Infinite integral involving the spheroidal function, a class of polynomials multivariable Aleph-functions VII

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
In the present paper we evaluate a generalized infinite integral involving the product of the spheroidal function, the multivariable Aleph-functions and general class of polynomials of several variables with general arguments. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.

Keywords:Multivariable Aleph-function, general class of polynomials, spheroidal function, multivariable I-function, Aleph-function of two variable, I-function of two variables.

2010 Mathematics Subject Classification. 33C99, 33C60, 44A20

1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [4] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$$\left.\begin{array}{c}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k) = \frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)} s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)} s_k)]} \quad (1.3)$$

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \quad (1.4)$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \cdots, r, i = 1, \cdots, R \text{ , } i^{(k)} = 1, \cdots, R^{(k)} \quad (1.5)$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1, \cdots, y_r) = \sum_{G_1, \cdots, G_r = 0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} \psi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$$

$$\times \ \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) y_1^{-\eta_{G_1,g_1}} \cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(., \cdots, .), \theta_i(.), i = 1, \cdots, r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1} = \frac{d_{g_1}^{(1)} + G_1}{\delta_{g_1}^{(1)}}, \cdots, \ \eta_{G_r,g_r} = \frac{d_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i + p_i] \neq \delta_j^{(i)}[d_{g_i}^i + G_i]$ (1.7)

for $j \neq m_i, m_i = 1, \cdots \eta_{G_i,g_i}; p_i, n_i = 0, 1, 2, \cdots, ; y_i \neq 0, i = 1, \cdots, r$ (1.8)

Consider the Aleph-function of s variables

$$\aleph(z_1, \cdots, z_s) = \aleph_{P_i,Q_i,\iota_i;r':P_{i^{(1)}},Q_{i^{(1)}},\iota_{i^{(1)}};r^{(1)};\cdots;P_{i^{(s)}},Q_{i^{(s)}};\iota_{i^{(s)}};r^{(s)}}^{0,N:M_1,N_1,\cdots,M_s,N_s} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s \end{array} \middle| \right.$$

$$\begin{array}{ll} [(\mathrm{u}_j; \mu_j^{(1)}, \cdots, \mu_j^{(r')})_{1,N}] & ,[\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(r')})_{N+1,P_i}] : \\ \cdots\cdots\cdots\cdots\cdots\cdots & ,[\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots, v_{ji}^{(r')})_{M+1,Q_i}] : \end{array}$$

$$\left. \begin{array}{l} [(\mathrm{a}_j^{(1)}); \alpha_j^{(1)})_{1,N_1}], [\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}]; \cdots; [(\mathrm{a}_j^{(s)}); \alpha_j^{(s)})_{1,N_s}], [\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}] \\ [(\mathrm{b}_j^{(1)}); \beta_j^{(1)})_{1,M_1}], [\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}]; \cdots; [(\mathrm{b}_j^{(s)}); \beta_j^{(s)})_{1,M_s}], [\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}] \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \zeta(t_1, \cdots, t_s) \prod_{k=1}^{s} \phi_k(t_k) z_k^{t_k} \, \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.9}$$

with $\omega = \sqrt{-1}$

$$\zeta(t_1, \cdots, t_s) = \frac{\prod_{j=1}^{N} \Gamma(1 - u_j + \sum_{k=1}^{s} \mu_j^{(k)} t_k)}{\sum_{i=1}^{r'} [\iota_i \prod_{j=N+1}^{P_i} \Gamma(u_{ji} - \sum_{k=1}^{s} \mu_{ji}^{(k)} t_k) \prod_{j=1}^{Q_i} \Gamma(1 - v_{ji} + \sum_{k=1}^{s} v_{ji}^{(k)} t_k)]} \tag{1.10}$$

and $$\phi_k(t_k) = \frac{\prod_{j=1}^{M_k} \Gamma(b_j^{(k)} - \beta_j^{(k)} t_k) \prod_{j=1}^{N_k} \Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}} [\iota_{i^{(k)}} \prod_{j=M_k+1}^{Q_{i^{(k)}}} \Gamma(1 - b_{ji^{(k)}}^{(k)} + \beta_{ji^{(k)}}^{(k)} t_k) \prod_{j=N_k+1}^{P_{i^{(k)}}} \Gamma(a_{ji^{(k)}}^{(k)} - \alpha_{ji^{(k)}}^{(k)} s_k)]} \tag{1.11}$$

