Analysis of Wave Propagation at an Imperfect Boundary between Micropolar Elastic Solid and Micropolar Porous Elastic Solid

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Abstract

The purpose of the present paper is to represent a mathematical model to look into the propagation of the waves at an imperfect boundary between micropolar elastic solid and micropolar porous elastic solid. The variation of modulus of amplitudes ratios of various reflected and refracted waves against the angle of incidence are computed numerically for obliquely incident wave travelling at high frequency as well as at low frequency. Discussed the corresponding derivation for the normal force stiffness, transverse force stiffness and welded contact. Stiffness effects on the amplitude ratios with the angle of incidence has been observed and depicted graphically.

Keywords: Amplitude ratio, longitudinal wave, reflection, refraction, micropolar elastic solid, porous, frequency, normal force stiffness, transverse force stiffness.

I. INTRODUCTION

A linear theory as a special case of the nonlinear theory of micro-elastic solids was first constructed by Eringen and Suhubi (1964 a, b). Later, Eringen (1965) and (1966) recognized and extended this theory.

Eringen's theory of micropolar elasticity keep on significance because of its applications in many physical substance for example material particles having rigid directors, chopped fibers composites, platelet composites, aluminium epoxy, liquid crystal with side chains, a large class of substance like liquid crystal with rigid molecules, rigid suspensions, animal blood with rigid cells, foams, porous materials, bones, magnetic fields, clouds with dust, concrete with sand and muddy fluids are example of micropolar materials.

Cowin and co-workers developed the theories of non-linear and linear elastic material with voids. The linear theory of elastic material with voids is a special class of the nonlinear theory in which the change in void volume fraction and the strain are taken as independent kinematic variables. Material may be called porous material which has the properties of small distributed pores.

Analysis of propagation of waves at an interface has long been of interest to researcher in the fields of geophysics, acoustics and nondestructive evaluation. Common to all these studies is the exploration of the degrees of interaction among the interfaces that manifest themselves in the form of reflection and refraction agents and give rise to geometric scattering.

An interface between two different medium is much more complicated and has physical properties different from those of the substratum. A generalization of this concept is that of an imperfect bonded interface. In this case displacement across the surfaces need not be continuous.

Imperfect bonding mull over in the present exploration is to mean that the stress components are incessantly, but the displacement field is not. The small vector variation in the displacement is considered to depend linearly on the traction vector. Momentous work has been done to illustrate the physical conditions on the interface by different mechanical boundary conditions by different researchers.

Many problems of waves and vibrations concerning the micropolar elasticity and material with voids have been discussed by many researchers in the past, e.g., chandersekhariah (1987), Wright (1998), Golamhossen (2000), Iesan and Nappa (2003), Tomar and Singh (2005, 06), Kumar and Singh (2009). Recently, such problems discussed by Tomar and Khurana (2011), Chong and Wei (2013), Kumari (2014), Shekhar and Parvez (2015), Zhang et al. (2016) and Merkel and Luding (2017). The present paper is concerned with reflection and refraction of longitudinal waves at an imperfect interface between and micropolar elastic solid half and micropolar elastic solid half space with porous.

II. BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

1) For medium M_1 (Micropolar elastic solid)

The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$({c_1}^2+{c_3}^2)\nabla^2\phi=\frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

$$({c_2}^2+{c_3}^2)\nabla^2 U+{c_3}^2\nabla\times\Phi=\frac{\partial^2 U}{\partial t^2}, \tag{2}$$

$$({c_4}^2\nabla^2-2{\omega_0}^2)\Phi+{\omega_0}^2\nabla\times U=\frac{\partial^2\Phi}{\partial t^2}, \tag{3}$$

where

$$c_1^2=\frac{\lambda+2\mu}{\rho},\qquad c_2^2=\frac{\mu}{\rho},\qquad c_3^2=\frac{\kappa}{\rho},\qquad c_4^2=\frac{\gamma}{\rho j},\qquad \omega_0^2=\frac{\kappa}{\rho j}, \tag{4}$$

Parfitt and Eringen (1969) have shown that eq. (1) corresponds to longitudinal wave propagating with velocity V_1, given by ${V_1}^2={c_1}^2+{c_3}^2$ and equations (2) - (3) are coupled equations in vector potentials U and Φ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{{\omega_0}^2}>20$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$.

where

$${\lambda_1}^2=\frac{1}{2}\left[B-\sqrt{B^2-4C}\right],\quad {\lambda_2}^2=\frac{1}{2}\left[B+\sqrt{B^2-4C}\right], \tag{5}$$

and

$$B=\frac{q(p-2)}{\omega^2}+\frac{1}{({c_2}^2+{c_3}^2)}+\frac{1}{{c_4}^2},\qquad C=\left(\frac{1}{{c_4}^2}-\frac{2q}{\omega^2}\right)\frac{1}{({c_2}^2+{c_3}^2)},$$

$$p=\frac{\kappa}{\mu+\kappa},\quad q=\frac{\kappa}{\gamma}. \tag{6}$$

where symbols $\lambda,\mu,\gamma,\kappa,$ j have their usual meaning. We consider a two dimensional problem by taking the following components of displacement and micro- rotation as

$$U=(u,0,w),\qquad \Phi=(0,\Phi_2,0), \tag{7}$$

where

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{8}$$

and components of stresses as

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2 \phi}{\partial z^2} + \lambda \frac{\partial^2 \phi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2 \psi}{\partial x \partial z}, \tag{9}$$

$$t_{zx} = (2\mu + \kappa)\frac{\partial^2 \phi}{\partial x \partial z} - (\mu + \kappa)\frac{\partial^2 \psi}{\partial z^2} + \mu \frac{\partial^2 \psi}{\partial x^2} - \kappa \Phi_2, \tag{10}$$

$$m_{zy} = \gamma \frac{\partial \Phi_2}{\partial z}. \tag{11}$$

2) For medium M_2 (Micropolar elastic solid with porous)

The constitutive and field equations of micropolar porous elastic solid (see figure 1), in the absence of body force density and body couple density, can be written as

$$(\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\nabla(\nabla.\mathbf{u}) - (\bar{\mu} + \bar{\kappa})\nabla \times (\nabla \times \mathbf{u}) + \bar{\kappa}\, \nabla \times \bar{\Phi} + \beta^* \nabla \bar{\psi} = \bar{\rho}\, \ddot{\mathbf{u}}, \tag{12}$$

$$(\bar{\alpha} + \bar{\beta} + \bar{\gamma})\nabla(\nabla.\bar{\Phi}) - \bar{\gamma}\, \nabla \times (\nabla \times \bar{\Phi}) + \bar{\kappa}\, \nabla \times \mathbf{u} - 2\,\bar{\kappa}\, \bar{\Phi} = \bar{\rho}\, \bar{j}\, \ddot{\bar{\Phi}}, \tag{13}$$

$$\alpha^* \nabla^2 \bar{\psi} - \xi^* \bar{\psi} - \omega^* \bar{\psi} - \beta^* \nabla.\mathbf{u} = \bar{\rho}\, \kappa^*\, \ddot{\Psi}. \tag{14}$$

where $\bar{\lambda}$ and $\bar{\mu}$; $\bar{\kappa}$, $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$; α^*, β^*, ξ^*, ω^* and κ^* are Lame's constant; elastic constants of micropolarity and elastic constants due to presence of voids, respectively; $\mathbf{u}$ (x, t) and $\bar{\Phi}$(x, t) are the displacement and microrotation vectors, $\bar{\psi}$ is the change in the void volume fraction from that of in the reference state; $\bar{j}$ is the micro-inertia and $\bar{\rho}$ is the density of the medium. The superposed dots on the right hand side of these equations denote second ordered partial derivatives with respect to time.

