# 4-Difference Cordial Labeling of Cycle and Wheel Related Graphs

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***Abstact* —Let $G$ be a $(p, q)$ graph. Let $k$ be an integer with $2 \leq k \leq p$ and $f : V(G) \to \{1, 2, \ldots k\}$ be a map. For each edge $uv$, assign the label $|f(u) - f(v)|$. The function $f$ is called a $k$-difference cordial labeling of $G$ if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with $x$ $(x \in \{1, 2, \ldots, k\})$, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a $k$-difference cordial labeling is called a $k$-difference cordial graph. In this paper we discuss 4-difference cordial labeling for cycle, wheel, crown, helm and gear graph.**

**Key words : Difference cordial labeling, 4-difference cordial labeling.**
**Subject classification number: 05C78.**

## I. Introduction

We consider simple, finite, undirected graph $G = (V, E)$. R. Ponraj, M. Maria Adaickalam and R. Kala [6] introduced $k$-difference cordial labeling of graphs. In [6], they investigated $k$-difference cordial labeling behavior of star, $m$ copies of star and proved that every graph is a subgraph of a connected $k$-difference cordial graph. In [7], R. Ponraj and M. Maria Adaickalam discussed the 3-difference cordial labeling behavior of path, cycle, star, bistar, complete graph, complete bipartite graph, comb, double comb, quadrilateral snake. For the standard terminology and notations we follow Harary[1].

## II. Main Results

In this paper we have proved that cycle, wheel, helm, crown and gear graph are 4-difference cordial graphs.

**Definition II.1.** *A cycle $C_n (n \in \mathbb{N}, n \geq 3)$ is closed path with $n$ vertices.*

**Theorem II.1.** *Cycle $C_n$ is a 4-difference cordial graph.*

*Proof.* Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$. We define labeling function $f : V(C_n) \to \{1, 2, 3, 4\}$ as follows.
**Case 1:** $n$ is odd.

$$\begin{aligned}
f(v_{4i+1}) &= 1;\ 0 \leq i \leq \lfloor \frac{n-1}{4} \rfloor.\\
f(v_{4i+2}) &= 2;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor.\\
f(v_{4i}) &= 3;\ 1 \leq i \leq \lfloor \frac{n-1}{4} \rfloor.\\
f(v_{4i+3}) &= 4;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor.
\end{aligned}$$

**Case 2:** $n$ is even.
**Subcase 1:** $n \equiv 0 (mod4)$.

$$\begin{aligned}
f(v_{4i}) &= 1;\ 1 \leq i \leq \frac{n}{4}.\\
f(v_{4i+3}) &= 2;\ 0 \leq i \leq \frac{n-4}{4}.\\
f(v_{4i+1}) &= 3;\ 0 \leq i \leq \frac{n-4}{4}.\\
f(v_{4i+2}) &= 4;\ 0 \leq i \leq \frac{n-4}{4}.
\end{aligned}$$

**Subcase 2:** $n \equiv 2(mod4)$.

$$\begin{aligned} f(v_1) &= 2, \\ f(v_2) &= 1. \\ f(v_{4i+1}) &= 1; \quad 1 \leq i \leq \frac{n-2}{4}. \\ f(v_{4i+2}) &= 2; \quad 1 \leq i \leq \frac{n-2}{4}. \\ f(v_{4i}) &= 3; \quad 1 \leq i \leq \frac{n-2}{4}. \\ f(v_{4i+3}) &= 4; \quad 0 \leq i \leq \frac{n-6}{4}. \end{aligned}$$

In each case cycle $C_n$ satisfies the conditions for 4-difference cordial labeling. Hence $C_n$ is a 4-difference cordial graph. □