Suppose , as usual , that the parameters

$u_j, j=1,\cdots,P; v_j, j=1,\cdots,Q;$

$a_j^{(k)}, j=1,\cdots,N_k; a_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)}, j=m_k+1,\cdots,Q_{i^{(k)}}; b_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,s, i=1,\cdots,r'$, $i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}+\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j^{(k)}+\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}-\sum_{j=1}^{M_k}\beta_j^{(k)}$$

$$-\iota_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.12}$$

The reals numbers τ_i are positives for $i=1,\cdots,r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)}=1\cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)$ with $j=1$ to M_k are separated from those of $\Gamma(1-u_j+\sum_{i=1}^{s}\mu_j^{(k)}t_k)$with $j=1$ to N and $\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}t_k)$ with $j=1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where

$$B_i^{(k)}=\sum_{j=1}^{N}\mu_j^{(k)}-\iota_i\sum_{j=N+1}^{P_i}\mu_{ji}^{(k)}-\iota_i\sum_{j=1}^{Q_i}\upsilon_{ji}^{(k)}+\sum_{j=1}^{N_k}\alpha_j^{(k)}-\iota_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\alpha_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\beta_j^{(k)}-\iota_{i^{(k)}}\sum_{j=M_k+1}^{q_{i^{(k)}}}\beta_{ji^{(k)}}^{(k)}>0,\quad \text{with } k=1\cdots,s, i=1,\cdots,r\ , i^{(k)}=1,\cdots,r^{(k)} \tag{1.13}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_s)=0(|z_1|^{\alpha'_1},\cdots,|z_s|^{\alpha'_s}), max(|z_1|,\cdots,|z_s|)\rightarrow 0$

$\aleph(z_1,\cdots,z_s)=0(|z_1|^{\beta'_1},\cdots,|z_s|^{\beta'_s}), min(|z_1|,\cdots,|z_s|)\rightarrow\infty$

where, $k=1,\cdots,z:\alpha'_k=min[Re(b_j^{(k)}/\beta_j^{(k)})], j=1,\cdots,M_k$ and

$$\beta'_k=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j=1,\cdots,N_k$$

We will use these following notations in this paper

$$U = P_i, Q_i, \iota_i; r'; \; V = M_1, N_1; \cdots; M_s, N_s \tag{1.15}$$

$$\mathrm{W} = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}, \cdots, P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i^{(s)}}; r^{(s)} \tag{1.16}$$

$$A = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(s)})_{N+1,P_i}\} \tag{1.17}$$

$$B = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \cdots, v_{ji}^{(s)})_{M+1,Q_i}\} \tag{1.18}$$

$$C = (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1}, \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots, (a_j^{(s)}; \alpha_j^{(s)})_{1,N_s}, \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.19}$$

$$D = (b_j^{(1)}; \beta_j^{(1)})_{1,M_1}, \iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)}; \beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}}, \cdots, (b_j^{(s)}; \beta_j^{(s)})_{1,M_s}, \iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)}; \beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.20}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \cdots, z_s) = \aleph_{U:W}^{0,N:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A : C} \\ \cdot & \cdot\;\cdot\;\cdot \\ \cdot & \mathrm{B : D} \\ \mathrm{z}_s & \end{array}\right) \tag{1.21}$$

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}[y_1, \cdots, y_t] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_t)_{M_tK_t}}{K_t!}$$

$$A[N_1, K_1; \cdots; N_t, K_t] y_1^{K_1} \cdots y_t^{K_t} \tag{1.22}$$

Where $M_1, \cdots, M_s$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_t, K_t]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_t)_{M_tK_t}}{K_t!} A[N_1, K_1; \cdots; N_t, K_t] \tag{1.23}$$

In the document , we note :

$$G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) \tag{1.24}$$

where $\phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}), \theta_1(\eta_{G_1,g_1}), \cdots, \theta_r(\eta_{G_r,g_r})$ are given respectively in (1.2) and (1.3)