For time harmonic plane wave propagation (i.e., $\propto \exp\{-i\bar{\omega}t\}$), the equations of motion (12) – (14) reduced to

$$(\bar{c}_1^2 + \bar{c}_3^2)\nabla(\nabla.\mathbf{u}) - (\bar{c}_2^2 + \bar{c}_3^2)\nabla \times (\nabla \times \mathbf{u}) + \bar{c}_3^2\, \nabla \times \bar{\Phi} + \bar{c}_6^2 \nabla \bar{\psi} + \bar{\omega}^2 \mathbf{u} = 0, \tag{15}$$

$$(\bar{c}_4^2 + \bar{c}_5^2)\nabla(\nabla.\bar{\Phi}) - \bar{c}_4^2 \nabla \times (\nabla \times \bar{\Phi}) + \bar{\omega}_0^2\, \nabla \times \mathbf{u} - 2\bar{\omega}_0^2 \bar{\Phi} + \bar{\omega}^2 \bar{\Phi} = 0, \tag{16}$$

$$(\alpha^* \nabla^2 - \xi^* + i\bar{\omega}\omega^* + \bar{\rho}\, \kappa^*\, \bar{\omega}^2)\bar{\psi} - \beta^* \nabla.\mathbf{u} = 0. \tag{17}$$

where

$$\bar{c}_1^2 = \frac{\bar{\lambda} + 2\bar{\mu}}{\bar{\rho}}, \quad \bar{c}_2^2 = \frac{\bar{\mu}}{\bar{\rho}}, \quad \bar{c}_3^2 = \frac{\bar{\kappa}}{\bar{\rho}}, \quad \bar{c}_4^2 = \frac{\bar{\gamma}}{\bar{\rho}\,\bar{j}}, \quad \bar{c}_5^2 = \frac{\bar{\alpha} + \bar{\beta}}{\bar{\rho}\,\bar{j}},$$

$$\bar{\omega}_0^2 = \frac{\bar{c}_3^2}{\bar{j}} = \frac{\bar{\kappa}}{\bar{\rho}\,\bar{j}}, \quad \bar{c}_6^2 = \frac{\beta^*}{\bar{\rho}}. \tag{18}$$

The constitutive relations for the micropolar porous elastic solid are given by (see Iesan, 1985)

$$\bar{t}_{kl} = \bar{\lambda}\bar{u}_{r,r}\delta_{kl} + \bar{\mu}(\bar{u}_{k,l} + \bar{u}_{l,k}) + \bar{\kappa}(\bar{u}_{l,k} - \epsilon_{klr}\bar{\phi}_r) + \delta_{kl}\beta^* \bar{\psi}, \tag{19}$$

$$\bar{m}_{kl} = \bar{\alpha}\bar{\phi}_{r,r}\delta_{kl} + \bar{\beta}\bar{\phi}_{k,l} + \bar{\gamma}\bar{\phi}_{l,k}, \tag{20}$$

$$\bar{h}_k = \alpha^* \bar{\psi}_{,k} \,. \tag{21}$$

where $\bar{t}_{kl}$, $\bar{m}_{kl}$, and $\bar{h}_k$ are the force stress tensor, couple stress tensor and equilibrated force vector, respectively.

Introducing the scalar potentials $\bar{q}$ and ξ, the vector potentials $\bar{U}$ and Π, through the Helmholtz's decomposition of vectors as

$$\mathbf{u} = \nabla\,\bar{q} + \nabla \times \bar{U}\,, \qquad \bar{\Phi} = \nabla\xi + \nabla \times \Pi\,, \qquad \nabla.\bar{U} = \nabla.\Pi = 0\,, \tag{22}$$

and employing these relations into equations of motion (15) – (17), we obtain the following system of equations

$$\left[(\bar{c}_1^2 + \bar{c}_3^2)\nabla^2 + \bar{\varpi}^2\right]\bar{q} + \bar{c}_6^2\bar{\psi} = 0\,, \tag{23}$$

$$\left[(\bar{c}_2^2 + \bar{c}_3^2)\nabla^2 + \bar{\varpi}^2\right]\bar{U} + \bar{c}_3^2\nabla^2 \times \Pi = 0\,, \tag{24}$$

$$\left[(\bar{c}_4^2 + \bar{c}_5^2)\nabla^2 - 2\bar{\varpi}_0^2 + \bar{\varpi}^2\right]\xi = 0\,, \tag{25}$$

$$\left[\bar{c}_4^2\nabla^2 - 2\bar{\varpi}_0^2 + \bar{\varpi}^2\right]\Pi + \bar{\varpi}_0^2\nabla \times \bar{U} = 0\,, \tag{26}$$

$$(\alpha^*\nabla^2 - \xi^* + i\bar{\varpi}\omega^* + \bar{\rho}\,\kappa^*\,\bar{\varpi}^2)\bar{\psi} - \beta^*\nabla^2\bar{q} = 0\,. \tag{27}$$

Following the procedure adopted by Tomar and Singh (2006) for plane waves advancing along the positive direction of a unit vector, we can obtain the dispersion relations giving the phase speeds of an independent longitudinal microrotational wave and two sets of coupled transverse waves along with the following dispersion equation giving the phase speeds of two longitudinal waves

$$\left(\tau^2 - \frac{\bar{\varpi}^2}{S^2}\right)\left(\tau^2 - \frac{\bar{\varpi}^2}{T^2} + \frac{1}{l_2^2} - \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right) - \frac{H^*}{l_1^2}\tau^2 = 0\,, \tag{28}$$

where

$$S^2 = \bar{c}_1^2 + \bar{c}_3^2\,, \qquad T^2 = \frac{\alpha^*}{\bar{\rho}\,\kappa^*}\,, \qquad l_1^2 = \frac{\alpha^*}{\beta^*}\,, \qquad l_2^2 = \frac{\alpha^*}{\xi^*} \text{ and } H^* = \frac{\beta^*}{(\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})}.$$

The quantity S is velocity of longitudinal displacement wave discussed by (Parfitt and Eringen 1969), T is the velocity of wave carrying a change in void volume discussed by (Puri and Cowin 1985)and H^* is a dimensionless number similar to that introduced by (Puri and Cowin 1985) and reduces to it in the absence of micropolarity. Also, equation (28) can be written as

$$\left(\tau^2 - \frac{\bar{\varpi}^2}{S^2}\right)\left(\tau^2 - \frac{\bar{\varpi}^2}{T^2} + \frac{1}{l_2^2} - \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right) - \frac{N^*}{l_1^2}\tau^2 = 0\,, \qquad N^* = \frac{l_2^2\,H^*}{l_1^2} \tag{29}$$

where 0 □ N^* □ 1. It can be seen that in the absence of micropolarity, the dispersion relation (29) match with the dispersion relation (24) of (Ciarletta and Sumbatyan 2003). If we put the void parameter $\beta^* = 0$, then the dispersion relation (19) yields $\tau^2 = \frac{\bar{\varpi}^2}{S^2}$, which gives the velocity of longitudinal displacement wave.

The general solution of the dispersion relation (29) is complex valued, but it admits the real solutions for high limit and low limit frequencies. Rewriting the equation (29) as given below:

$$\tau^4 - \left(\frac{\bar{\varpi}^2}{S^2} + \frac{\bar{\varpi}^2}{T^2} - \frac{1}{l_2^2} + \frac{N^*}{l_2^2} + \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right)\tau^2 + \frac{\bar{\varpi}^2}{S^2}\left(\frac{\omega^2}{T^2} - \frac{1}{l_2^2} + \frac{i\bar{\varpi}\omega^*}{\alpha^*}\right) = 0\,. \tag{30}$$

For high frequency case ($l_2\bar{\varpi} \gg 1$), we obtain the following two roots of the equation (30) given as

$$\tau_1 = \frac{\varpi}{S}, \qquad \tau_2 = \frac{\varpi}{T} + \frac{\omega^* T}{2\alpha^*}, \tag{31}$$

Similarly for low limit frequency case ($l_2\varpi \ll 1$), we obtain the following two roots of the equation (30) given as

$$\tau_1 = \frac{\varpi}{S\sqrt{1-N^*}}, \qquad \tau_2 = \frac{\varpi}{c_4^*} + \frac{i\sqrt{1-N^*}}{l_2}, \tag{32}$$

where

$c_4^* = \frac{2\alpha^*\sqrt{1-N^*}}{\omega^* l_2}$. c_4^* is the phase speed of volume fractional wave discussed by (Puri and Cowin 1985).

III. FORMULATION OF THE PROBLEM

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the uniform micropolar elastic solid half space M_1 [z>0] and the micropolar elastic solid half space with porous M_2 [z<0]. A longitudinal wave propagates through the medium M_1 and incident at the plane z=0 and makes an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get three reflected waves in the medium M_1 and four refracted waves in medium M_2.

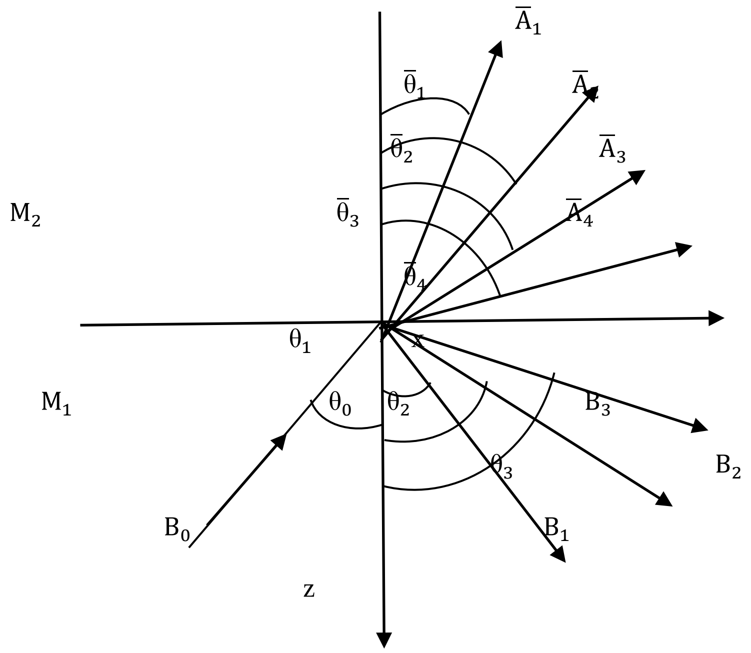


Fig.1 Geometry of the problem.