**Example 1.** *The 4-difference cordial labeling of $C_{18}$ is shown in Figure 1.*

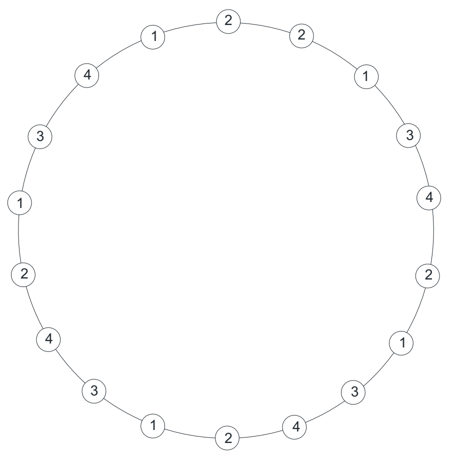


Fig. 1

**Definition II.2.** *The wheel $W_n(n \in \mathbb{N}, n \geq 3)$ is a join of the graphs $C_n$ and $K_1$. i.e $W_n = C_n + K_1$.*
*Here vertices corresponding to $C_n$ are called rim vertices and $C_n$ is called rim of $W_n$.*
*The vertex corresponding to $K_1$ is called apex vertex.*

**Theorem II.2.** *$W_n$ is a 4-difference cordial graph.*

*Proof.* Let $v_0$ be the apex vertex and $v_1, v_2, \ldots, v_n$ be the rim vertices of $W_n$. We define labeling function $f : V(W_n) \rightarrow \{1, 2, 3, 4\}$ as follows.

**Case 1:** $n$ is odd.

$$\begin{aligned} f(v_{4i}) &= 1 \quad ; \quad 1 \leq i \leq \lfloor\frac{n-1}{4}\rfloor. \\ f(v_{4i+1}) &= 2 \quad ; \quad 0 \leq i \leq \lfloor\frac{n-1}{4}\rfloor. \\ f(v_{4i+2}) &= 3 \quad ; \quad 0 \leq i \leq \lfloor\frac{n-3}{4}\rfloor. \\ f(v_{4i+3}) &= 4 \quad ; \quad 0 \leq i \leq \lfloor\frac{n-3}{4}\rfloor. \end{aligned}$$

**Case 2:** $n$ is even.

$$\begin{aligned} f(v_1) &= 2, \\ f(v_2) &= 3, \\ f(v_3) &= 4. \\ f(v_{4i+3}) &= 1; \quad 1 \leq i \leq \lfloor\frac{n-3}{4}\rfloor. \\ f(v_{4i}) &= 2; \quad 1 \leq i \leq \lfloor\frac{n}{4}\rfloor. \\ f(v_{4i+1}) &= 3; \quad 1 \leq i \leq \lfloor\frac{n-2}{4}\rfloor. \\ f(v_{4i+2}) &= 4; \quad 1 \leq i \leq \lfloor\frac{n-2}{4}\rfloor. \end{aligned}$$

In each case wheel graph $W_n$ satisfies the conditions of 4-difference cordial labeling. Hence $W_n$ is 4-difference cordial graph. □

**Example 2.** *4-difference cordial labeling of $W_{11}$ is shown in Figure 2.*

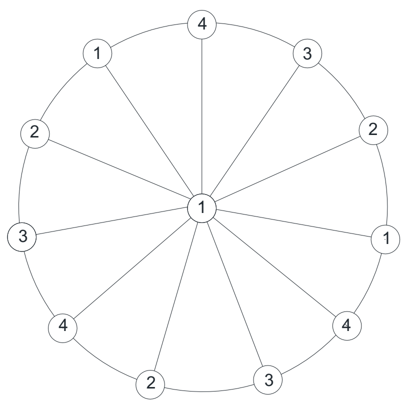


Fig. 2

**Definition II.3.** *[3] The crown $C_n \odot K_1 (n \in \mathbb{N}, n \geq 3)$ is obtained by joining a pendant edge to each vertex of $C_n$.*

**Theorem II.3.** *Crown $C_n \odot K_1$ is a 4-difference cordial graph.*

*Proof.* Let $V(C_n \odot K_1) = \{v_1, v_2, \ldots, v_n, v'_1, v'_2, \ldots, v'_n\}$, where $v_1, v_2, \ldots, v_n$ are rim vertices and $v'_1, v'_2, \ldots, v'_n$ are pendant vertices.
We define labeling function $f : V(C_n \odot K_1) \to \{1, 2, 3, 4\}$ as follows.
**Case 1:** $n$ is odd.