The spheroidal function $\psi_{\alpha n}(c, \eta)$ of general order $\alpha > -1$ can be expansed as ([3] an [8].

$$\psi_{\alpha n}(c, \eta) = \frac{i^n \sqrt{2\pi}}{V_{\alpha n(c)}} \sum_{k=0,or1}^{\infty_*} a_k(c|\alpha n)(c\eta)^{-\alpha-\frac{1}{2}} J_{k+\alpha+\frac{1}{2}}(c\eta) \tag{1.25}$$

which represents the function uniformly on (∞, ∞), where the coefficients $a_k(c|\alpha n)$ satisfy the recursion formula [14 ,eq.67]and the asterisk over the summation sign indicates that the sum is taken over only even or odd values of k according as n is even or odd. As $c \to 0, a_k(c|\alpha n) \to 0, k \neq n$

2. Required integral

We have the following integral, see Marichev et al ([1], 2.2.9 23 page 311).

Lemme

$$\int_0^{+\infty}\frac{x^{\alpha-1}}{(x^2+2xycos\gamma+y^2)^\rho}\text{d}x=\frac{1}{2}y^{\alpha-2\rho}\left\{B\left(\frac{\alpha}{2},\rho-\frac{\alpha}{2}\right){}_2F_1\left[\frac{\alpha}{2},\rho-\frac{\alpha}{2};\frac{1}{2};cos^2\gamma\right]-\right.$$

$$\left.(\alpha-1)|cos\gamma|\ B\left(\frac{\alpha-1}{2},\rho-\frac{\alpha-1}{2}\right){}_2F_1\left[\frac{\alpha-1}{2},\rho-\frac{\alpha-1}{2};\frac{3}{2};cos^2\gamma\right]\right\} \tag{2.1}$$

with $|argy|<\pi,0<|\gamma|<\pi,0<Re\alpha<2Re\rho$

3. Main integral

Let $X_{\alpha,\rho}=\dfrac{x^\alpha}{(x^2+2xycos\gamma+y^2)^\rho}$, we have the following generalized infinite integral :

Theorem

$$\int_0^{+\infty}\frac{x^{\alpha-1}}{(x^2+2xycos\gamma+y^2)^\rho}\psi_{\alpha n}(c^\sigma,X_{\beta,\gamma})S^{M_1,\cdots,M_t}_{N_1,\cdots,N_t}\begin{pmatrix}\text{y}_1X_{\gamma_1,\mu_1}\\ \cdot\ \cdot\ \cdot\\ \text{y}_tX_{\gamma_t,\mu_t}\end{pmatrix}\aleph^{0,\text{n}:v}_{u:w}\begin{pmatrix}\text{z}_1X_{\alpha_1,\beta_1}\\ \cdot\ \cdot\ \cdot\\ \text{z}_rX_{\alpha_r,\beta_r}\end{pmatrix}$$

$$\aleph^{0,N:V}_{U:W}\begin{pmatrix}\text{Z}_1X_{\eta_1,\epsilon_1}\\ \cdot\ \cdot\ \cdot\\ \text{Z}_sX_{\eta_s,\epsilon_s}\end{pmatrix}\text{d}x=\frac{1}{2}y^{\alpha-2\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\frac{cos^{2n'}\gamma}{n'!\left(\frac{1}{2}\right)_{n'}}$$

$$\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}c^{\sigma(2m+k)}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}\quad 2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-2\mu_i)+\sum_{i=1}^r\eta_{G_i,g_i}(\alpha_i-2\beta_i)}\aleph^{0,N+2:V}_{U_{21}:W}\left(\begin{matrix}\text{Z}_1z^{\eta_1-2\epsilon_1}\\ \cdot\ \cdot\ \cdot\\ \cdots\\ \text{Z}_sz^{\eta_s-2\epsilon_s}\end{matrix}\ \middle|\right.$$

$$\left.\begin{matrix}(\text{1-n'-}\frac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),A_1,A:C\\ \cdot\ \cdot\ \cdot\\ (\text{1-}(\rho+\delta RA+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r\eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s),B:D\end{matrix}\right)$$

$$-|cos\gamma|\frac{1}{2}y^{\alpha-2\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\frac{cos^{2n'}\gamma}{n'!\left(\frac{3}{2}\right)_{n'}}c^{\sigma(2m+k)}$$

