4-Difference Cordial Labeling of Cycle and Wheel Related Graphs

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Abstact **—Let G be a (p, q) graph. Let k be an integer with $2 \leq k \leq p$ and $f : V(G) \to \{1, 2, \dots k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. The function f is called a k-difference cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x ($x \in \{1, 2, \dots, k\}$), $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we discuss 4-difference cordial labeling for cycle, wheel, crown, helm and gear graph.**

Key words : Difference cordial labeling, 4-difference cordial labeling.
Subject classification number: 05C78.

I. Introduction

We consider simple, finite, undirected graph $G = (V, E)$. R. Ponraj, M. Maria Adaickalam and R. Kala [6] introduced k-difference cordial labeling of graphs. In [6], they investigated k-difference cordial labeling behavior of star, m copies of star and proved that every graph is a subgraph of a connected k-difference cordial graph. In [7], R. Ponraj and M. Maria Adaickalam discussed the 3-difference cordial labeling behavior of path, cycle, star, bistar, complete graph, complete bipartite graph, comb, double comb, quadrilateral snake. For the standard terminology and notations we follow Harary[1].

II. Main Results

In this paper we have proved that cycle, wheel, helm, crown and gear graph are 4-difference cordial graphs.

Definition II.1. *A cycle $C_n (n \in \mathbb{N}, n \geq 3)$ is closed path with n vertices.*

Theorem II.1. *Cycle C_n is a 4-difference cordial graph.*

Proof. Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$. We define labeling function $f : V(C_n) \to \{1, 2, 3, 4\}$ as follows.
Case 1: n is odd.

$$
\begin{aligned}
f(v_{4i+1}) &= 1;\ 0 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \\
f(v_{4i+2}) &= 2;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\
f(v_{4i}) &= 3;\ 1 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \\
f(v_{4i+3}) &= 4;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor.
\end{aligned}
$$

Case 2: n is even.
Subcase 1: $n \equiv 0 (mod 4)$.

$$
\begin{aligned}
f(v_{4i}) &= 1;\ 1 \leq i \leq \frac{n}{4}. \\
f(v_{4i+3}) &= 2;\ 0 \leq i \leq \frac{n-4}{4}. \\
f(v_{4i+1}) &= 3;\ 0 \leq i \leq \frac{n-4}{4}. \\
f(v_{4i+2}) &= 4;\ 0 \leq i \leq \frac{n-4}{4}.
\end{aligned}
$$

Subcase 2: $n \equiv 2(mod 4)$.

$$\begin{aligned} f(v_1) &= 2, \\ f(v_2) &= 1. \\ f(v_{4i+1}) &= 1; \quad 1 \leq i \leq \frac{n-2}{4}. \\ f(v_{4i+2}) &= 2; \quad 1 \leq i \leq \frac{n-2}{4}. \\ f(v_{4i}) &= 3; \quad 1 \leq i \leq \frac{n-2}{4}. \\ f(v_{4i+3}) &= 4; \quad 0 \leq i \leq \frac{n-6}{4}. \end{aligned}$$

In each case cycle C_n satisfies the conditions for 4-difference cordial labeling. Hence C_n is a 4-difference cordial graph. □

Example 1. *The 4-difference cordial labeling of C_{18} is shown in Figure 1.*

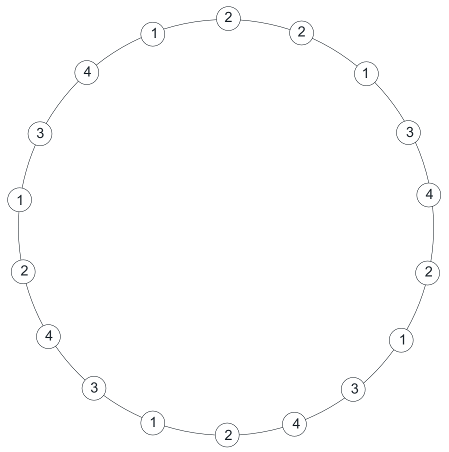

Fig. 1

Definition II.2. *The wheel $W_n(n \in \mathbb{N}, n \geq 3)$ is a join of the graphs C_n and K_1. i.e $W_n = C_n + K_1$.*
Here vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n.
The vertex corresponding to K_1 is called apex vertex.

