Closed Domination in Jump Graphs

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ABSTRACT

In this paper, we introduce the closed domination in jump graphs. Some interesting relationship aree known between domination and closed domination and relation between closed domination and independent domination.
Key words: *closed dominating set and closed domination number, Jump graph*

I. Introduction:

The concept of domination in graphs evolved from a chess board problem known as the queens problem, to find the minimum number of queens needed on 8 x 8 chess board, such that each sequence is either occupied or attempted by a queen C. Berge [3] in 1958 and 1962 and

O. Ore [8] in 1962 started the normal study on the theory of dominating set. There after several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1,200 papers related o domination in graphs with variations.

All throughout this paper, we only consider undirected graphs without loops. The basic definitions and concepts used in this study are adopted from [4]. Given jump graph J (G) =(V(j(G)), E(J(G)) the cardinality

| V (J(G)| of the vertex set V(J(G)) is the order of J(G).

The distance $d_{J(G)}(u, v)$ between two vertices u and v of the jump graph J(G) is the length of the shortest path joining u and v if $d_{J(G)}(u, v) = 1$ then u and v are said to be adjacent.

For a given vertex v of a jump graph J(G), the open neighborhood of v in J(G) is the set $N_{J(G)}(v)$ of all vertices of J(G) that are adjacent to v. The degree $deg_{J(G)}$ of v refers to $| N_{J(G)}(v) |$ and $\Delta (J(G)) = \max \{ deg_{J(G)} : v \in V((JG)) \}$. The closed neighborhood of v in the set $N_{J(G)}[v] = N_{J(G)}(v) \cup v$

For some $S \subseteq V(J(G))$ $N_{J(G)}(S) = \underbrace{\cup}_{v \in S} N_{J(G)}(v)$ and $N_{J(G)}[S] = N_{j(G)}(S) \cup S$.

If $N_{J(G)} = V(J(G))$ then s is the dominating set in J(G). The minimum cardinality among dominating set is called the domination number of J(G) denoted by $\gamma(J(G))$. The reader may refer to [12] for the fundamental concepts in domination in jump graphs.

A dominating set S in a jump graph J(G) is an independent dominating set if for every pair of distinct vertices u and v in S. u and v are non-adjacent in J(G). The minimum cardinality $\gamma_i (J(G))$ of an independent dominating set in jump graph J(G) is called the independent domination number of J(G). For the purpose of the present study, we define a minimum independent dominating set (resp. maximum independent dominating set) to be any independent dominating set of minimum (resp. maximum) cardinality.

Given a graph J(G) choose $v_1 \in V(J(G))$ and put $S_1 = \{v_1\}$.

If $N_{J(G)}[S] \neq V(J(G))$ choose $v_2 \in V(J(G))\backslash S_1$ and put $S_2 = \{v_1, v_2\}$ where possible, for $k \geq 3$ choose $v_k \in V(JG))\backslash N_{J(G)}[s_{k-1}]$ and put $S_k = \{ v_1, v_2, v_3 \ldots v_k\}$ There exist a positive integer k such that $N_{J(G)}[S_k] = V(J(G))$.

A dominating set obtained in the way given above is called a closed dominating set is called a closed domination number of jump graph J(G) and is denoted by $\bar{\gamma}(J(G))$. A closed dominating set S is said to be in its canonical form if it is written as $S = \{ v_1, v_{2 \ldots, \ldots} v_k \}$ where the vertices V_j

Satisfy the properties given above by minimum closed dominating set (resp .maximum closed dominating set) we mean a closed dominating set of minimum(resp. maximum) cardinality.

For all positive integer n, $\bar{\gamma}(J(P_n)) = \lceil \frac{n}{3} \rceil$. If $n \geq 3$ then

$\bar{\gamma}(J(C_n)) = \lceil \frac{n}{3} \rceil$

Since any independent dominating set is closed dominating set, so follows the inequality $\gamma(J(G)) \leq \bar{\gamma}(J(G)) \leq \gamma_i (J(G))$.

2. Some relationship with domination and independent domination numbers.

In this section we determine some relationship between numbers $\bar{\gamma}(J(G))$ and $\gamma(J(G))$ and between $\bar{\gamma}(J(G))$ and $\gamma_i (J(G))$.

Lemma: J(G) be any jump graph and $S \subseteq V(J(G))$ a dominating set in J(G). Then for every component C of J(G) $S \cap V(J(C))$ is a dominating set in C.