$$a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}$$

$$2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-2\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-2\beta_i)}\aleph_{U_{32}:W}^{0,N+3:V}\left(\begin{array}{c}\mathrm{Z}_1 z^{\eta_1-2\epsilon_1}\\ \cdot\cdot\cdot\\ \cdots\\ \mathrm{Z}_s z^{\eta_s-2\epsilon_s}\end{array}\right|$$

$$(1\text{-}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),$$
$$\cdot\cdot\cdot$$
$$(2\text{-}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),$$

$$\left.\begin{array}{c}(\frac{3}{2}\text{-n'-}\frac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),A_2,A:C\\ \cdot\cdot\cdot\\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r\eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s),B:D\end{array}\right) \quad (3.1)$$

Where

$$\mathrm{A}_1=\Big(1-n'+\tfrac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i)-\rho-\gamma RA+\sum_{i=1}^t K_i\mu_i-$$

$$-\sum_{i=1}^r\eta_{G_i,g_i}\beta_i;\epsilon_1-\frac{\eta_1}{2},\cdots,\epsilon_s-\frac{\eta_s}{2}\Big) \quad (3.2)$$

$$\mathrm{A}_2=\Big(\tfrac{1}{2}-n'+\tfrac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i)-\rho-\gamma RA+\sum_{i=1}^t K_i\mu_i-$$

$$-\sum_{i=1}^r\eta_{G_i,g_i}\beta_i;\epsilon_1-\frac{\eta_1}{2},\cdots,\epsilon_s-\frac{\eta_s}{2}\Big) \quad (3.3)$$

We use the notations : $U_{21}=P_i+2;Q_i+1;\iota_i;r'$ and $U_{32}=P_i+3;Q_i+2;\iota_i;r'$

Provided that

a) $min\{\gamma_i,\mu_i,\alpha_j,\beta_j,\eta_k,\epsilon_k\}>0,i=1,\cdots,t,j=1,\cdots,r,k=1,\cdots,s$

b) $|argy|<\pi,0<|\gamma|<\pi,$

c) $0<Re(\alpha+(2m+k)\beta)+\sum_{i=1}^r\alpha_i\min_{1\leqslant j\leqslant m_i}Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+\sum_{i=1}^s\eta_i\min_{1\leqslant j\leqslant M_i}Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)<$

$$< 2Re(\rho+(2m+k)\gamma)+\sum_{i=1}^{r}\beta_i \min_{1\leqslant j\leqslant m_i} Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)+\sum_{i=1}^{s}\epsilon_i \min_{1\leqslant j\leqslant M_i} Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)$$

d)$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) ; $i=1,\cdots,r$

e)$|argZ_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (1.13) ; $i=1,\cdots,s$

e) The series occuring on the right-hand side of (3.1) is absolutely and uniformly convergent.

Proof : Let $M\{\} = \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\zeta(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)\{\}$ (3.4)

Expressing the spheroidal function with the help of equation (1.25), the Aleph-function of r variables in series with the help of equation (1.6), the general class of polynomial of several variables $S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}[y_1,\cdots,y_t]$ with the help of equation (1.22) and the Aleph-function of s variables in Mellin-Barnes contour integral with the help of equation (1.9), we obtain

$$\text{L.H.S} = \int_0^{+\infty}\frac{x^{\alpha-1}}{(x^2+2xycos\gamma+y^2)^\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}$$

$$a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}c^{\sigma(2m+k)}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}$$

$$M\left[Z_1^{t_1}\cdots Z_s^{t_s}\frac{x^{\beta Ra+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i+\sum_{i=1}^s\eta_i t_i}}{(x^2+2xycos\gamma+y^2)^{\gamma Ra+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r\eta_{G_i,g_i}i+\sum_{i=1}^s\epsilon_i t_i}}\right]\mathrm{d}t_1\cdots\mathrm{d}t_s\ \mathrm{d}x \tag{3.6}$$

