Optimal Solution of a Degenerate Transportation Problem

Reena.G.patel, Dr.P.H.Bhathawala
[1]Assistant professor, Department of applied science and Humanities,S.P.C.E,Visnagar,North Gujarat.
[2]Professor and Head (Retired) Veer Narmad South Gujarat University, Surat

ABSTRACT: *The Transportation Problem is criticaltool for real life problem. Mathematically it is an application of Linear Programming problem. At the point when the analysts are doing some work on Transportation problem has a typical inquiry that, how we can way to deal with the optimality of Transportation problem. Optimality gives us the optimal route that prompts the either most extreme benefit or least aggregate cost whichever is required. Since last numerous years, there was so much research has been improved the situation for Non-Degenerate Transportation problem, however here we are acquainting the new approach to get the optimality when the Transportation problem facing the degeneracy.so , here in this paper, the algorithm tries to clarify the optimal solution of Degenerate Transportation Problem, or close to the optimal solution.*

KEY WORDS: *IBFS, Degeneracy, optimality, L-Shape of Transportation problem, Degrees of freedom for optimality.*

INTRODUCTION: Transportation problem is exceptionally powerful crucial part of linear programming problem which can be connected for required sources of supply to corresponding destination of demand, with the end goal that the aggregate transportation cost ought to be limited. The essential phase of any transportation problem as Initial Basic Feasible Solution is gotten by any of the techniques as North West Corner strategy, Least Cost Method, Vogel's Approximation technique, and the remaining and most vital work is to be for optimality of the Transportation problem is Verified by MODI.

The Transportation Problem was first settled by F.L. Hitchcock. At that point after T. C. Koopmans worked again on the hypothesis of F.L. Hitchcock in the following paper-(2).These two research work are extremely useful in the development of transportation techniques. The linear programming with fuzzy numbers and its optimal solution presented by Bazarra, Jarvis and Sherali in 1990. On the other hand Lai and Huang in 1992, accepted the circumstances in which all the parameters are fuzzy number. Additionally they have utilized the triangular possibility distribution on fuzzy numbers. Since most recent couple of years prior, in 2006, Swarup, Gupta and Mohan disclosed the strategy to reach up to the sensitivity analysis or post optimality analysis of the different parameters in the linear programming problems. In corporate division so many producers took after the Optimization basics as often as possible in the linear programming problem for any kind of real world problem. For this imperative reason, it is exceptionally pivotal to build up the new methodologies that can prompt the model to "best fit" in to the real world as much as possible.

Here we have built up another way to deal with the optimal solution or close to the optimal solution. Additionally, the new calculation depicted here gives the best way for the optimality with stepwise methodology with numerical cases for the justification.

MATHEMATICAL ASPECTS RELATED TO TRANSPORTATION PROBLEM:

Balanced Transportation Problem:
A Transportation Problem is said to be balanced transportation problem if total number of supply is same as total number of demand.

Unbalanced Transportation Problem:
A Transportation Problem is said to be unbalanced transportation problem if total number of supply is not same as total number of demand.

Objective Function: It is a linear function of decision variables expressing the objective of the decision maker.

Feasible solution: Any solution $X_{ij} \geq 0$ is said to be a feasible solution of a transportation problem if it satisfies the constraints.

Basic Feasible Solution:
Any solution $X_{ij} \geq 0$ is said to be a feasible solution of a transportation problem if it satisfies the constraints. The feasible solution is said to be basic feasible solution if the number of nonnegative allocations is equal to (m+n-1) while satisfying all rim requirements, i.e., it must satisfy requirement and availability constraint. There are three ways to get basic feasible solution.

Initial basic feasible solution:
The initial solution obtained by any of the three methods must satisfy the following conditions:

1) The solution must be feasible, i.e. it must satisfy all the supply and demand constraints (also called rim conditions)
2) The number of positive allocations must be equal to (m+n-1), where m is the number of rows and n is the number of columns.

Non - Degenerate Basic Feasible Solution: A basic feasible solution is said to be non-degenerate if it has exactly (m+n-1) positive allocations in the Transportation Problem. If the allocations are less than the required number of (m+n-1) then it is called the Degenerate Basic Feasible Solution. Then this type of solution is not easy to modify because at this stage it is impossible to draw a closed loop for each occupied cell. Thus degeneracy needs to be removed for the improvement in the obtained solution. Thus the degeneracy occurs at two different stages.

1) At initial Basic feasible solution, in which the number of occupied cells may be less than (m+n-1).
2) At any stage while moving towards optimal solution, in which two or more occupied cells may become unoccupied simultaneously.

MATHEMATICAL BACKGROUND:

Let us consider the standard balanced transportation problem with m sources A_i (with supplies a_i), i ∈ I={1,2,3........,m} and n destinations B_j (with demands b_j), j∈ J={1,2,3,......,n} .
If X_{ij} = the number of load units moving from A_i to B_j , the feasible solution (x) and set of feasible solutions (X) is:
$X=\{x/\sum_{j\in J} X_{ij} = a_i, \forall i \in I; \equiv \sum_{i\in I} X_{ij} = b_j, \forall j \in J; X_{ij} \geq 0\ \forall (i,j);\ \sum a_i = \sum b_j\}$.
Mathematically the problem can be stated as minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}\, x_{ij}$ subject to
$\sum_{j=1}^{n} x_{ij} = a_i; for\ i = 1,2, \ldots\ldots, m\ (supply\ constraints)$ And
$\sum_{i=1}^{n} x_{ij} = b_j$ For j=1, 2... n (demand constraints) $X_{ij} \geq 0$ for all i & j.
A transportation problem is said to be balanced if the total supply from all sources equals to the total demands in all destinations i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, otherwise it is called the unbalanced transportation problem.

Transportation Problem:

Origins (i)	Destinations (j)				Supply (a_i)
	1	2	....	n	
1	X_{11} C_{11}	X_{12} C_{12}	....	X_{1n} C_{1n}	a_1
2	X_{21} C_{21}	X_{22} C_{22}	....	X_{2n} C_{2n}	a_2
3	X_{31} C_{31}	X_{32} C_{32}		X_{3n} C_{3n}	a_3
....	....	....	....	....	....
M	X_{m1} C_{m1}	X_{m2} C_{m2}	....	X_{mn} C_{mn}	a_m
Demand (b_j)	b_1	b_2	....	b_n	$\sum a_i = \sum b_j$

First, let us convert the constraints of the transportation problem into our standard matrix form for a linear programming problem, Ax=B.

X= $[x_{11},\ldots\ldots, x_{1n,}, x_{21}\ldots\ldots, x_{2n},\ldots\ldots, x_{mn}]$

(1)

B= $[a_1,\ldots\ldots, a_{m,}, b_1\ldots\ldots, b_n]$

If the Constraints are written as,

$$x_{11} + x_{12} + \ldots\ldots + x_{1n} = a_1,$$
$$x_{21} + \ldots\ldots + x_{2n} = a_2,$$
$$\ddots \vdots$$
$$x_{m1} + \ldots\ldots + x_{mn} = a_m,$$
$$x_{11} + x_{21} + \ldots\ldots + x_{m1} = b_1,$$
$$\ddots$$
$$\ddots$$
$$x_{1n} + x_{2n} + \ldots\ldots + x_{mn} = b_n, \quad (2)$$

Now the equivalent reduced row echelon form of the above system – (2) is as follows,

$$x_{11} + x_{12} + .x_{13} + \ldots\ldots + x_{1n} = a_1,$$
$$x_{12} + \ldots\ldots + x_{22} + \ldots\ldots + x_{32} + \ldots\ldots + x_{m2} = b_2$$
$$x_{13} + \ldots\ldots + x_{23} + \ldots\ldots + ..x_{33} + \ldots\ldots + x_{m3} = b_3$$
$$x_{14} + \ldots\ldots + x_{24} + \ldots\ldots + x_{34} + \ldots\ldots + x_{m4} = b_4$$
$$\ddots \ldots\ldots \ddots \ldots\ldots \ddots \ldots\ldots \ddots \ldots = \vdots$$
$$x_{1n} + \ldots\ldots + x_{2n} + \ldots\ldots + x_{3n} + \ldots\ldots + x_{mn} = b_n$$

$$x_{21} + .x_{22} + x_{23}\ldots\ldots + x_{2n} = a_2,$$
$$\ddots \vdots$$
$$x_{m1} + x_{m2} + x_{m3} + \ldots\ldots + x_{mn} = a_m \quad (3)$$

The Degree of Freedom for Optimality:-

(1) Construct the Transportation matrix along with supply and demand equations of order (m+n) × (mn) corresponding to the system-(2).

(2) Find its reduced row echelon form of the matrix which corresponds to system-(3).

(3) Now check the pivot elements (row wise) of this matrix with corresponding allocations of the rows in the simple transportation matrix of order (mxn).

(4) Now the number of allocations must follow the relation between the pivot elements in the matrix of order (m+n) × (mn) and the corresponding allocations of rows in simple Transportation matrix of order (mxn) with the degree of freedom for optimality (m-1) as per the simple transportation matrix, such that (m-1) number of allocations must be from
(m+n-1) pivots from the matrix of order (m+n) × (mn).

ALGORITHM:

Step: 1 Construct the transportation matrix from the given transportation problem.

Step: 2 Find an IBFS using any one of the method as NWCM, LCM, VAM.

Step:3 If the degeneracy is found,i.e. (number of allocations are less than the (m+n-1).

So for resolution of degeneracy at initial solution, apply a negligible quantity close
To zero to one or more (if required) to unoccupied cells then it will converted to m+n-1
Occupied cells. This small quantity is denoted by(Δ).this small quantity will neither affect the total Cost nor the supply and demand values. In a minimization transportation problem,allocate the (Δ) to unoccupied cells which has minimum transportation cost, while in maximization problems allocate the (Δ) to unoccupied cells which has maximum transportation cost. In some of the problems Δ must be added in one of the unoccupied cells that may help in the following algorithm.

Step: 4 Then, skip or omit the minimum n-cost cells only from unoccupied cells (non basic variables) from the transportation matrix.
Step: 5 Assign $+\theta$ to the next minimum cost cell from unoccupied cells and start to make a loop with occupied cells if possible, otherwise move to the next to next cell from unoccupied cells. Then find
$\theta = min(-\theta)$and add that min (-θ) value at $+\theta$ and subtract that min (-θ) value from ($-\theta$).
Step: 6 Continue this process unless and until the loop made contains at least 2 cells from L-shape of the matrix having (m+n-1) pivots of the system-3. Then, find the cost of the matrix.

If we observe that this transportation cost is less than the cost obtained in Step: 2 then apply the Test for Optimality. Otherwise, go to Step-6.
Step: 7 Repeat Step-3, 4 and 5 until **The Test for Optimality** is satisfied.

The Test for Optimality:

(1) All minimum consecutive n-cost cells are allocated in the simple transportation matrix.
(2) All minimum consecutive n-cost cells are allocated in the simple transportation matrix with the degree of freedom for optimality (m-1).
(3) At least one of minimum consecutive n-cost cells are allocated in the simple transportation matrix with the degree of freedom for optimality (m-1)**.**

Step: 8now the total minimum cost is calculated as sum of the product of cost and corresponding allocated value of Supply/demand. I.e. total cost $=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij}$.

NUMERICAL EXAMPLES:

1) A manufacturer wants to ship 22 loads of his product as shown below. The matrix gives the kilometres from sources of supply to the destinations,

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	5	8	6	6	3	8
S_2	4	7	7	6	5	5
S_3	8	4	6	6	4	9
Demand	4	4	5	4	8	22 / 25

The shipping cost is Rs. 10 per load per km. what shipping schedule be used in order to minimize the total transportation cost?