In medium M_1

$$\phi = B_0 \exp\{ik_0 (x\sin\theta_0 - z\cos\theta_0) - i\omega_1 t\} + B_1 \exp\{ik_0 (x\sin\theta_1 + z\cos\theta_1) - i\omega_1 t\}, \tag{33}$$

$$\psi = B_2 \exp\{ik_1 (x\sin\theta_2 + z\cos\theta_2) - i\omega_2 t\} + B_3 \exp\{ik_2 (x\sin\theta_3 + z\cos\theta_3) - i\omega_3 t\}, \tag{34}$$

$$\Phi_2 = EB_2 \exp\{ik_1 (x\sin\theta_2 + z\cos\theta_2) - i\omega_2 t\} + FB_3 \exp\{ik_2 (x\sin\theta_3 + z\cos\theta_3) - i\omega_3 t\}, \tag{35}$$

Where
$$E = \frac{k_1^2\left(k_1^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{D}, \tag{36}$$

$$F = \frac{k_2^2\left(k_2^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{D}, \tag{37}$$

and

$$D = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), \quad k_1^2 = \lambda_1^2\omega^2, \quad k_2^2 = \lambda_2^2\omega^2 . \tag{38}$$

where B_0, B_1, B_2, B_3 are amplitudes of incident longitudinal displacement wave, reflected longitudinal wave, reflected coupled transverse and reflected micro-rotation waves respectively.

In medium $\mathbf{M_2}$

$$\bar{q} = \sum_{i=1,2} \bar{A}_i \bar{P}_i, \qquad \bar{U}_2 = \sum_{i=3,4} \bar{A}_i \bar{P}_i, \qquad \bar{\phi}_2 = \sum_{i=3,4} \bar{\eta}_{3,4} \bar{A}_i \bar{P}_i, \tag{39}$$

where $\bar{P}_i = \exp\{i\bar{k}_i(x \sin\bar{\theta}_i - z\cos\bar{\theta}_i) - i\bar{\omega}_1 t\}$ and $\bar{A}_i$ $(i = 1, 2, 3, 4)$ are the amplitudes ratios, can be determined using boundary conditions at the interface z = 0. The quantities $\bar{\eta}_{3,4}$ are the coupling parameters between $\bar{U}_2$ (the y-component of vector $\bar{U}$) and $\bar{\phi}_2$, are given by Parfitt and Eringen (1969) and can be rewritten as below

$$\bar{\eta}_{3,4} = \bar{\omega}_0^2\left[\bar{V}_{3,4}^2 - 2\frac{\bar{\omega}_0^2}{\bar{k}_{3,4}^2} - \bar{c}_4^2\right]^{-1}.$$

Using equations from (22) into equations (19)-(21), we can write the requisite components of stresses and displacements into potential form. The requisite components of stresses are given by

$$\bar{t}_{zz} = (\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\,\bar{q}_{,zz} + (2\bar{\mu} + \bar{\kappa})\,\bar{U}_{2,xz} + \bar{\lambda}\,\bar{q}_{,xx} + \beta^*\,\bar{\psi},$$

$$\bar{t}_{zx} = (2\bar{\mu} + \bar{\kappa})\,\bar{q}_{,xz} - (\bar{\mu} + \bar{\kappa})\,\bar{U}_{2,zz} + \bar{\mu}\,\bar{U}_{2,xx} - \bar{\kappa}\,\bar{\phi}_2,$$

$$\bar{m}_{zy} = \bar{\gamma}\,\bar{\phi}_{2,z},$$

$$\bar{h}_k = \alpha^*\,\bar{\psi}_{,z}. \tag{40}$$

where $\bar{\psi} = \left(\nabla^2 + \bar{\xi}_2^2\right)\bar{q}$, $\quad \bar{\xi}_1^2 = \frac{\omega^2}{\bar{c}_3^2 + 2\bar{c}_2^2}$, $\quad \bar{\xi}_2^2 = \frac{\omega^2}{\bar{c}_1^2 + \bar{c}_3^2}$.

and we consider a two dimensional problem in x-z plane by taking

$$\mathbf{u} = (u_1, 0, u_3), \qquad \Phi = (0, \bar{\phi}_2, 0), \qquad \bar{\psi} = \bar{\psi}(x, z).$$

and the requisite components of displacements are given by

$$u_1 = \bar{q}_{,x} - \bar{U}_{2,z}, \quad u_3 = \bar{q}_{,z} - \bar{U}_{2,x}.$$

IV. BOUNDARY CONDITIONS

At the interface between micropolar elastic solid half-space and micropolar porous elastic solid half-space, the appropriate boundary conditions are continuity of force stresses, couple stresses, force vector, displacements and microrotation. Mathematically, these boundary conditions at the interface $z = 0$, can be written as:

$$t_{zz} = \bar{t}_{zz}\,,\ t_{zx} = \bar{t}_{zx}\,,\ m_{zy} = \bar{m}_{zy}\,,\ \bar{h}_z = 0,$$

$$\bar{t}_{zx} = K_t(u - u_1), \qquad \bar{t}_{zz} = K_n(w - u_3), \qquad \Phi_2 = \bar{\phi}_2. \tag{41}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{\lambda_1^{-1}} = \frac{\sin\theta_3}{\lambda_2^{-1}} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2} = \frac{\sin\bar{\theta}_3}{\bar{V}_3} = \frac{\sin\bar{\theta}_4}{\bar{V}_4}\,, \tag{42}$$

For longitudinal wave,

$$V_0 = V_1, \theta_0 = \theta_1, \tag{43}$$

Also at $z = 0$

$$k_0 V_0 = k_1 V_1 = k_2 \lambda_1^{-1} = k_3 \lambda_2^{-1} = \bar{k}_1 \bar{V}_1 = \bar{k}_2 \bar{V}_2 = \bar{k}_3 \bar{V}_3 = \bar{k}_4 \bar{V}_4 = \omega. \tag{44}$$

Making the use of potentials given by equations (33)-(35) and (39) in the boundary conditions given by (41) and using (42)-(44), we get a system of seven non homogeneous equations which can be written as

$$\sum_{j=1}^{7} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3,4,5,6,7\ ) \tag{45}$$

where

$$Z_1 = \frac{B_1}{B_0}, Z_2 = \frac{B_2}{B_0}, Z_3 = \frac{B_3}{B_0}, Z_4 = \frac{\bar{A}_1}{B_0}, Z_5 = \frac{\bar{A}_2}{B_0}, Z_6 = \frac{\bar{A}_3}{B_0}, Z_7 = \frac{\bar{A}_4}{B_0}, \tag{46}$$

where Z_1 to Z_7 are the amplitude ratios of reflected longitudinal wave, reflected coupled wave (CD I) at an angle θ_2, reflected coupled-wave (CD II) at an angle θ_3, refracted longitudinal displacement wave (LD), refracted longitudinal volume fractional wave (LVM) and refracted two sets of coupled waves respectively. Also a_{ij} and Y_i in non-dimensional form are as

$$a_{11} = -\left\{\frac{\lambda}{\mu} + D_2\cos^2\theta_1\right\}, \qquad a_{12} = -D_2\sin\theta_2\cos\theta_2\frac{k_2^2}{k_0^2},$$

$$a_{13} = -D_2\sin\theta_3\cos\theta_3\,\frac{k_3^2}{k_0^2}\,, \qquad a_{14} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{\mu}\right)\left\{\frac{\bar{k}_1^2}{k_0^2}\sin^2\bar{\theta}_1 - \frac{\bar{\xi}_1^2}{k_0^2}\right\},$$

$$a_{15} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{\mu}\right)\left\{\frac{\bar{k}_2^2}{k_0^2}\sin^2\bar{\theta}_2 - \frac{\bar{\xi}_1^2}{k_0^2}\right\}, \qquad a_{16} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{\mu}\right)\frac{\bar{k}_3^2}{k_0^2}\,\sin\bar{\theta}_3\,\cos\bar{\theta}_3\,,$$

$$a_{17} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{\mu}\right)\frac{\bar{k}_4^2}{k_0^2}\,\sin\bar{\theta}_4\,\cos\bar{\theta}_4\,, \qquad Y_1 = a_{11}.$$