Now, change the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we thus find that :

$$\text{L.H.S} = \frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}c^{\sigma(2m+k)}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}$$

$$M\left[Z_1^{t_1}\cdots Z_s^{t_s}\int_0^{\infty}\frac{x^{\alpha+\beta Ra+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i+\sum_{i=1}^s\eta_i t_i-1}}{(x^2+2xycos\gamma+y^2)^{\rho+\gamma Ra+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r\eta_{G_i,g_i}i+\sum_{i=1}^s\epsilon_i t_i}}dx\right]\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{3.7}$$

Evaluating the inner integral with the help of the equation (2.1). Now expressing the Gauss hypergeometric function ${}_2F_1$ in serie, use the relation $\Gamma(a)(a)_n = \Gamma(a+n)$ with $Re(a) > 0$ for the first term and use the relations $\Gamma(a)(a)_n = \Gamma(a+n)$ with $Re(a) > 0$ and $\alpha - 1 = \dfrac{\Gamma(\alpha)}{\Gamma(\alpha-1)}$ with $Re(\alpha) > 0$ for the second term. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

4. Multivariable I-function

If $\iota_i, \iota_{i^{(1)}}, \cdots, \iota_{i^{(s)}} \rightarrow 1$, the Aleph-function of several variables degenere to the I-function of several variables. The generalized simple integral have been derived in this section for multivariable I-functions defined by Sharma et al [3].

Corollary 1

$$\int_0^{+\infty} \frac{x^{\alpha-1}}{(x^2+2xycos\gamma+y^2)^{\rho}} \psi_{\alpha n}(c^{\sigma}, X_{\beta,\gamma}) S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c} \mathrm{y}_1 X_{\gamma_1,\mu_1} \\ \cdots \\ \mathrm{y}_t X_{\gamma_t,\mu_t} \end{array}\right) \aleph_{u:w}^{0,\mathrm{n}:v}\left(\begin{array}{c} \mathrm{z}_1 X_{\alpha_1,\beta_1} \\ \cdots \\ \mathrm{z}_r X_{\alpha_r,\beta_r} \end{array}\right)$$

$$I_{U:W}^{0,N:V}\left(\begin{array}{c} \mathrm{Z}_1 X_{\eta_1,\epsilon_1} \\ \cdots \\ \mathrm{Z}_s X_{\eta_s,\epsilon_s} \end{array}\right) \mathrm{d}x = \frac{1}{2} y^{\alpha-2\rho} \frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^{\sigma})} \sum_{k=0,or1}^{\infty_*} \sum_{m=0}^{\infty} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \frac{cos^{2n'}\gamma}{n'!\left(\frac{1}{2}\right)_{n'}}$$

$$\sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} a_1 \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) \frac{(-)^m a_k(c^{\sigma}|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})} y_1^{K_1} \cdots y_t^{K_t} c^{\sigma(2m+k)}$$

$$z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} \quad 2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-2\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-2\beta_i)} I_{U_{21}:W}^{0,N+2:V}\left(\begin{array}{c} \mathrm{Z}_1 z^{\eta_1-2\epsilon_1} \\ \cdots \\ \cdots \\ \mathrm{Z}_s z^{\eta_s-2\epsilon_s} \end{array}\right|$$

$$\left.\begin{array}{c} (1\text{-n'-}\frac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s), A_1, A : C \\ \cdots \\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s), B : D \end{array}\right)$$

$$\text{-}|cos\gamma| \frac{1}{2} y^{\alpha-2\rho} \frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^{\sigma})} \sum_{k=0,or1}^{\infty_*} \sum_{m=0}^{\infty} \sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{cos^{2n'}\gamma}{n'!\left(\frac{3}{2}\right)_{n'}} c^{\sigma(2m+k)}$$

$$a_1 \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) \frac{(-)^m a_k(c^{\sigma}|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})} y_1^{K_1} \cdots y_t^{K_t} z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}}$$

$$2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^{t}K_i(\gamma_i-2\mu_i)+\sum_{i=1}^{r}\eta_{G_i,g_i}(\alpha_i-2\beta_i)}\ I_{U_{32}:W}^{0,N+3:V}\left(\begin{array}{c} Z_1 z^{\eta_1-2\epsilon_1} \\ \cdot\cdot\cdot \\ \cdots \\ Z_s z^{\eta_s-2\epsilon_s} \end{array}\right|$$