$$
\begin{aligned}
f(v_{2i+1}) &= 1;\ 0 \leq i \leq \frac{n-1}{2}. \\
f(v_{2i}) &= 3;\ 1 \leq i \leq \frac{n-1}{2}. \\
f(v'_{2i+1}) &= 2;\ 0 \leq i \leq \frac{n-1}{2}. \\
f(v'_{2i}) &= 4;\ 1 \leq i \leq \frac{n-1}{2}.
\end{aligned}
$$

**Case 2:** $n$ is even.

$$
\begin{aligned}
f(v_{2i+1}) &= 1;\ 0 \leq i \leq \frac{n-2}{2}. \\
f(v_{2i}) &= 3;\ 1 \leq i \leq \frac{n}{2}. \\
f(v'_{2i+1}) &= 2;\ 0 \leq i \leq \frac{n-2}{2}. \\
f(v'_{2i}) &= 4;\ 1 \leq i \leq \frac{n}{2}.
\end{aligned}
$$

In each case the crown graph $C_n \odot K_1$ satisfies the conditions of 4-difference cordial labeling. Hence it is 4-difference cordial graph. □

**Example 3.** *4-difference cordial labeling of crown $C_9 \odot K_1$ is shown in Figure 3.*

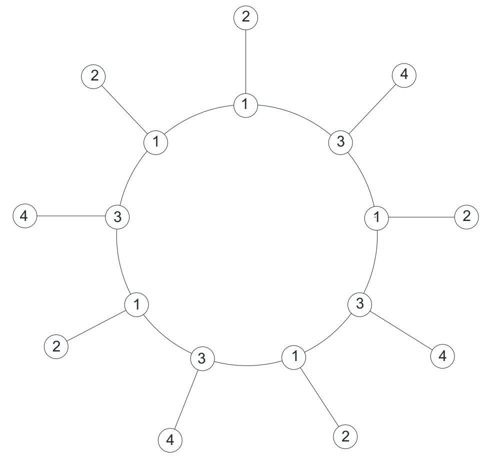

Fig. 3

**Definition II.4.** *A helm $H_n (n \geq 3)$ is the graph obtained from the wheel $W_n$ by adding a pendant edge at each vertex on the rim of $W_n$.*

**Theorem II.4.** *$H_n$ is a 4-difference cordial graph.*

*Proof.* Let $V(H_n) = \{v_0, v_1, \ldots, v_n, v'_1, v'_2, \ldots, v'_n\}$, where $v_0$ is apex vertex, $\{v_1, v_2, \ldots, v_n\}$ are rim vertices and $\{v'_1, v'_2, \ldots, v'_n\}$ are pendant vertices.
We define labeling function $f : V(H_n) \to \{1, 2, 3, 4\}$ as follows.
**Case 1:** $n$ is odd.

$$
\begin{aligned}
f(v_{4i}) &= 1;\ 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor. \\
f(v_{4i+1}) &= 2;\ 0 \leq i \leq \lfloor \frac{n-1}{2} \rfloor. \\
f(v_{4i+2}) &= 3;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\
f(v_{4i+3}) &= 4;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\
f(v'_{4i+3}) &= 1;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\
f(v'_{4i+2}) &= 2;\ 0 \leq i \leq \lfloor \frac{n-3}{4} \rfloor. \\
f(v'_{4i+1}) &= 3;\ 0 \leq i \leq \lfloor \frac{n-1}{4} \rfloor. \\
f(v'_{4i}) &= 4;\ 1 \leq i \leq \lfloor \frac{n-1}{4} \rfloor.
\end{aligned}
$$

**Case 2:** $n$ is even.

$$\begin{aligned} f(v_{2i+1}) &= 2;\ 0 \leq i \leq \frac{n-2}{2}. \\ f(v_{2i}) &= 4;\ 1 \leq i \leq \frac{n}{2}. \\ f(v'_{2i+1}) &= 1;\ 0 \leq i \leq \frac{n-2}{2}. \\ f(v'_{2i}) &= 3;\ 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