$$a_{21} = D_2\sin\theta_1\cos\theta_1\,, \qquad a_{22} = -\left\{\left(D_1\cos^2\theta_2 - \sin^2\theta_2\right) - \frac{\kappa}{\mu}\frac{E}{k_2^2}\right\}\frac{k_2^2}{k_0^2},$$

$$a_{23} = -\left\{\left(D_1\cos^2\theta_3 - \sin^2\theta_3\right) - \frac{\kappa}{\mu}\frac{F}{k_3^2}\right\}\frac{k_3^2}{k_0^2}, \qquad a_{24} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{\mu}\right)\frac{\bar{k}_1^2}{k_0^2}\sin\bar{\theta}_1\cos\bar{\theta}_1,$$

$$a_{25} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{\mu}\right)\frac{\bar{k}_2^2}{k_0^2}\sin\bar{\theta}_2\cos\bar{\theta}_2, \qquad a_{26} = -\left(\frac{\bar{\mu}+\bar{\kappa}}{\mu}\right)\frac{\bar{k}_3^2}{k_0^2}\sin\bar{\theta}_3\cos\bar{\theta}_3 - \frac{\bar{\kappa}\,\bar{\eta}_3}{\mu\,k_0^2},$$

$$a_{27} = -\left(\frac{\bar{\mu}+\bar{\kappa}}{\mu}\right)\frac{\bar{k}_4^2}{k_0^2}\sin\bar{\theta}_4\cos\bar{\theta}_4 - \frac{\bar{\kappa}\,\bar{\eta}_4}{\mu\,k_0^2}, \qquad Y_2 = a_{12}.$$

$$a_{31} = 0, \qquad a_{32} == E\gamma\cos\theta_2\frac{\delta_1}{k_0}, \qquad a_{33} = F\gamma\cos\theta_3\frac{\delta_2}{k_0}, \qquad a_{34} = a_{35} = 0,$$

$$a_{36} = \bar{\eta}_3\,\bar{\gamma}\cos\bar{\theta}_3\frac{\bar{k}_3}{k_0}, \qquad a_{37} = \bar{\eta}_4\,\bar{\gamma}\cos\bar{\theta}_4\frac{\bar{k}_4}{k_0}, \qquad Y_3 = a_{31}.$$

$$a_{41} = a_{42} = a_{43} = a_{46} = a_{47} = 0, \qquad a_{44} = \frac{\bar{k}_1}{k_0}\left(\bar{k}_1^2 - \bar{\xi}_2^2\right)\cos\bar{\theta}_1,$$

$$a_{45} = \frac{\bar{k}_2}{k_0}\left(\bar{k}_2^2 - \bar{\xi}_2^2\right)\cos\bar{\theta}_2, \qquad Y_4 = a_{41}.$$

$$a_{51} = -i\sin\theta_1, \qquad a_{52} = \frac{ik_2}{k_0}\cos\theta_2, \qquad a_{53} = \frac{ik_3}{k_0}\cos\theta_3,$$

$$a_{54} = (2\bar{\mu}+\bar{\kappa})\frac{\bar{k}_1^2}{K_t\,k_0}\sin\bar{\theta}_1\cos\bar{\theta}_1 + \frac{i\bar{k}_1}{k_0}\sin\bar{\theta}_1,$$

$$a_{55} = (2\bar{\mu}+\bar{\kappa})\frac{\bar{k}_2^2}{K_t\,k_0}\sin\bar{\theta}_2\cos\bar{\theta}_2 + \frac{i\bar{k}_2}{k_0}\sin\bar{\theta}_2,$$

$$a_{56} = -\left\{(\bar{\mu}+\bar{\kappa})\frac{\bar{k}_3^2}{K_t\,k_0}\sin\bar{\theta}_3\cos\bar{\theta}_3 - \frac{i\bar{k}_3}{k_0}\cos\bar{\theta}_3 + \frac{\bar{\kappa}\,\bar{\eta}_3}{K_t\,k_0}\right\},$$

$$a_{57} = -\left\{(\bar{\mu}+\bar{\kappa})\frac{\bar{k}_4^2}{K_t\,k_0}\sin\bar{\theta}_4\cos\bar{\theta}_4 - \frac{i\bar{k}_4}{k_0}\cos\bar{\theta}_4 + \frac{\bar{\kappa}\,\bar{\eta}_4}{K_t\,k_0}\right\}, \qquad Y_5 = -a_{51}.$$

$$a_{61} = -i\cos\theta_1, \qquad a_{62} = \frac{ik_2}{k_0}\sin\theta_2, \qquad a_{63} = \frac{ik_3}{k_0}\sin\theta_3,$$

$$a_{64} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{K_n}\right)\left\{\frac{\bar{k}_1^2}{k_0^2}\sin^2\bar{\theta}_1 - \frac{\bar{\xi}_1^2}{k_0^2}\right\} + \frac{i\bar{k}_1}{k_0}\cos\bar{\theta}_1,$$

$$a_{65} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{K_n}\right)\left\{\frac{\bar{k}_2^2}{k_0^2}\sin^2\bar{\theta}_2 - \frac{\bar{\xi}_1^2}{k_0^2}\right\} + \frac{i\bar{k}_2}{k_0}\cos\bar{\theta}_2,$$

$$a_{66} = \left(\frac{2\bar{\mu}+\bar{\kappa}}{K_n}\right)\frac{\bar{k}_3}{k_0}\sin\bar{\theta}_3\cos\bar{\theta}_3 - \frac{i\bar{k}_3}{k_0}\sin\bar{\theta}_3,$$

$$a_{67}=\left(\frac{2\bar{\mu}+\bar{\kappa}}{K_n}\right)\frac{\bar{k}_4}{k_0}\sin\bar{\theta}_4\cos\bar{\theta}_4-\frac{i\bar{k}_4}{k_0}\sin\bar{\theta}_4\,,\qquad Y_6=a_{61}.$$

$$a_{71}=0,\qquad a_{72}=-E,\qquad a_{73}=-F,\qquad a_{74}=a_{75}=0,\qquad a_{76}=\bar{\eta}_3\,,$$

$$a_{77}=\bar{\eta}_4\,,\qquad Y_7=a_{71}. \tag{47}$$

where

$$D_1=1+\frac{\lambda}{\mu},\qquad D_2=1+D_1.$$

V. PARTICULAR CASES

Case I: Normal force stiffness ($K_n\neq 0, K_t\to\infty$)

In this case, we obtain a system of seven non homogeneous equations as those given by equation (45) with changed a_{ij} as

$$a_{54}=\frac{i\bar{k}_1}{k_0}\sin\bar{\theta}_1\,,\qquad a_{55}=\frac{i\bar{k}_2}{k_0}\sin\bar{\theta}_2\,,\qquad a_{56}=\frac{i\bar{k}_3}{k_0}\cos\bar{\theta}_3\,,\qquad a_{57}=\frac{i\bar{k}_4}{k_0}\cos\bar{\theta}_4\,. \tag{48}$$

Case II: Transverse force stiffness ($K_n\to\infty, K_t\neq 0$)

In this case also, a system of seven non homogeneous equations as those given by equation (45) is gained with changed a_{ij} as given below

$$a_{64}=\frac{i\bar{k}_1}{k_0}\cos\bar{\theta}_1\,,\qquad a_{65}=\frac{i\bar{k}_2}{k_0}\cos\bar{\theta}_2\,, a_{66}=-\frac{i\bar{k}_3}{k_0}\sin\bar{\theta}_3\,,\qquad a_{67}=-\frac{i\bar{k}_4}{k_0}\sin\bar{\theta}_4\,. \tag{49}$$

Case III: Welded contact ($K_n\to\infty, K_t\to\infty$)

Again in this case, also, a system of seven non homogeneous equations as those given by equation (45) is obtained with changed a_{ij} as

$$a_{54}=\frac{i\bar{k}_1}{k_0}\sin\bar{\theta}_1\,,\qquad a_{55}=\frac{i\bar{k}_2}{k_0}\sin\bar{\theta}_2\,,\qquad a_{56}=\frac{i\bar{k}_3}{k_0}\cos\bar{\theta}_3\,,\qquad a_{57}=\frac{i\bar{k}_4}{k_0}\cos\bar{\theta}_4\,,$$

$$a_{64}=\frac{i\bar{k}_1}{k_0}\cos\bar{\theta}_1\,,\qquad a_{65}=\frac{i\bar{k}_2}{k_0}\cos\bar{\theta}_2\,, a_{66}=-\frac{i\bar{k}_3}{k_0}\sin\bar{\theta}_3\,,\qquad a_{67}=-\frac{i\bar{k}_4}{k_0}\sin\bar{\theta}_4\,. \tag{50}$$