$$(1\text{-}(\alpha+\beta RA+\textstyle\sum_{i=1}^{t}K_i\gamma_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),$$
$$\cdot\cdot\cdot$$
$$(2\text{-}(\alpha+\beta RA+\textstyle\sum_{i=1}^{t}K_i\gamma_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),$$

$$\left.\begin{array}{c}(\frac{3}{2}\text{-n'-}\frac{1}{2}(\alpha+\beta RA+\sum_{i=1}^{t}K_i\gamma_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),A_2,A:C \\ \cdot\cdot\cdot \\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^{t}K_i\mu_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s),B:D\end{array}\right) \quad (4.1)$$

under the same notations and conditions that (4.1) with $\iota_i,\iota_{i^{(1)}},\cdots,\iota_{i^{(s)}}\rightarrow 1$

5. Aleph-function of two variables

If $s=2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following simple integrals.

Corollary 2

$$\int_0^{+\infty}\frac{x^{\alpha-1}}{(x^2+2xycos\gamma+y^2)^\rho}\psi_{\alpha n}(c^\sigma,X_{\beta,\gamma})S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c} \text{y}_1X_{\gamma_1,\mu_1} \\ \cdot\cdot\cdot \\ \text{y}_tX_{\gamma_t,\mu_t}\end{array}\right)\aleph_{u:w}^{0,\text{n}:v}\left(\begin{array}{c} \text{z}_1X_{\alpha_1,\beta_1} \\ \cdot\cdot\cdot \\ \text{z}_rX_{\alpha_r,\beta_r}\end{array}\right)$$

$$\aleph_{U:W}^{0,N:V}\left(\begin{array}{c} \text{Z}_1X_{\eta_1,\epsilon_1} \\ \cdot\cdot\cdot \\ \text{Z}_2X_{\eta_2,\epsilon_2}\end{array}\right)\text{d}x=\frac{1}{2}y^{\alpha-2\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\frac{cos^{2n'}\gamma}{n'!\left(\frac{1}{2}\right)_{n'}}$$

$$\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}c^{\sigma(2m+k)}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}\quad 2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^{t}K_i(\gamma_i-2\mu_i)+\sum_{i=1}^{r}\eta_{G_i,g_i}(\alpha_i-2\beta_i)}\aleph_{U_{21}:W}^{0,N+2:V}\left(\begin{array}{c} Z_1 z^{\eta_1-2\epsilon_1} \\ \cdot\cdot\cdot \\ \cdots \\ Z_2 z^{\eta_2-2\epsilon_2} \end{array}\right|$$

$$\left.\begin{array}{c}(1\text{-n'-}\frac{1}{2}(\alpha+\beta RA+\sum_{i=1}^{t}K_i\gamma_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),A_1,A:C \\ \cdot\cdot\cdot \\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^{t}K_i\mu_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\beta_i);\epsilon_1,\epsilon_2),B:D\end{array}\right)$$

$$-|cos\gamma|\frac{1}{2}y^{\alpha-2\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\frac{cos^{2n'}\gamma}{n'!\left(\frac{3}{2}\right)_{n'}}c^{\sigma(2m+k)}$$

$$a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}$$

$$2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-2\mu_i)+\sum_{i=1}^r\eta_{G_i,g_i}(\alpha_i-2\beta_i)}\aleph_{U_{32}:W}^{0,N+3:V}\left(\begin{array}{c|}Z_1z^{\eta_1-2\epsilon_1}\\ \cdots\\ \cdots\\ Z_2z^{\eta_2-2\epsilon_2}\end{array}\right.$$

$$(1\text{-}(\alpha+\beta RA+\textstyle\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),$$
$$\cdots$$
$$(2\text{-}(\alpha+\beta RA+\textstyle\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),$$

$$\left.\begin{array}{c}(\frac{3}{2}\text{-n'-}\frac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r\eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),A_2,A:C\\ \cdots\\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r\eta_{G_i,g_i}\beta_i);\epsilon_1,\epsilon_2),B:D\end{array}\right) \quad (5.1)$$

under the same conditions and notation that (3.1) with $s=2$

6. I-function of two variables

If $\iota_i,\iota_i',\iota_i''\rightarrow 1$, then the Aleph-function of two variables degenere in the I-function of two variables defined by sharma et al [4] and we obtain the same formula with the I-function of two variables.