Solution: Here the given problem is unbalanced Transportation Problem. So the initial basic feasible solution is developed after making the balance transportation problem by adding dummy row is given by,

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	5	8	[5] 6	6	[3] 3	8
S_2	[4] 4	7	7	[1] 6	5	5
S_3	8	[4] 4	6	6	[5] 4	9
Dummy row	0	0	0	[3] 0	0	3
Demand	4	4	5	4	8	25

Here at this stage, we get the Degeneracy as the IBFS does not have required number of m+n-1 =4+5-1=8 occupied cells, therefore the IBFS is degenerate. So for removing the degeneracy, apply theΔ to unoccupied cell (S_2, D_5) which has the minimum transportation cost among the unoccupied cells, as the given problem is for minimization problem.

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	5	8	[5] 6	6	[3] 3	8
S_2	[4] 4	7	7	[1] 6	5	5
S_3	8	[4]	6	6	[5]	9

		4			4	
Dummy row	0	0	0	**3** 0	0	3
Demand	4	4	5	4	8	25

Now by our new algorithm, we get the optimal solution as given below,

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	5	8	6	6	**8** 3	8
S_2	**4** 4	7	7	**1** 6	5	5
S_3	8	**4** 4	**2** 6	**3** 6	4	9
Dummy row	0	0	**3** 0	0	0	3
Demand	4	4	5	4	8	25

Total Cost obtained by new method is as follows,

Total Minimum Cost = ((8*3)+(4*4)+(1*6)+(4*4)+(2*6)+(3*6)+(0*6))*10

= Rs. 920

CONCLUSION: The fundamental point of this paper is to reach up to the optimal solution of a Transportation problem particularly when it experiences to the Degeneracy with well-ordered process. The above algorithm gives the optimal or close to the optimal solution for a Degenerate Transportation Problem with less number of steps with accuracy of making the decision of optimality. The future extent of this method is that the decision maker put some conceivable augmentations of less number of steps contrast with this algorithm. Consequently this method is most capable method to this present real world issues.

ACKNOWLEDGEMENTS:

I might want to offer my true thanks to Dr.Bhavin. S. Patel, Assistant Professor, Applied Science and Humanities Department at Sankalchand Patel College of Engineering, Visnagar for his priceless direction, collaboration, consistent consolation, bolster, supportive remarks, sagacious proposals and suggestions. I genuinely value his regarded direction and consolation, his knowledge and company at the time of crisis would be remembered lifelong.

REFERENCES:

1) Bazaraa M.S., Jarvis J.J., Sherali H.D., 1990. Linear Programming and Network Flows, John Weily, Second Edition, New York.
2) Lai Y.J., Hwang C.L., 1992. Mathematical Programming Methods and Applications, Springer, Berlin.
3) Swarup K., Gupta P.K., Mohan M., 2006. Operations Research, Sultan Chand and Sons, New Delhi.
4) Charnes, Cooper (1954). The Stepping-Stone method for explain linear programming. Calculation in transportation problems. Management Science 1(1)49-69
5) Dantzing G.B (1963).Linear Programming and Extensions, New Jersey: Princeton University Press.
6) Hitchcock FL (1941). The distribution of a product from several sources to Numerous Localities, Journal of Mathematical Physics 20 1941 224-230.
7) Koopmans TC (1947). Optimum Utilization of the Transportation system proceeding of the International statistical conference, Washington D.C.
8) A Charnes, W.W. Cooper and A. Henderson. An Introduction to Linear Programming, Wiley, New York, 1953.
9) Taha. H. A., Operation s Research –Introduction, Prentice Hall Of India (PVT), New Delhi, 2004.
10) Sharma J.K., Operations Research –Theory and Applications, Macmillan India (LTD), New Delhi, 2005.
11) Sudhakar V.J., Arunasankar .N., Karpagam .T., A New Approach for finding an optimal solution for Transportation problems, European Journal of Scientific Research ,ISSN 1450-216X Vol.68 No.2(2012), pp 254-257
12) Hadley. G., Linear Programming, University of Chicago, Addison-Wesley Publishing Company, Inc. Reading, Massachusetts. Palo Alto. London