VI. NUMERICAL RESULTS AND DISCUSSION

The theoretical results obtained above indicate that the amplitudes ratios $Z_i(i=1,2,3,4,5,6,7)$ depend on the angle of incidence of incident wave and elastic properties of half spaces. In order to study in more detail the behavior of various amplitudes ratios. Following Gauthier (1982), the physical constants for micropolar elastic solid are

$$\lambda=7.59\times10^{11}\text{dyne/cm}^2,\qquad \mu=1.89\times10^{11}\text{dyne/cm}^2,$$

$$\kappa=0.0149\times10^{11}\text{dyne/cm}^2,\qquad \rho=2.19\text{gm/cm}^3$$

$$\gamma=0.0268\times10^{11}\text{ dyne}\,,\qquad j=0.0196\text{ cm}^2,\qquad \frac{\omega^2}{{\omega_0}^2}=20. \tag{51}$$

For a particular modal microplar porous elastic solid, the physical constants are given as

$$\bar{\lambda} = 5.5 \times 10^{11}\text{dyne/cm}^2, \quad \bar{\mu} = 2.14 \times 10^{11}\text{dyne/cm}^2,$$

$$\bar{\kappa} = 0.129 \times 10^{11}\text{dyne/cm}^2, \quad \bar{\gamma} = 1.88 \times 10^{11}\text{dyne/cm}^2.$$

$$\bar{j} = 0.0166\ \text{cm}^2, \quad \bar{\rho} = 2.2\ \text{gm/cm}^3, \quad \xi^* = 10 \times 10^{11}\text{dyne/cm}^2,$$

$$\beta^* = 8 \times 10^{11}\text{dyne/cm}^2, \quad \omega^* = 0.01 \times 10^{11}\text{dyne/cm}^2,$$

$$\alpha^* = 0.002 \times 10^{11}\text{dyne/cm}^2, \tag{52}$$

A computer programme in MATLAB has been evolved to calculate the modulus of amplitude ratios of various reflected and refracted waves for the particular model and to depict graphically. $Z_i(i = 1,2,3)$ and $Z_i(i = 4,5,6,7)$ represents the modulus of amplitude ratios for reflected and refracted waves respectively. The variations in all the figures are shown for the range $0^o \leq \theta_0 \leq 90^o$. Figures (2) – (34) represent the variations of the amplitude ratios of reflected and refracted waves with angle of incidence of incident longitudinal wave.

In all these figures dashed line represent the general case GEN of imperfect boundary, bold dotted line represent the normal force stiffness case NFS, bold dashed line represents the transverse force stiffness case TFS and solid line represents the welded contact case WD.

For the case of high frequency:

Figures (2) – (4), shows the variations of amplitude ratios $Z_i(i = 1,2,3)$ with respect to angle of incidence, when a longitudinal wave (LD wave) is incident obliquely at the interface with the frequency $\bar{\omega}$=500rad/s. The behavior of distribution of all curves is different. Figure (2) shows the variation of amplitude ratio of reflected LD wave. The amplitude ratio first rapidly increases to their maximum value at the angle of incidence 17^o and after that rapidly decrease at the angle of incidence near by 18^o for the case WD. For the cases GEN and TFS, firstly decreases with the corresponding angle and getting minimum value at angle 65^o and after that increases with the corresponding angle, for the case (NFS), firstly decreases with the corresponding angle and getting minimum value at angle 71^o and after that increases with the corresponding angle. Figure (3), shows the variation of amplitude ratio of reflected CD I, the values for WD sharply increases with the corresponding angle and getting maximum value at angle 18^o and then sharply decreases at the same angle and approaches to zero and after that increases and decreases values are corresponding with the angle. The maximum values for the welded contact are large than all other cases. Figure (4), shows the variation of amplitude ratio of reflected CD II. The maximum values for the welded contact are large than all other cases. Also effect of stiffness is very clear.

In figures (5) – (12), shows the variations of amplitude ratios $Z_i(i = 4,5,6,7)$ with angle of incidence of incident longitudinal wave (LD wave). The behavior of all curves is different. In figures (5) – (6), shows the variations of amplitude ratios of refracted LD wave and the behavior for GEN and NFS is oscillatory. Figure (6), the values for WD increases with the corresponding angle and getting maximum value at angle 18^o and sharply decreases at the same angle. After that it decreases with the corresponding angle and approaches to zero. The value for TFS decreases with the corresponding angle.

In figures (7) – (8), shows the variations of amplitude ratios of refracted LVM wave, the behaviour in figures (7) – (8) are approximately same as in the figures (5) and (6). In figures (9) – (10), shows the variations of amplitude ratios of refracted CD I wave, the behavior in all cases is same i.e. first increases and takes maximum values and then decreases and approaches to zero. In figures (11) – (12), shows the variations of amplitude ratios of refracted CD II wave, discussions about the figures (11) – (12) are same as in the figures (9) and (10).

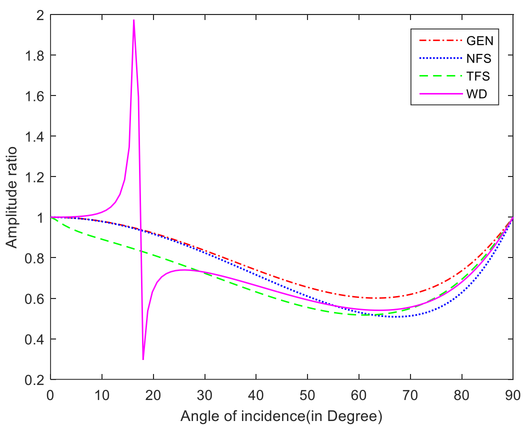
For the case of low Frequency:

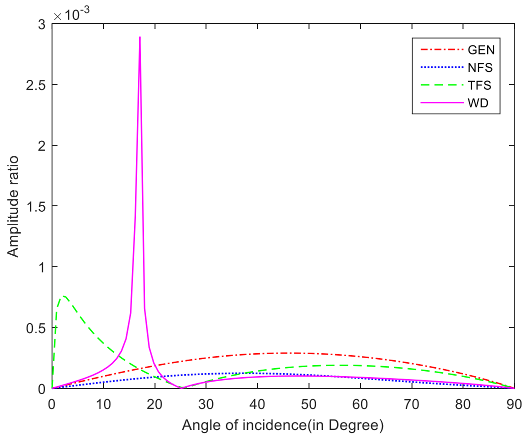
In figures (13) – (23), shows the variation of modulus values of amplitudes ratios of various reflected and refracted waves with respect to angle of incidence, when a longitudinal wave (LD wave) is incident obliquely at the interface with the frequency ϖ=10rad/s. The figures (13) – (23), for the low frequency are approximately same as high frequency.

Absence of micropolarity in the case of low Frequency:

In figures (24) – (34), shows the variations of amplitude ratios $Z_i (i = 1,2,3,4,5,6,7)$ with respect to angle of incidence, when a longitudinal wave (LD wave) is incident obliquely at the interface with the frequency ϖ=10rad/s. The behavior of distribution of all curves is different. In figures (24) – (25), show the variation of amplitude ratio of reflected LD wave, the values for WD and NFS, smoothly decreases to their minimum value at the angle of incidence 65^o and then increases and takes maximum values. The values of $|Z_1|$ for GEN case are oscillatory with angle of incidence. The value for TFS are rapidly increases and takes maximum value at the angle of 8^o and sharply decreases and takes minimum value at the angle of 9^o and lastly decreases with corresponding the angle. In figure (26), show the variation of amplitude ratio of reflected CD I wave, the values for GEN and NFS are increases with increase in angle of incidence and takes maximum value and then starts to decrease and takes minimum value. For TFS case the value of modulus of amplitude ratio sharply increases at an approximate angle 10^o and then decreases and tends to zero value. The values are small for WD case. Figure (27), shows the variation of amplitude ratio of reflected CD II wave, values of all parameter are same as the figure (26).

Figures (28) – (29) show the variation of amplitude ratios of reflected LD wave, the values for GEN, NFS and WD are monotonic with corresponding angle and the values for TFS are getting maximum value at the angle of 9^o and after that decreases with corresponding the angle and approaches to zero value. Figures (30) – (31) show the variations of amplitude ratios of refracted LVM wave, behavior of figures (30) – (31) are same as the figures (28) and (29) but values are different behavior of modulus of amplitude ratio. Figures (32), shows the variations of amplitude ratios of refracted CD I wave, the values for GEN, NFS and WD cases are increases as well as decreases with corresponding angle and getting maximum value at the angle of 20^o approximately. Behaviour of figure (33) is same like as figure (29). Figure (34) shows the variation of amplitude ratio of refracted CD II wave, values of all parameter are same as the figure (26). Comparing the figures (13) – (23) to corresponding figures from (14) – (24), the effect of micropolarity is very clear.





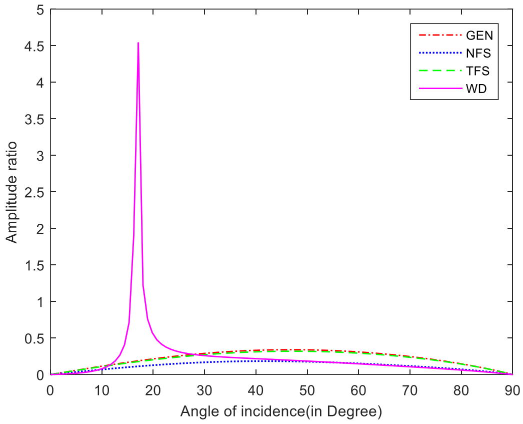
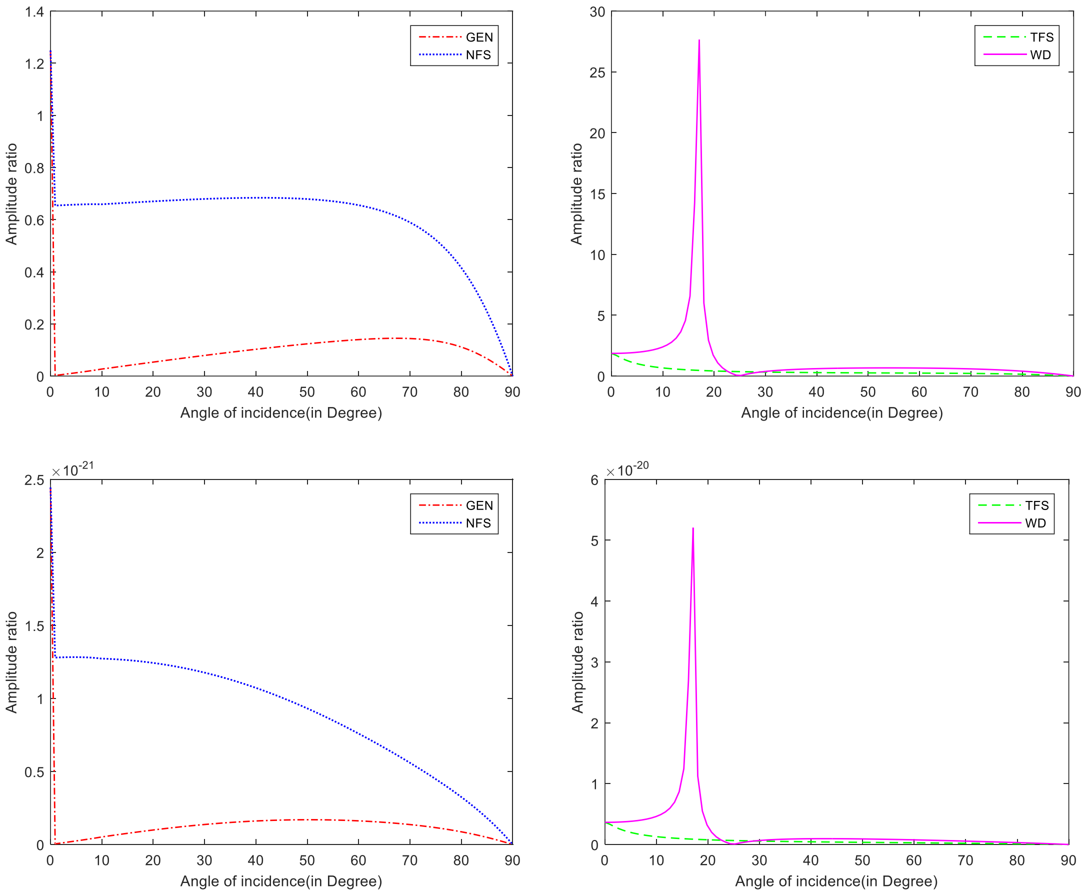


Fig.2-4. (High frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident LD wave.



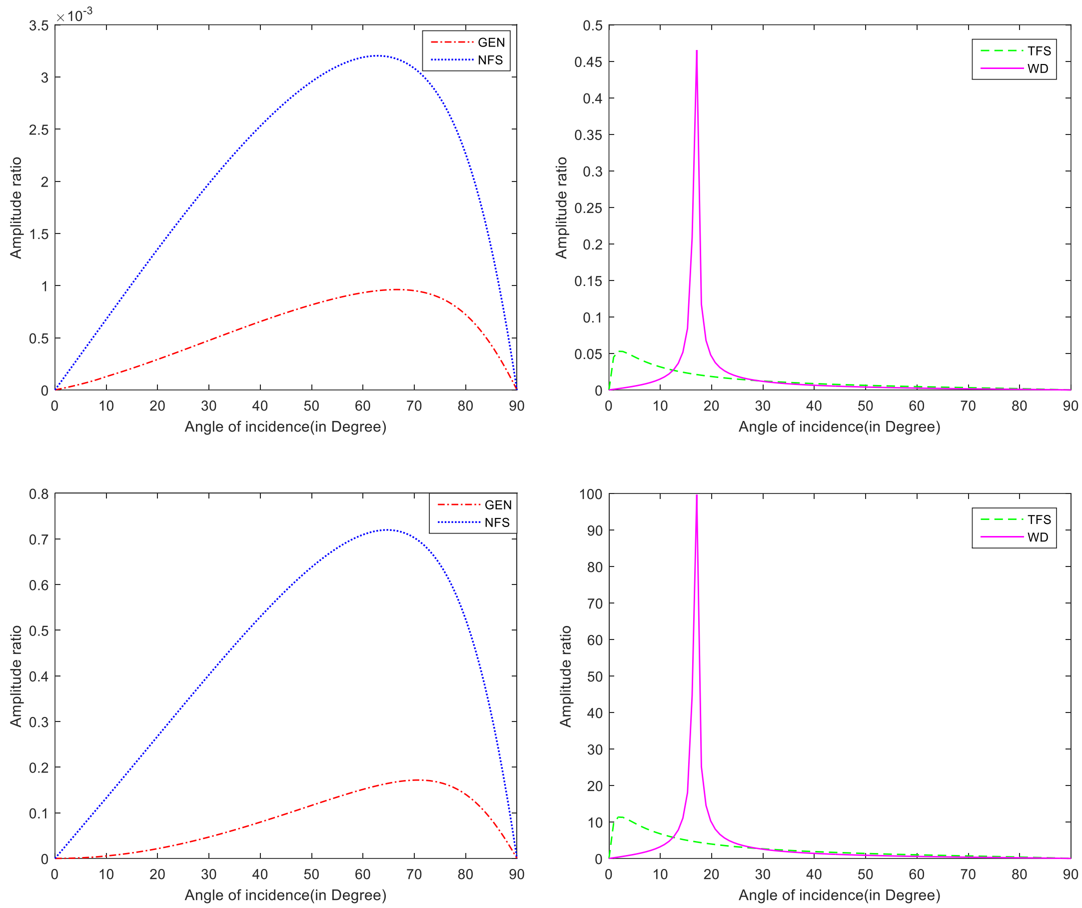
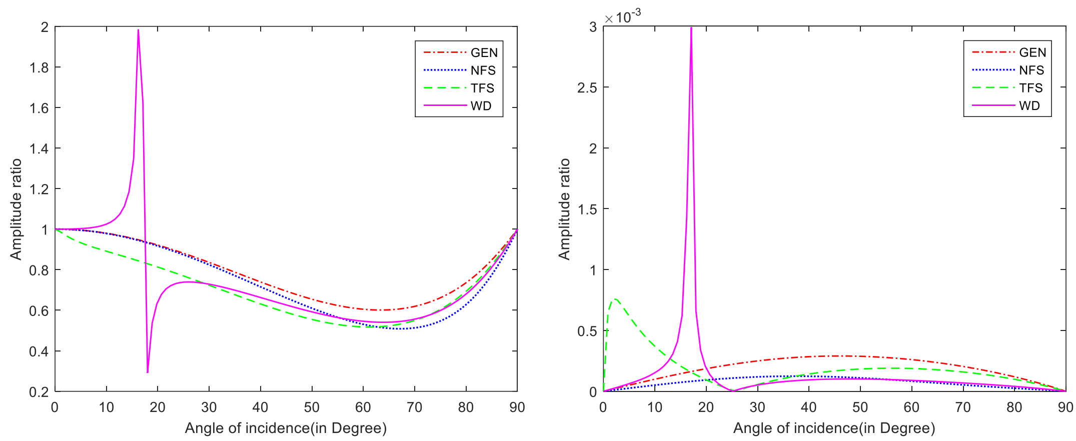


Fig.5-12. (High frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 4,5,6,7)$ with angle of incidence of the incident LD wave.



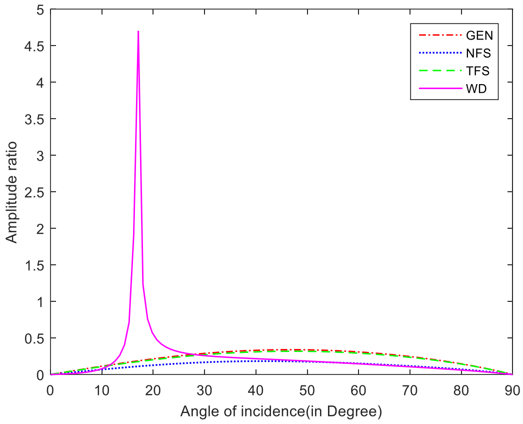
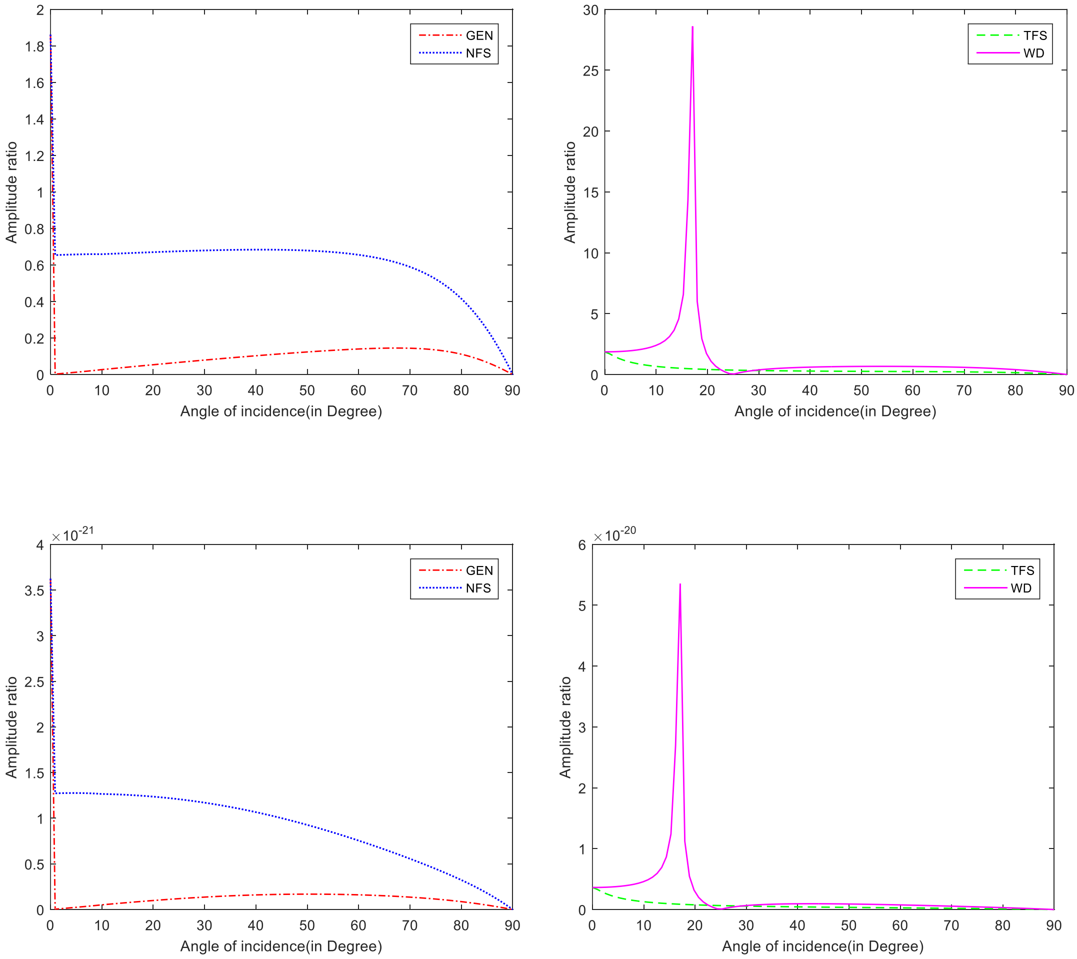


Fig.13-15. (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident LD wave.



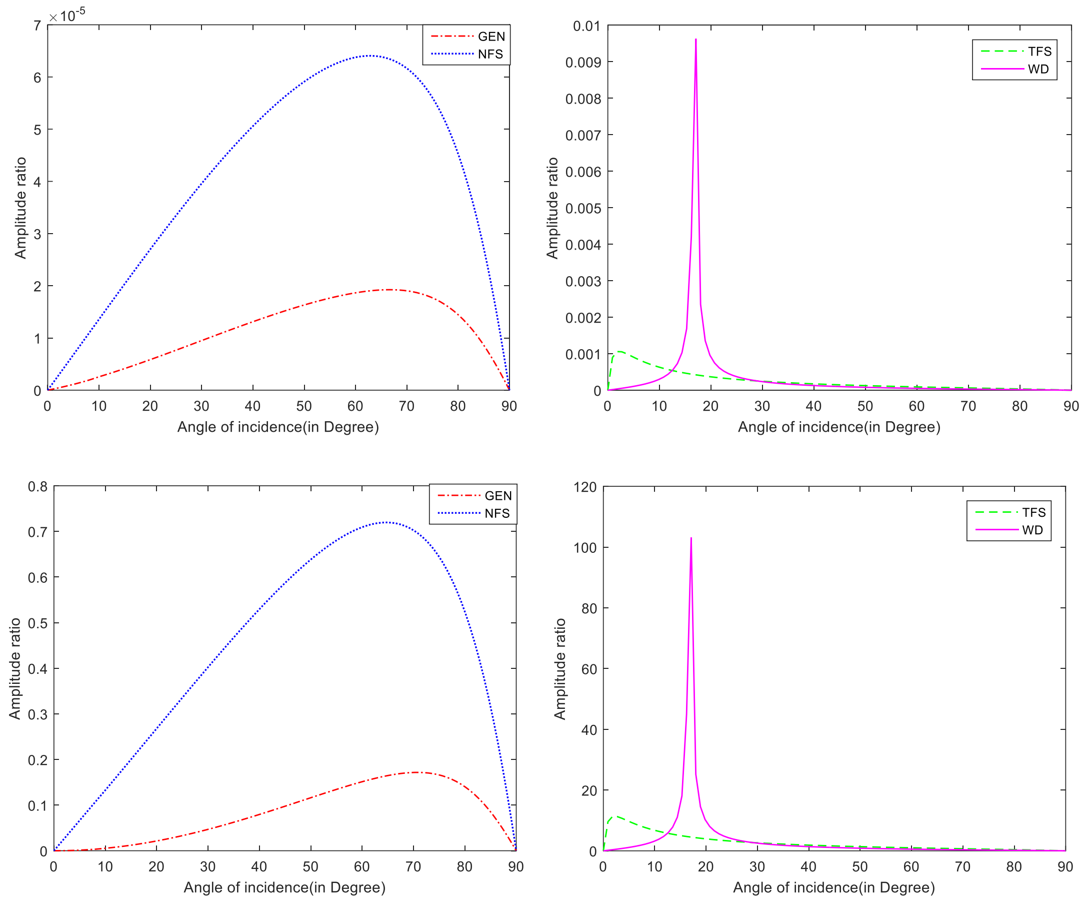
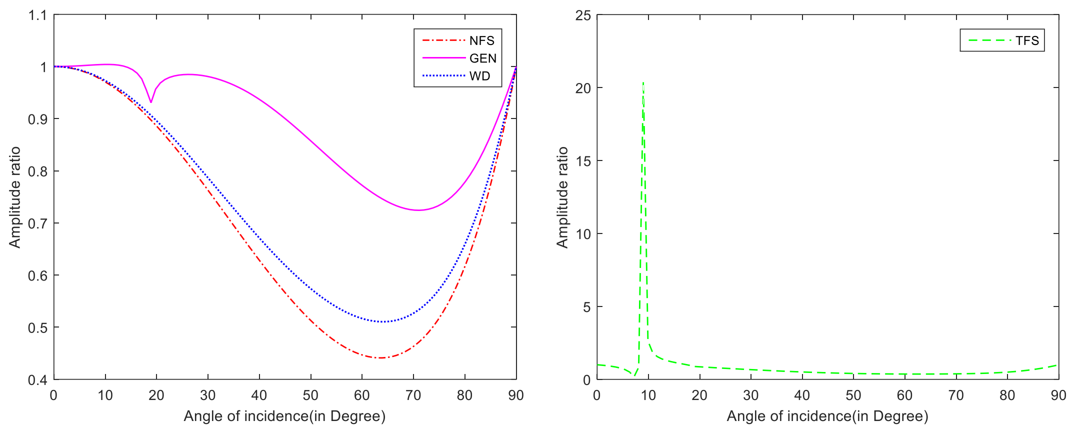


Fig.16-23. (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 4,5,6,7)$ with angle of incidence of the incident LD wave.



