Fuzzy Numerical Results Derived From Crashing CPM/PERT Networks of Padma Bridge in Bangladesh

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Abstract

In this announcement, a method is analyzed to discover solution of fuzzy CPM and PERT problem by using LR-type fuzzy numbers and their rankingfunction of Padma Bridge in Bangladesh.LR-type fuzzy number with some constructive and simple approximation arithmetic operators defined by Dubois and Prade [1].Abbasbandy and Hajjari [8] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and the right spreads at some α-levels of trapezoidal fuzzy numbersMazlum, Gumeri[16] are solved fuzzy CPM and PERT problem is most recent practicable subject by using triangular fuzzy number. In this project, a method is analyzed to find solution of fuzzy CPM and PERT problem by using LR-type fuzzy numbers and their ranking functions of Padma Bridge in Bangladesh. To illustrate our method, a numerical example is solved.

Keywords: *PERT and CPM, LR-type,Fuzzy, ranking function,triangular fuzzy number.*

I. INTRODUCTION

At first Fuzzy Set Theory, proposed in 1965 by LA Zadeh, is not only paying attention in well-defined and specific data, but also in indecisive and fuzzy data.. The strict boundaries bent by classical mathematics have been impassive, and ambiguity took place in the decision-making processes Steyn&Stokar[19], the theory provided the real world tribulations to be expressed mathematically. In roughly every field of science and technology, the eminent use of fuzzy set theory with the new perspectives abounding to decision-making in industrial systems to the issue of the classic operations research studies have prolonged the area Aziz[12]. This speculationbroadly used in game theory and network problems in operations research, linear programming, nonlinear programming, goal programming, dynamic programming, transportation models Junior, Carvalho[13]

In this communication the authors intend to investigate a method is analyzed to discover solution of fuzzy CPM and PERT problem by using LR-type fuzzy numbers and their ranking function of Padma Bridge in Bangladesh.

II. LITERATURE REVIEW AND HYPOTHESES

2.1. Fuzzy Set Theory

(a_1, a_2, a_3) is represents triangular fuzzy numbers, here a_2 is a number indicating the size is maximize, a_1 and $a_3$size are upper and bottom limit shows the acceptable values. Elements of $A$with "x" is defined as shown a universal set, which is conventional for a subset of the membership function characteristic represented by μ, and [0, 1] as the following ranges p. wang [17]

$$\mu A(x) = \begin{matrix} 1 & if\ x\ \varepsilon\ A \\ 0 & otherwise \end{matrix}$$

Definition 2.2. [5] The characteristic function μ_M of a crisp set $M \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\widetilde{M}}$such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\widetilde{M}}: X \to [0,1]$.The assigned value indicates the membership grade of the element in the set M. The function $\mu_{\widetilde{M}}$is called the membership function and the set $\widetilde{M} = \{(x, \mu_{\widetilde{M}}(x)); x\epsilon X\}$.Defined by $\mu_{\widetilde{M}}$for each $x \in X$ is called a fuzzy set.

Definition 2.3. [18] A fuzzy set $\widetilde{M}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\widetilde{M}}: R \to [0,1]$. is continuous.

2. $\mu_{\widetilde{M}}(x)$ for all $x \in (-\infty, a] \cup [d, \infty)$
3. $\mu_{\widetilde{M}}(x)$strictly increasing on [a, b] and strictly decreasing on [c, d].
4. $\mu_{\widetilde{M}}(x) = 1$for all $x \in [b, c]$ where $a < b < c < d$.

Definition 2.4. [4] A function, usually denoted by L or R, is a reference function of a fuzzy number if and only if $L(x) = L(-x), L(0) = 1$and L is nonincreasing on [0,+∞) .

Definition 2.5. [4] A fuzzy number $\widetilde{M}$ is of left, right-type (LR-type), if there exist reference functions L (for left), R (for right), and scalars $\alpha > 0, \beta > 0$ with

$$\mu_{\widetilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \leq m \\ R\left(\frac{x-m}{\beta}\right), x \geq m \end{cases}$$

The variable (m) is called value of $\widetilde{M}$,which is a real number and α and β are called the left and right spreads, respectively and $\widetilde{M}$is denoted $(m, \alpha, \beta)_{LR}$

Definition 2.6. [6].Two LR-type fuzzy numbers$\widetilde{M} = (m, \alpha, \beta)_{LR}$and $\widetilde{N} = (n, \gamma, \delta)_{LR}$ are said to be equal if and only if $m = n$ and $\alpha = \gamma, \beta = \delta$.