Theorem II.2. *W_n is a 4-difference cordial graph.*

Proof. Let v_0 be the apex vertex and $v_1, v_2, \ldots, v_n$ be the rim vertices of W_n. We define labeling function $f : V(W_n) \to \{1, 2, 3, 4\}$ as follows.
Case 1: n is odd.

$$\begin{aligned} f(v_{4i}) &= 1 \quad ; \quad 1 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \\ f(v_{4i+1}) &= 2 \quad ; \quad 0 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \\ f(v_{4i+2}) &= 3 \quad ; \quad 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\ f(v_{4i+3}) &= 4 \quad ; \quad 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \end{aligned}$$

Case 2: n is even.

$$\begin{aligned} f(v_1) &= 2, \\ f(v_2) &= 3, \\ f(v_3) &= 4. \\ f(v_{4i+3}) &= 1; \quad 1 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\ f(v_{4i}) &= 2; \quad 1 \leq i \leq \lfloor \frac{n}{4} \rfloor. \\ f(v_{4i+1}) &= 3; \quad 1 \leq i \leq \lfloor \frac{n-2}{4} \rfloor. \\ f(v_{4i+2}) &= 4; \quad 1 \leq i \leq \lfloor \frac{n-2}{4} \rfloor. \end{aligned}$$

In each case wheel graph W_n satisfies the conditions of 4-difference cordial labeling. Hence W_n is 4-difference cordial graph. □

Example 2. *4-difference cordial labeling of W_{11} is shown in Figure 2.*

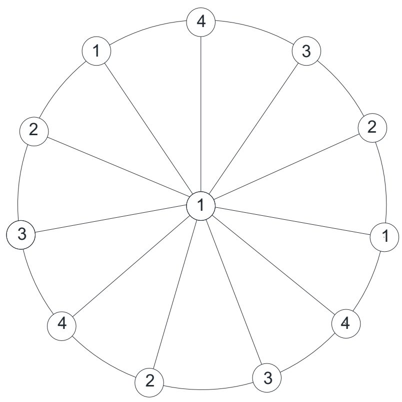

Fig. 2

Definition II.3. *[3] The crown $C_n \odot K_1 (n \in \mathbb{N}, n \geq 3)$ is obtained by joining a pendant edge to each vertex of C_n.*

Theorem II.3. *Crown $C_n \odot K_1$ is a 4-difference cordial graph.*

Proof. Let $V(C_n \odot K_1) = \{v_1, v_2, \ldots, v_n, v'_1, v'_2, \ldots, v'_n\}$, where $v_1, v_2, \ldots, v_n$ are rim vertices and $v'_1, v'_2, \ldots, v'_n$ are pendant vertices.
We define labeling function $f : V(C_n \odot K_1) \rightarrow \{1, 2, 3, 4\}$ as follows.
Case 1: n is odd.

$$\begin{aligned} f(v_{2i+1}) &= 1;\ 0 \leq i \leq \frac{n-1}{2}. \\ f(v_{2i}) &= 3;\ 1 \leq i \leq \frac{n-1}{2}. \\ f(v'_{2i+1}) &= 2;\ 0 \leq i \leq \frac{n-1}{2}. \\ f(v'_{2i}) &= 4;\ 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

Case 2: n is even.

$$\begin{aligned} f(v_{2i+1}) &= 1;\ 0 \leq i \leq \frac{n-2}{2}. \\ f(v_{2i}) &= 3;\ 1 \leq i \leq \frac{n}{2}. \\ f(v'_{2i+1}) &= 2;\ 0 \leq i \leq \frac{n-2}{2}. \\ f(v'_{2i}) &= 4;\ 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

In each case the crown graph $C_n \odot K_1$ satisfies the conditions of 4-difference cordial labeling. Hence it is 4-difference cordial graph. □

Example 3. *4-difference cordial labeling of crown $C_9 \odot K_1$ is shown in Figure 3.*

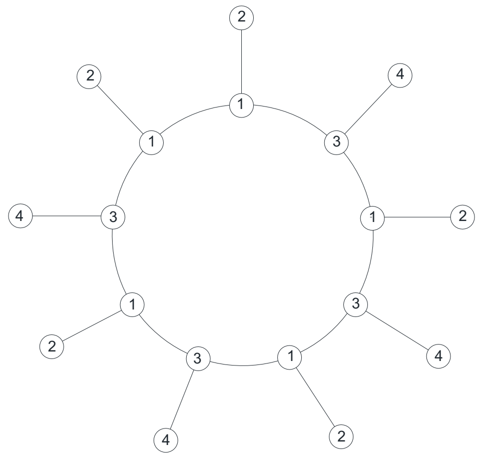

Fig. 3

Definition II.4. *A helm $H_n (n \geq 3)$ is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the rim of W_n.*

Theorem II.4. *H_n is a 4-difference cordial graph.*

Proof. Let $V(H_n) = \{v_0, v_1, \ldots, v_n, v'_1, v'_2, \ldots, v'_n\}$, where v_0 is apex vertex, $\{v_1, v_2, \ldots, v_n\}$ are rim vertices and $\{v'_1, v'_2, \ldots, v'_n\}$ are pendant vertices.
We define labeling function $f : V(H_n) \rightarrow \{1, 2, 3, 4\}$ as follows.
Case 1: n is odd.