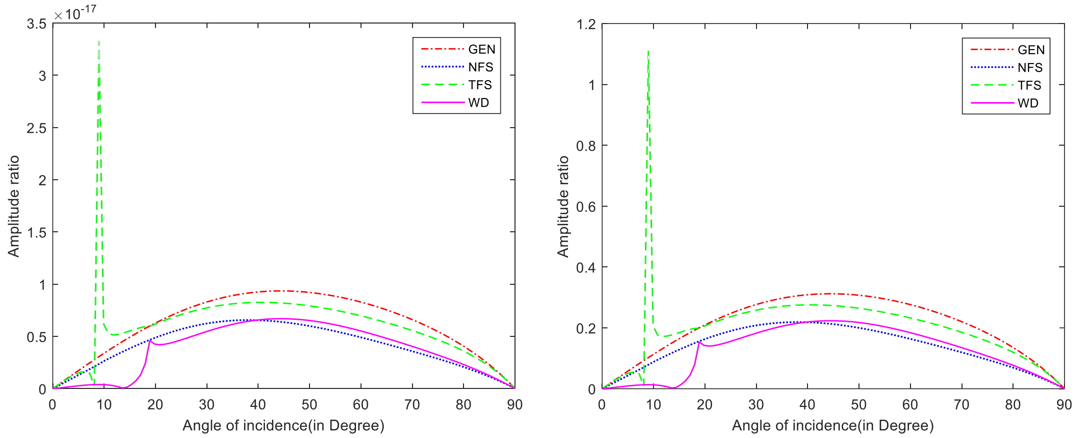
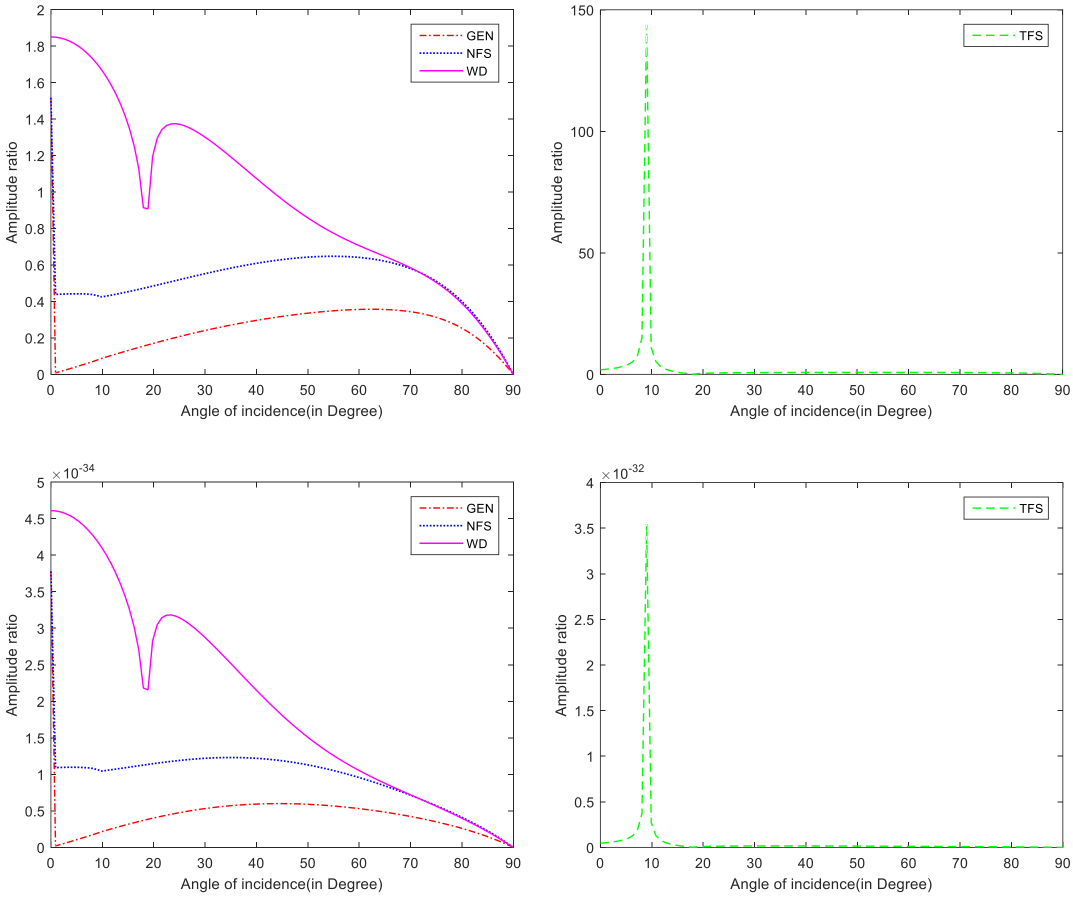


Fig.24-27. (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident LD wave (micropolarity approaches to zero).



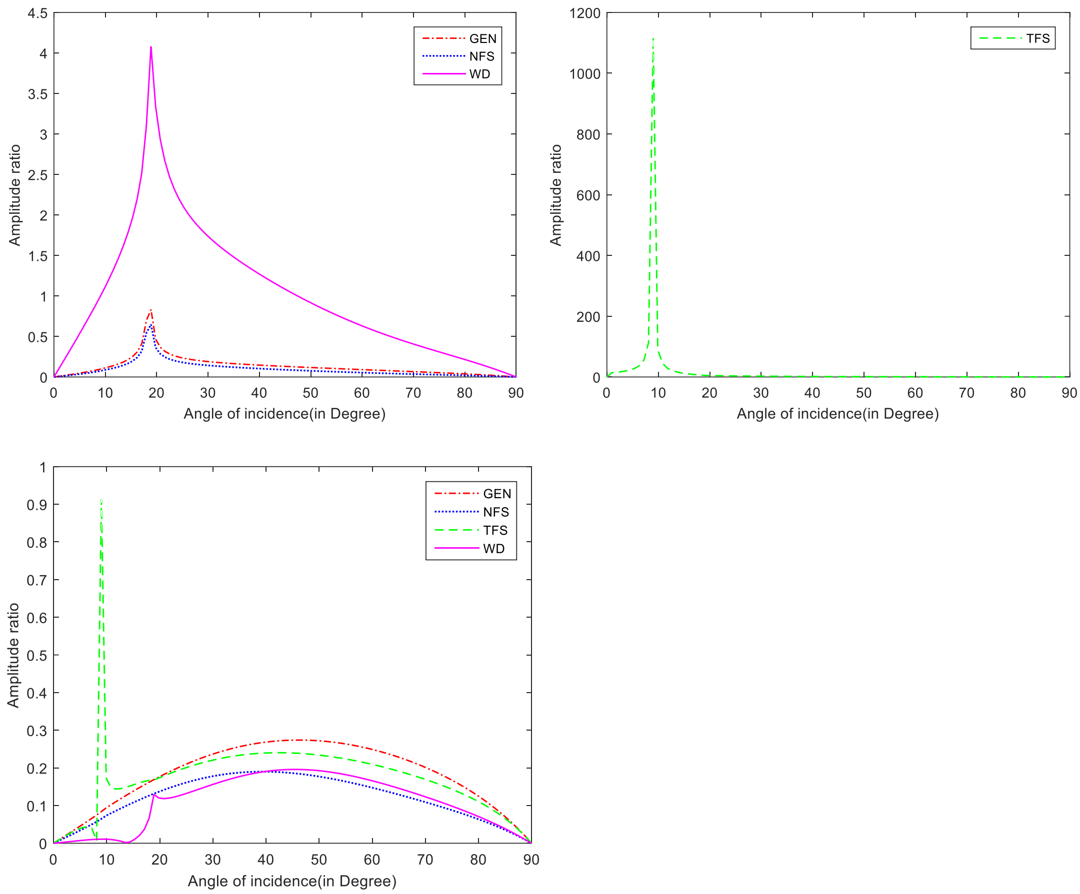


Fig.28-34. (Low frequency case) Variation of modulus amplitudes ratios $|Z_i|$, $(i = 4, 5, 6, 7)$ with angle of incidence of the incident LD wave (micropolarity approaches to zero).

VII. CONCLUSION

The analytic expression for the reflection and refraction coefficients of various reflected wave and refracted waves has been derived for the normal force stiffness, transverse force stiffness and welded contact. The results are consider to be useful in further theoretical and observational studies of propagation of waves in more realistic models of micropolar elastic solid present in the interior of earth. Making the use of appropriate set of boundary conditions, the system of simultaneous equations giving the amplitudes of various reflected and refracted waves are obtained.

(a) The amplitudes of various reflected and refracted waves are found to be complex valued.
(b) The modulus of amplitudes of various reflected and refracted waves depend upon angle of incidence, frequency, stiffness of forces and elastic properties of materials of the medium.
(c) Maximum amount of incident energy is carried along the reflected and refracted longitudinal displacement wave.
(d) For limit high and low frequency cases, the void volume fractional wave is more influenced by the micropolarity of the medium.

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