Theorem 2.2 : Let J(G) be a jump graph of order n, then

(i) $\bar{\gamma}(J(G))=1$ if and only if if $J(G)= k_1$ or $k_1 + \cup_{j=1}^{k} H_j$ for some $k \geq 1$ and connected graphs $h_1, H_2, \ldots\ldots H_k$

(ii) $\bar{\gamma}(J(G))=n$ if and only if $J(G)= \bar{k}_{n.}$

(iii) $\bar{\gamma}(J(G)) = n - 1$ if and only if $J(G)=k_{2=}$ or $J(G)= k_2 \cup \bar{k}_{n-2}$

(iv) $\bar{\gamma}(J(G)) = n - 2$ if and only if if J(G) is one of the following p_3, k_3, p_4, C_4 , $k_2 \cup k_2$, $k_2 \cup k_2 \cup \bar{k}_{n-4}$, $p_3 \cup \bar{k}_{n-3}$, $k_3 \cup \bar{k}_{n-3}$,
$C_4 \cup \bar{k}_{n-4}$, $P_4 \cup \bar{k}_{n-4}$

Proof: (i) Suppose that $J(G) = k_1 + \cup_{j=1}^{k} k_j$ for some connected graphs $H_1, H_2, \ldots\ldots\ldots H_k$ choose $v_i \in V(J(K_1))$ Since $V(J(G)) \leq N_{J(G)}[v_1]$, $\bar{\gamma}(J(G)) = 1$. Conversely, suppose that $\bar{\gamma}(J(G)) = 1$ Let $v \in V(J(G))$ such that {v} is a closed dominating set in J(G).
If $J(G) \neq K_1$ then $V(J(G))\setminus \{v\}= N_{J(G)} [v]$. Consequently

$$J(G) = \{v\} + \cup_{j=1}^{k} H_j$$

For some $k \geq 1$, and connected graphs $H_1, H_2, \ldots\ldots\ldots H_k$

(ii) In view of Lemma 2.1 if $J(G) = \bar{k}_n$ then $\bar{\gamma}(J(G))=n$ Suppose that $J(G) \neq \bar{k}_n$ If $J(G) = K_2$ then $\bar{\gamma}(J(G))=1 \neq 2$, Suppose that $J(G) \neq K_2$ and let v and u are adjacent vertices in J(G). Construct a closed dominating set $\{v_1, v_2, \ldots\ldots v_k\}$ in J(G) such that $v_1=v$ and $v_2 \neq u$ then $k \leq n - 1$. Consequently $\bar{\gamma}(J(G)) \leq n$. This completely proves (ii)

(iii) if n=2 then by (i) $\bar{\gamma}(J(G))=1$ if $J(G)= k_2$ Now proceed with $n \geq 3$ suppose that $\bar{\gamma}(J(G))=n -1$ by (ii) $\Delta (J(G)) \geq 1$. Suppose that $\Delta (J(G)) > 1$.and let $v \in V(J(G))$ with $| N_{J(G)}(v) | = \Delta (J(G))$. Construct a closed dominating set $\{ v_1, v_2, \ldots\ldots v_k\}$ in J(G) such that $v_1=v$ and $v_2 \in V(J(G))\setminus N_{J(G)}[v]$ then $k \leq n - 2$ a contradiction. Thus $\Delta (J(G)) = 1$ therefore $J(G) = J(k_2) \cup J(k_{n-2})$ The converse follows from lemma 2.1

(v) Suppose that $\bar{\gamma}(J(G)) = n- 2$ then $n \geq 3$. Also $\Delta (J(G)) \leq 2$ (otherwise a closed dominating set can be constructed with cardinality of at most n-3 a contradiction). Clearly if Thus $\Delta (J(G)) = 1$ then either $J(G) = J(k_2) \cup J(\bar{k}_2)$ or $J(G) = J(k_2) \cup J(k_2) \cup J(\bar{k}_{n-4})$. Now suppose that $\Delta (J(G)) = 2$ and Let $v \in V(J(G))$ with $| N_{J(G)}(v)| = 2$. Suppose that [u, v, w] is a geodesic In J(G) We consider two cases,

(1) [u, v, w] lies in a sub graph C_k in J(G). and (2) [u, v, w] lies in a sub graph P_k of J(G) suppose that [u, v, w] lies in a cycle C_k in J(G). If $k \geq 5$ then $k - \bar{\gamma}(J(G)) \geq 3$ and $\bar{\gamma}(J(G)) \leq n-3$. A contradiction, Thus either k=3 or k=4. If k=3 and $J(G) \neq C_3$ then $N_{J(G)}[x]=\{x\}$ for all $x \in V(J(G))\backslash V(J(C_3))$.

This yields the graph $J(G) = J(k_3) \cup J(\bar{k}_{n-3})$

If k=4 and $J(G) \neq J(C_4)$ then $N_{J(G)}[x] = \{x\}$ for all $x \in V(J(G))\backslash V(J(C_4))$.

So that $J(G)= J(C_4) \cup J(\bar{k}_{n-4})$. Similarly if [u, v, w] lies in a path P_k in J(G) then either k=3 or k=4. If k=3 and $J(G) \neq J(P_3)$ then

$J(G) = J(P_3) \cup J(\bar{k}_{n-3})$. If k=4 and $J(G) \neq J(P_4)$ then

$J(G)= J(P_4) \cup J(\bar{k}_{n-4})$.

The converse follows from lemma 2.1.

Following similar proof, theorem 2.2 also holds if $\bar{\gamma}(J(G))$ is replaced by $\gamma_i(J(G))$.

Corollary 2.3 Let G be any graph of order n, Then for k=1, n-1, n-2, n

i) If γ (J(G)) if and only if $\bar{\gamma}(J(G))=k$

ii) γ_i (J(G)) =k if and only if $\bar{\gamma}(J(G))=k$

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