In each case the helm graph $H_n$ satisfies the conditions of 4-difference cordial labeling. Hence $H_n$ is 4-difference cordial graph. □

**Example 4.** *4-difference cordial labeling of helm $H_9$ is shown in Figure 4.*

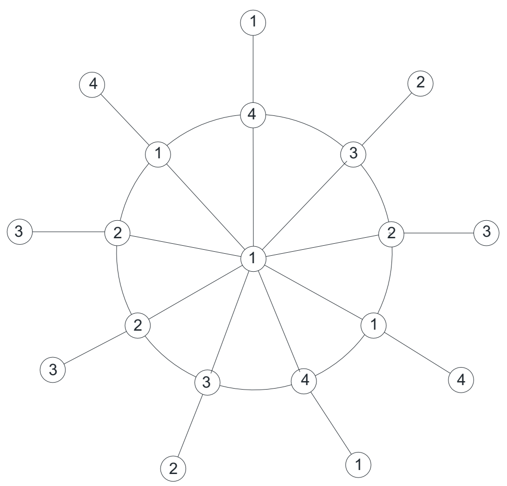


Fig. 4

**Definition II.5.** *A gear graph $G_n(n \geq 3)$ is obtained from the wheel $W_n$ by adding a vertex between every pair of adjacent vertices of rim of $W_n$.*

**Theorem II.5.** *Gear $G_n$ is a 4-difference cordial graph.*

*Proof.* Let $G_n$=$\{v_0, v_1, \ldots, v_{2n}\}$, where $v_0$ is apex vertex, $\{v_1, v_3, \ldots, v_{2n-1}\}$ are the vertices of degree 3 and $\{v_2, v_4, \ldots, v_{2n}\}$ are the vertices of degree 2.
We define labeling function $f : V(G_n) \rightarrow \{1, 2, 3, 4\}$ as follows.

**Case 1:** $n$ is odd.

$$\begin{aligned} v_0 &= 3. \\ f(v_{4i+1}) &= 1;\ 0 \leq i \leq \frac{n-1}{2}. \\ f(v_{4i+2}) &= 2;\ 0 \leq i \leq \frac{n-1}{2}. \\ f(v_{4i+3}) &= 3;\ 0 \leq i \leq \frac{n-3}{2}. \\ f(v_{4i+4}) &= 4;\ 0 \leq i \leq \frac{n-3}{2}. \end{aligned}$$

**Case 2:** $n$ is even.

$$\begin{aligned} v_0 &= 1. \\ f(v_{4i+1}) &= 1;\ 0 \leq i \leq \frac{n}{2} - 1. \\ f(v_{4i+2}) &= 2;\ 0 \leq i \leq \frac{n}{2} - 1. \\ f(v_{4i+3}) &= 3;\ 0 \leq i \leq \frac{n}{2} - 1. \\ f(v_{4i+4}) &= 4;\ 0 \leq i \leq \frac{n}{2} - 1. \end{aligned}$$

In each case the gear graph $G_n$ satisfies the conditions of 4-difference cordial labeling. Hence $G_n$ is 4-difference cordial graph. □

**Example 5.** *4-difference cordial labeling of $G_5$ is shown in Figure 5.*

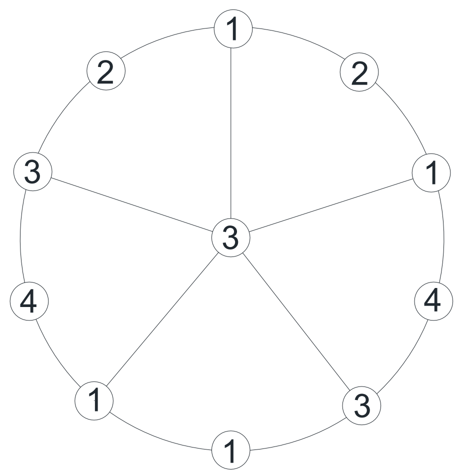


Fig. 5

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