C_4)$	16	8	8
3	$S(3,C_4)$	64	24	40
4	$S(4,C_4)$	256	88	168
5	$S(5,C_4)$	1024	344	680
	--------	-----	------	-----
n	$S(n,C_4)$	2^{2n}	{4 times of no. of vertices of degree 2 in $S(n-1,C_4)\}-8$	{4 times of no. of vertices of degree 3 in $S(n-1,C_4)\}+8$

Conclusion:

The vertex domination number of Sierpiński 4-cycle graph $S(n,C_4)$ is given by 2^{2n-2}. Its edgedomination number is calculated by $|E(S(n-1,C_4))|$. It contain $[4\times\{\text{no. of vertices of degree 2 in } S(n-1,C_4)\}-8]$ edges of degree 2 and $[4\times\{\text{no. of vertices of degree 3 in } S(n-1,C_4)\}+8]$ edges of degree 3.

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