1.3: Operation of L-R Type Fuzzy Number:

Theorem 2.6. [6]. $\widetilde{M} = (m, \alpha, \beta)_{LR}, \widetilde{N} = (n, \gamma, \delta)_{LR}$ be two fuzzy numbers of LR-type.
Then, we have:

i. $(m, \alpha, \beta)_{LR} \oplus (n, \gamma, \delta)_{LR} = (m + n, \alpha + \gamma, \beta + \delta)_{LR}$
ii. $(m, \alpha, \beta)_{LR} \ominus (n, \gamma, \delta)_{LR} = (m - n, \alpha + \delta, \beta + \gamma)_{LR}$
iii. $-(m, \alpha, \beta)_{LR} = (-m, \alpha, \beta)_{LR}$

Definition 2.7, [5]: A fuzzy number $(m, \alpha, \beta)_{LR}$ is said to be non-negative fuzzy number if and only if $0. m - \alpha \geq 0$

Definition 2.8. [5] A fuzzy number $(m, \alpha, \beta)_{LR}$ is said to be non-positive fuzzy number if and only if $0. m - \alpha \leq 0$

Theorem 2.9. [6]. Let $\widetilde{M}$ and $\widetilde{N}$ are two fuzzy numbers of LR- type: $\widetilde{M} = (m, \alpha, \beta)_{LR}$ and $\widetilde{N} = (n, \gamma, \delta)_{LR}$
Then:

i. If $\widetilde{M}$ and $\widetilde{N}$ are positive:$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} = (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}$
ii. If $\widetilde{N}$ positive and $\widetilde{M}$ negative:$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} = (mn, n\alpha - m\gamma, n\beta - m\delta)_{LR}$
iii. If $\widetilde{M}$ and $\widetilde{N}$ are negative:$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} = (mn, -m\delta - n\beta, -n\alpha - m\gamma)_{LR}$

Definition: Scalar multiplication of fuzzy number: [35].

$$\lambda \otimes \widetilde{M} = \lambda \otimes (m, \alpha, \beta)_{LR} = \begin{cases} (m\lambda, \alpha\lambda, \beta\lambda)_{LR}, \lambda > 0, \\ (m\lambda, -\beta\lambda, -\alpha\lambda)_{LR}, \lambda < 0, \end{cases}$$

We restrict our attention to linear ranking function,that is a ranking function D such that : For any $\widetilde{M}, \widetilde{N}$belonging to $F(\mathfrak{R})$ and any k $\in \mathfrak{R}$.

Here we introduce a linear ranking function that is similar to the ranking function [1]. For any arbitrary fuzzy number $\widetilde{M} = (\underline{M}(r), \overline{M}(r))$, we use ranking function as follows:

$$D(\widetilde{M}) = \int_{[0,1]} (\underline{M}(r)) + \int_{[0,1]} (\overline{M}(r))$$

In fact, an efficient approach for ordering the elements is to define a ranking function D: F(x) → which maps for each fuzzy number into the real line,where a natural order exists.

III. FUZZY CRITICAL PATH METHOD (FCPM)

CPM (Critical Path Method) is a technique that is used to analysis on the network planning. This technique is worked towards the realization of a project. The duration of activity is assumed to be constant by the deterministic method of CPM that the membership functions is used to interest based merging method. The formula of LR-type fuzzy numbers of the following:
Each pessimistic, optimistic and median optimal value is represented by the value of lowest, highest and median of triangular fuzzy number. BaykasogluGokcen [14]

$$\left(\frac{a+2b+c}{4}\right)$$

Fuzzy PERT Method (FPERT)

In this study, known as the method of FPERT benchmarking method is used in this method assumes that each job is known of fuzzy time ChanasZielinski[9]. Benchmarking method to find the time to complete the project in the early start-to-finish in a blur in transition forward and backward pass, pass a blur start-to-finish time is calculated in the following way.

$$E\tilde{S}_i = \max_{\mathrm{V_j \in P_i}}[E\tilde{S}_j \oplus \tilde{A}_j]$$

$$E\tilde{F}_i = E\tilde{S}_i \oplus \tilde{A}_j$$

$$L\tilde{F}_i = \min_{\mathrm{V_j \in S_i}}[L\tilde{F}_j \ominus \tilde{A}_j]$$

$$L\tilde{S}_i = L\tilde{F}_i \ominus \tilde{A}_i$$

3.4: Data Collection from Padma Bridge

Although there are many methods used in project management methods, PERT and CPM are most popular methods [18]. In this study, the classical PERT and CPM in project management used in project management with a fuzzy PERT (FPERT) and fuzzy CPM (FCPM) techniques to be used, and the results will be analyzed. All analysis and observations are made in a new online internet branch project. For these project activities and activities time are obtained by X, Y and Z consulting firm's project team. The results are

Table 1: Project Schedule.

Activity Code	Previous Activity	Definition of activity	Date of starting	Date of ending	Normal time (Year)
A	-	The creation of the project plan and team	4/1/2010	1/12012	2
B	A	Approach road and Service area-2	1/10/2013	*1/10/2017*	4
C	A	Environment Land acquisition	1/1/2012	1/12/2012	1
D	C	Submitted the financial proposal of tenders	1/4/2014	1/6/2014	0.12
E	C	Janjira approach road	1/10/2013	*1/6/ 2017*	3.67
F	C	Mawa approach road	1/1/2014	*1/7/2016*	2.5
G	B	service area-2	1/1/2014	*1/7/2016*	2.5
H	E, F,G	Main bridge	1/11/2014	*1/11/2018*	4
I	D	Engineering support, sefty team and work for river management issued	1/10/2013	*1/2/2019*	5.25
J	H,I	Complete	*1/11/2018*	*1/2/2019*	0.25

3.4. Fuzzy PERT Calculation of the Project Duration

In the Table 1, X Y and Z are taken from normal time has been considered as triangular fuzzy number. This activity period on the basis of the average values were calculated for fuzzy PERT. In calculating the average value of the weight of each company by 1/3 and by summing the values are assumed to be average values were obtained.

Average value = $(a\ (x,y,z), m\ (x,y,z), B\ (x,y,z))$

a$(x,y,z) = X, Y$, and Z companies optimistic activity time's average values a(x, y, z)= ax+ay+az is calculated by formula.

Here, $ax = X1\ /\ 3, ay = Y1\ /\ 3, az = Z1\ /\ 3,$

$X1 = X, Y1 = Y = Z$and$Z1$ are the company's optimistic activity time period refers to the activities of the firm optimistic.

$m\ (x,y,z) = X, Y$, and Z companies considered the optimal operation time and the average value was calculated in the same method.

$b\ (x,y,z) = X, Y$, and Z companies pessimistic activity time has been considered as the average value was calculated in the same method. [16]

Table 2: X, Y and Z Company received from Operations Fuzzy And Average Value of Time

Activity Code	Previous Activity	Fuzzy Average Value $(a(x,y,z), m(x,y,z), b(x,y,z))$	LR-type fuzzy number $(m(x,y,z), a(x,y,z), b(x,y,z))$	Ranking function $D(\tilde{M})$
A	-	(1,2,3)	(2,1, 3)	2
B	A	(3.5,4,5)	(4,3.5, 5)	4
C	A	(0.5,1,1.5)	(1,0.5, 1.5)	1
D	C	(0,0.12,1)	(0.12,0 ,1)	0.31
E	C	(3,3.67,4)	(3.67,3, 4)	3.59
F	C	(2,2.5,3)	(2.5,2, 3)	2.5
G	B	(2,2.5,3)	(2.5,2, 3)	2.5
H	E, F,G	(3.5,4,5)	(4,3.5, 5)	4
I	D	(5,5.25,6)	(5.25,5, 6)	5.38
J	H,I	(0,0.25,0.5)	(0.25,0, 0.5)	0.25

There are 4 alternative ways; these routes, (A-B-G-H-J), (A-C-D-I-J), (A-C-E-H-J) and (A-C-F-H-J).

All activity durations is expressed as rank of fuzzy numbers, fuzzy path length is calculated in the following way.

Route 1 $(A-B-G-H-J)$: $2+2.5+4+0.25=8.75$

Route 2 $(A-C-D-I-J)$: $2+0.31+5.38+0=7.69$

Route 3 $(A-C-E-H-J)$: $2+3.59+4+0.25=9.84$

Route 4 $(A-C-F-H-J)$: $2+2.5+4+0.25=8.75$

After calculating alternative routes calculated degree of criticality of each path.. Each of the roads is calculated and the degree of criticality is shown in Table-2. According to the results, Routes 3 which is the critical path of the project (A-C-E-H-J) and the project completion time is 9.84 year and the fuzzy number is $(9.68, 9.84, 10)$.

IV. RESULT AND CONCLUSION

CPM and PERT is widely used in construction, IT, manufacturing and defense like as contemporary planning and scheduling techniques. The solutions of many problems are solved from these methods and the programming is large-scale projects are used to these. These two techniques provide great benefit to the decision makers with being analytical. In this research, the Padma bridge project completion time is compared after 9.84 years that is 9 years and 10 months from the beginning time 2010 by using fuzzy and classical implementations of the two methods. That is it will be completed after 2019 but the project is speedily running with some parallel designed work by the governing techniques. So that the project completion time is 2018 is possible.

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