$$\begin{aligned} f(v_{4i}) &= 1;\ 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor. \\ f(v_{4i+1}) &= 2;\ 0 \leq i \leq \lfloor \frac{n-1}{2} \rfloor. \\ f(v_{4i+2}) &= 3;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\ f(v_{4i+3}) &= 4;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\ f(v'_{4i+3}) &= 1;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\ f(v'_{4i+2}) &= 2;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\ f(v'_{4i+1}) &= 3;\ 0 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \\ f(v'_{4i}) &= 4;\ 1 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \end{aligned}$$

Case 2: n is even.

$$f(v_{2i+1}) = 2;\ 0 \leq i \leq \frac{n-2}{2}.$$
$$f(v_{2i}) = 4;\ 1 \leq i \leq \frac{n}{2}.$$
$$f(v'_{2i+1}) = 1;\ 0 \leq i \leq \frac{n-2}{2}.$$
$$f(v'_{2i}) = 3;\ 1 \leq i \leq \frac{n}{2}.$$

In each case the helm graph H_n satisfies the conditions of 4-difference cordial labeling. Hence H_n is 4-difference cordial graph. □

Example 4. *4-difference cordial labeling of helm H_9 is shown in Figure 4.*

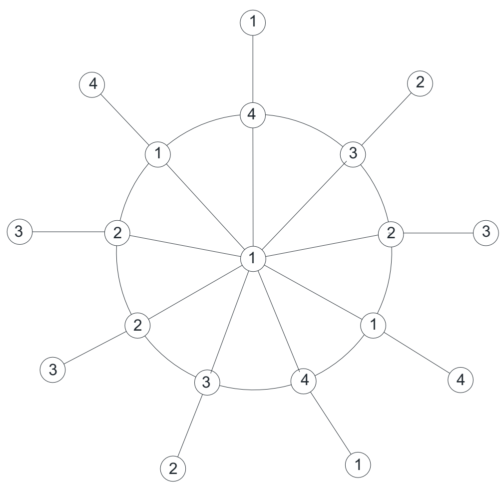

Fig. 4

Definition II.5. *A gear graph $G_n(n \geq 3)$ is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of rim of W_n.*

Theorem II.5. *Gear G_n is a 4-difference cordial graph.*

Proof. Let G_n=$\{v_0, v_1, \ldots, v_{2n}\}$, where v_0 is apex vertex, $\{v_1, v_3, \ldots, v_{2n-1}\}$ are the vertices of degree 3 and $\{v_2, v_4, \ldots, v_{2n}\}$ are the vertices of degree 2.
We define labeling function $f : V(G_n) \rightarrow \{1, 2, 3, 4\}$ as follows.

Case 1: n is odd.

$$v_0 = 3.$$
$$f(v_{4i+1}) = 1;\ 0 \leq i \leq \frac{n-1}{2}.$$
$$f(v_{4i+2}) = 2;\ 0 \leq i \leq \frac{n-1}{2}.$$
$$f(v_{4i+3}) = 3;\ 0 \leq i \leq \frac{n-3}{2}.$$
$$f(v_{4i+4}) = 4;\ 0 \leq i \leq \frac{n-3}{2}.$$

Case 2: n is even.

$$v_0 = 1.$$
$$f(v_{4i+1}) = 1;\ 0 \leq i \leq \frac{n}{2} - 1.$$
$$f(v_{4i+2}) = 2;\ 0 \leq i \leq \frac{n}{2} - 1.$$
$$f(v_{4i+3}) = 3;\ 0 \leq i \leq \frac{n}{2} - 1.$$
$$f(v_{4i+4}) = 4;\ 0 \leq i \leq \frac{n}{2} - 1.$$

In each case the gear graph G_n satisfies the conditions of 4-difference cordial labeling. Hence G_n is 4-difference cordial graph. □

Example 5. *4-difference cordial labeling of G_5 is shown in Figure 5.*

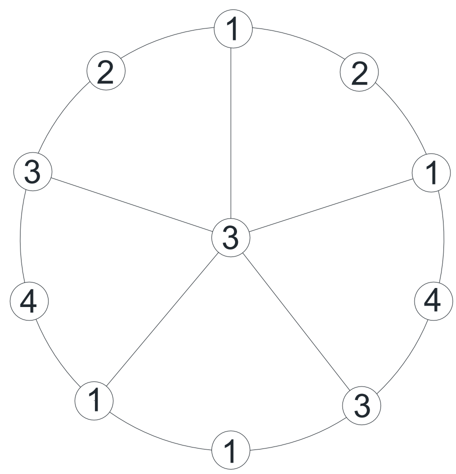

Fig. 5

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