Corollary 3

$$\int_0^{+\infty}\frac{x^{\alpha-1}}{(x^2+2xycos\gamma+y^2)^\rho}\psi_{\alpha n}(c^\sigma,X_{\beta,\gamma})S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c}y_1X_{\gamma_1,\mu_1}\\ \cdots\\ y_tX_{\gamma_t,\mu_t}\end{array}\right)\aleph_{u:w}^{0,n:v}\left(\begin{array}{c}z_1X_{\alpha_1,\beta_1}\\ \cdots\\ z_rX_{\alpha_r,\beta_r}\end{array}\right)$$

$$I_{U:W}^{0,N:V}\left(\begin{array}{c}Z_1X_{\eta_1,\epsilon_1}\\ \cdots\\ Z_2X_{\eta_2,\epsilon_2}\end{array}\right)\mathrm{d}x=\frac{1}{2}y^{\alpha-2\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\frac{cos^{2n'}\gamma}{n'!\left(\frac{1}{2}\right)_{n'}}$$

$$\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}c^{\sigma(2m+k)}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}} \quad 2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-2\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-2\beta_i)} I_{U_{21}:W}^{0,N+2:V}\left(\begin{array}{c} Z_1 z^{\eta_1-2\epsilon_1} \\ \cdots \\ \cdots \\ Z_2 z^{\eta_2-2\epsilon_2} \end{array}\right|$$

$$\left.\begin{array}{c}(1\text{-n'-}\tfrac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),A_1,A:C \\ \cdots \\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\epsilon_1,\epsilon_2),B:D\end{array}\right)$$

$$-|cos\gamma|\frac{1}{2}y^{\alpha-2\rho}\frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)}\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\frac{cos^{2n'}\gamma}{n'!\left(\frac{3}{2}\right)_{n'}}c^{\sigma(2m+k)}$$

$$a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}y_1^{K_1}\cdots y_t^{K_t}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}$$

$$2^{(\beta-2\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-2\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-2\beta_i)} I_{U_{32}:W}^{0,N+3:V}\left(\begin{array}{c} Z_1 z^{\eta_1-2\epsilon_1} \\ \cdots \\ \cdots \\ Z_2 z^{\eta_2-2\epsilon_2} \end{array}\right|$$

$$\begin{array}{c}(1\text{-}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2), \\ \cdots \\ (2\text{-}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),\end{array}$$

$$\left.\begin{array}{c}(\tfrac{3}{2}\text{-n'-}\tfrac{1}{2}(\alpha+\beta RA+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\eta_2),A_2,A:C \\ \cdots \\ (1\text{-}(\rho+\delta RA+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\epsilon_1,\epsilon_2),B:D\end{array}\right) \qquad (6.1)$$

under the same conditions and notation that (3.1) with $s=2$ and $\iota_i,\iota_i',\iota_i''\to 1$

7. Conclusion

In this paper we have evaluated a unified generalized infinite integral involving the multivariable Aleph-functions, a class of polynomials of several variables and the spheroidal function and general arguments.The integral established in this paper is of very general nature as it contains Multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1]Marichev O.I. Prudnikov A.P. And Brychkow Y.A. Elementay functions. Integrals and series Vol 1. USSR Academy of sciences . Moscow 1986.

[2] Rhodes D.R. On the spheroidal functions. J. Res. Nat. Bur. Standards. Sect. B 74(1970), page187-209.

[3] Sharma C.K.and Ahmad S.S.: On the multivariable I-function. Acta ciencia Indica Math , 1994 vol 20,no2, p 113-116.

[4] C.K. Sharma and P.L. mishra : On the I-function of two variables and its properties. Acta Ciencia Indica Math , 1991 Vol 17 page 667-672.

[5] Sharma K. On the integral representation and applications of the generalized function of two variables , International Journal of Mathematical Engineering and Sciences , Vol 3 , issue1 (2014) , page1-13.

[6] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.

[7] Stratton J.A. And Chu L.J. Elliptic and spheroidal wave function J. Math. And Phys. 20 (1941), page 259-309.

Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE