Infinite integral involving the spheroidal function, a class of polynomials and multivariable I-functions I

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ABSTRACT
In the present paper we evaluate a infinite integral involving the product of the spheroidal function, multivariable I-functions defined by Prasad [2] and general class of polynomials with general arguments. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.

Keywords:Multivariable I-function, general class of polynomials,spheroidal function, multivariable H-function.

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1.Intoduction

In the present paper we evaluate a infinite integral involving the product of the spheroidal function, multivariable I-functions defined by Prasad [2] and general class of polynomials with general arguments.
The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,N_2;0,N_3;\cdots;0,N_r:m',n';\cdots;M^{(s)},N^{(r)}}_{P_2,Q_2,P_3,Q_3;\cdots;P_r,Q_r:P',Q';\cdots;P^{(r)},Q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{A}_{2j};\gamma'_{2j},\gamma''_{2j})_{1,P_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{B}_{2j};\delta'_{2j},\delta''_{2j})_{1,Q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{A}_{rj};\gamma'_{rj},\cdots,\gamma^{(r)}_{rj})_{1,P_r}:(A'_j,\gamma'_j)_{1,P'};\cdots;(A^{(r)}_j,\gamma^{(r)}_j)_{1,P^{(r)}} \\ \\ (\mathrm{B}_{rj};\delta'_{rj},\cdots,\delta^{(r)}_{rj})_{1,Q_r}:(B'_j,\delta'_j)_{1,Q'};\cdots;(B^{(r)}_j,\delta^{(r)}_j)_{1,Q^{(r)}} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=\sum_{k=1}^{N^{(i)}}\gamma_k^{(i)}-\sum_{k=N^{(i)}+1}^{P^{(i)}}\gamma_k^{(i)}+\sum_{k=1}^{M^{(i)}}\delta_k^{(i)}-\sum_{k=M^{(i)}+1}^{Q^{(i)}}\delta_k^{(i)}+\left(\sum_{k=1}^{N_2}\alpha_{2k}^{(i)}-\sum_{k=N_2+1}^{P_2}\gamma_{2k}^{(i)}\right)+\cdots+$$

$$\left(\sum_{k=1}^{N_r}\gamma_{rk}^{(i)}-\sum_{k=N_r+1}^{P_r}\gamma_{rk}^{(i)}\right)-\left(\sum_{k=1}^{Q_2}\delta_{2k}^{(i)}+\sum_{k=1}^{Q_3}\delta_{3k}^{(i)}+\cdots+\sum_{k=1}^{Q_r}\delta_{rk}^{(i)}\right) \tag{1.3}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\gamma_1'},\cdots,|z_r|^{\gamma_r'}\,)\,,max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\ =0(\,|z_1|^{\delta_1'},\cdots,|z_r|^{\beta_r'}\,)\,,min(\,|z_1,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\gamma_k'=min[Re(B_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,M_k$ and

$$\delta_k'=max[Re((A_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,N_k$$

We will use these following notations in this paper :

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

Serie representation of multivariable I-function of several variables is given by

$$I(y_1,\cdots,y_r)=\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})y_1^{-\eta_{G_1,g_1}}\cdots y_r^{-\eta_{G_r,g_r}} \tag{1.4}$$

Where $\psi(.,\cdots,.),\theta_i(.),i=1,\cdots,r$ are defined by Prasad (see integral (1.2))

$$\eta_{G_1,g_1}=\frac{B_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \ \eta_{G_r,g_r}=\frac{B_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[B_j^i+p_i]\neq\delta_j^{(i)}[B_{g_i}^i+G_i]$ (1.5)

for $j\neq M_i,M_i=1,\cdots\eta_{G_i,g_i};P_i,N_i=0,1,2,\cdots,;y_i\neq 0,i=1,\cdots,r$ (1.6)

In the document , we will note :

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})=\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}) \tag{1.7}$$

where $\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),\theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are given in (1.2)

We will use these following notations in this paper :

$$U_1 = P_2, Q_2; P_3, Q_3; \cdots ; P_{r-1}, q_{r-1}; V_1 = 0, N_2; 0, N_3; \cdots ; 0, N_{r-1} \tag{1.8}$$

$$W_1 = (P', Q'); \cdots ; (P^{(r)}, Q^{(r)}); X_1 = (M', N'); \cdots ; (M^{(r)}, N^{(r)}) \tag{1.9}$$

$$A_1 = (A_{2k}, \gamma_{2k}^{(1)}, \gamma_{2k}^{(2)}); \cdots ; (A_{(r-1)k}, \gamma_{(r-1)k}^{(1)}, \gamma_{(r-1)k}^{(2)}, \cdots , \gamma_{(r-1)k}^{(r-1)}) \tag{1.10}$$

$$B_1 = (B_{2k}, \delta_{2k}^{(1)}, \delta_{2k}^{(2)}); \cdots ; (B_{(r-1)k}, \delta_{(r-1)k}^{(1)}, \delta_{(r-1)k}^{(2)}, \cdots , \delta_{(r-1)k}^{(r-1)}) \tag{1.11}$$

$$\mathfrak{A}_1 = (A_{rk}; \gamma_{rk}^{(1)}, \gamma_{rk}^{(2)}, \cdots , \gamma_{rk}^{(r)}) : \mathfrak{B}_1 = (B_{rk}; \delta_{rk}^{(1)}, \delta_{rk}^{(2)}, \cdots , \delta_{rk}^{(r)}) \tag{1.12}$$

$$A_1' = (A_k', \gamma_k')_{1,P'}; \cdots ; (a_k^{(r)}, \gamma_k^{(r)})_{1,P^{(r)}}; B_1' = (B_k', \delta_k')_{1,Q'}; \cdots ; (B_k^{(r)}, \delta_k^{(r)})_{1,Q^{(r)}} \tag{1.13}$$

The multivariable I-function of r-variables write :

$$I(z_1, \cdots , z_r) = I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}_1; \mathfrak{A}_1; A_1' \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & \mathrm{B}_1; \mathfrak{B}_1; B_1' \end{array}\right) \tag{1.14}$$

The multivariable I-function of s-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, ...z_s) = I_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p',q';\cdots;p^{(s)},q^{(s)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(s)},n^{(s)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j}; \alpha_{2j}', \alpha_{2j}'')_{1,p_2}; \cdots ; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_{2j}; \beta_{2j}', \beta_{2j}'')_{1,q_2}; \cdots ; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{sj}; \alpha_{sj}', \cdots , \alpha_{sj}^{(s)})_{1,p_s} : (a_j', \alpha_j')_{1,p'}; \cdots ; (a_j^{(s)}, \alpha_j^{(s)})_{1,p^{(s)}} \\ \\ (\mathrm{b}_{sj}; \beta_{sj}', \cdots , \beta_{sj}^{(s)})_{1,q_s} : (b_j', \beta_j')_{1,q'}; \cdots ; (b_j^{(s)}, \beta_j^{(s)})_{1,q^{(s)}} \end{array}\right) \tag{1.15}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1} \cdots \int_{L_s} \xi(t_1, \cdots , t_s)\prod_{i=1}^{s} \phi_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.16}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega'_i\pi$, where

$$\Omega'_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)}\right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)}\right) \tag{1.17}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_s) = 0(\,|z_1|^{\gamma'_1}, \cdots, |z_s|^{\gamma'_s}\,)\,, max(\,|z_1|, \cdots, |z_s|\,) \rightarrow 0$$

$$I(z_1, \cdots, z_s)\; = 0(\,|z_1|^{\delta'_1}, \cdots, |z_s|^{\beta'_s}\,)\,, min(\,|z_1, \cdots, |z_s|\,) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.18}$$

$$W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \tag{1.19}$$

$$A = (a_{2k}, \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)}); \cdots; (a_{(s-1)k}, \alpha_{(s-1)k}^{(1)}, \alpha_{(s-1)k}^{(2)}, \cdots, \alpha_{(s-1)k}^{(s-1)}) \tag{1.20}$$

$$B = (b_{2k}, \beta_{2k}^{(1)}, \beta_{2k}^{(2)}); \cdots; (b_{(s-1)k}, \beta_{(s-1)k}^{(1)}, \beta_{(s-1)k}^{(2)}, \cdots, \beta_{(s-1)k}^{(s-1)}) \tag{1.21}$$

$$\mathfrak{A} = (a_{sk}; \alpha_{sk}^{(1)}, \alpha_{sk}^{(2)}, \cdots, \alpha_{sk}^{(s)}) : \mathfrak{B} = (b_{sk}; \beta_{sk}^{(1)}, \beta_{sk}^{(2)}, \cdots, \beta_{sk}^{(s)}) \tag{1.22}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b'_k, \beta'_k)_{1,q'}; \cdots; (b_k^{(s)}, \beta_k^{(s)})_{1,q^{(s)}} \tag{1.23}$$

The multivariable I-function write :

$$I(z_1,\cdots,z_s) = I^{V;0,n_s;X}_{U:p_s,q_s;W}\left(\begin{array}{c|c} \mathrm{z}_1 & \text{A };\mathfrak{A};\text{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & \text{B}; \mathfrak{B};\text{ B'} \end{array}\right) \tag{1.24}$$

The generalized polynomials defined by Srivastava [4], is given in the following manner :

$$S^{M_1,\cdots,M_t}_{N_1,\cdots,N_t}[y_1,\cdots,y_t] \; = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_t)_{M_tK_t}}{K_t!}$$

$$A[N_1,K_1;\cdots;N_t,K_t]y_1^{K_1}\cdots y_t^{K_t} \tag{1.25}$$

Where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_t,K_t]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_t)_{M_tK_t}}{K_t!}A[N_1,K_1;\cdots;N_t,K_t] \tag{1.26}$$

In the document , we note :

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}) \tag{1.27}$$

where $\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),\theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are given respectively by (1.2)

The spheroidal function $\psi_{\alpha n}(c,\eta)$ of general order $\alpha > -1$ can be expansed as ([3] an [6].

$$\psi_{\alpha n}(c,\eta)= \frac{i^n\sqrt{2\pi}}{V_{\alpha n(c)}}\sum_{k=0,or1}^{\infty *} a_k(c|\alpha n)(c\eta)^{-\alpha-\frac{1}{2}}J_{k+\alpha+\frac{1}{2}}(c\eta) \tag{1.28}$$

which represents the function uniformly on (∞,∞), where the coefficients $a_k(c|\alpha n)$ satisfy the recursion formula [14 ,eq.67]and the asterisk over the summation sign indicates that the sum is taken over only even or odd values of k according as n is even or odd. As $c\rightarrow 0, a_k(c|\alpha n)\rightarrow 0, k\neq n$

2. Required integral

We have the following result , see Marichev et al ([1], 2.2.11, eq.26 page 316)

Lemme

$$\int_0^{+\infty}\frac{x^{\alpha-1}}{\sqrt{(x^2+z^2)(x^2y^2+z^2)}\left(\sqrt{x^2+z^2}+\sqrt{(x^2y^2+z^2)}\right)^{\mu}}\mathrm{d}x = 2^{-\mu-1}z^{\alpha-\mu-2}B\left(1+\frac{\mu-\alpha}{2},\frac{\alpha}{2}\right)$$

$$\times{}_2F_1\left[\frac{\alpha}{2},\frac{\mu+1}{2};1+\mu;1-y^2\right] \tag{2.1}$$

where : $Rez, Rey > 0, 0 < Re(\alpha) < Re(\mu)+2$

3.Main integral

Let $X_{\alpha,\beta} = \dfrac{x^{\alpha-1}}{\left(\sqrt{x^2+z^2}+\sqrt{(x^2y^2+z^2)}\right)^{\mu}}$, we have the following generalized infinite integral

Theorem

$$\int_0^{+\infty} \frac{x^{\alpha-1}}{\sqrt{(x^2+z^2)(x^2y^2+z^2)}\left(\sqrt{x^2+z^2}+\sqrt{(x^2y^2+z^2)}\right)^{\mu}} \psi_{\alpha n}(c^{\sigma}, X_{\beta,\gamma}) S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c} \mathrm{y}_1 X_{\gamma_1,\mu_1} \\ \cdot\cdot\cdot \\ \mathrm{y}_t X_{\gamma_t,\mu_t} \end{array}\right)$$

$$I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{array}{c} \mathrm{z}_1 X_{\alpha_1,\beta_1} \\ \cdot\cdot\cdot \\ \mathrm{z}_r X_{\alpha_r,\beta_r} \end{array}\right) I_{U:p_s,q_s;W}^{V;0,n_s;X}\left(\begin{array}{c} \mathrm{Z}_1 X_{\eta_1,\epsilon_1} \\ \cdot\cdot\cdot \\ \mathrm{Z}_s X_{\eta_s,\epsilon_s} \end{array}\right) \mathrm{d}x = \frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^{\sigma})} 2^{-\mu} z^{\alpha-\mu-1} \sum_{k=0,or1}^{\infty_*} \sum_{m=0}^{\infty}$$

$$\sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} a_1 \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$$

$$\frac{(-)^m a_k(c^{\sigma}|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})} y_1^{K_1} \cdots y_t^{K_t} z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} c^{\sigma(2m+k)} 2^{-(\gamma(2m+k)+\sum_{i=1}^t K_i\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i)}$$

$$\frac{1}{n'!}(1-y^2)^{n'} z^{(\beta-\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-\beta_i)} I_{U:p_s+4,q_s+3;W}^{V;0,n_s+4;X}\left(\begin{array}{c} \mathrm{Z}_1 z^{\eta_1-\epsilon_1} 2^{-\epsilon_1} \\ \cdot\cdot\cdot \\ \cdots \\ \mathrm{Z}_s z^{\eta_s-\epsilon_s} 2^{-\epsilon_s} \end{array}\right|$$

A; (1-n'-$\frac{1}{2}(\alpha+\beta(2m+k)+\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i; \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2})$,

$\cdot\cdot\cdot$

B;

($\frac{1}{2}(\alpha-\mu+(\beta-\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-\beta_i)); \frac{\epsilon_1-\eta_1}{2}, \cdots, \frac{\epsilon_s-\eta_s}{2})$,

$\cdot\cdot\cdot$

(-$\frac{1}{2}(\mu+\gamma(2m+k)+\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i); \frac{\epsilon_1}{2}, \cdots, \frac{\epsilon_s}{2})$,

(-n'+$\frac{1}{2}(1-\mu-\delta RA-\sum_{i=1}^t K_i\mu_i-\sum_{i=1}^r \eta_{G_i,g_i}\beta_i); \epsilon_1, \cdots, \epsilon_s)$,

$\cdot\cdot\cdot$

(+$\frac{1}{2}(1-\mu-\delta RA-\sum_{i=1}^t K_i\mu_i-\sum_{i=1}^r \eta_{G_i,g_i}\beta_i); \epsilon_1, \cdots, \epsilon_s)$,

$$\left.\begin{matrix}(\text{-}\mu-\delta RA-\sum_{i=1}^{t}K_i\mu_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\beta_i;\epsilon_1,\cdots,\epsilon_s),\mathfrak{A}:A' \\ \cdots \\ (\text{-n'-}\mu-\delta RA-\sum_{i=1}^{t}K_i\mu_i-\sum_{i=1}^{r}\eta_{G_i,g_i}\beta_i;\epsilon_1,\cdots,\epsilon_s),\mathfrak{B}:B'\end{matrix}\right) \tag{3.1}$$

a) $min\{\gamma_i,\mu_i,\alpha_j,\beta_j,\eta_k,\epsilon_k\}>0, i=1,\cdots,t, j=1,\cdots,r,k=1,\cdots,s$

b) $Rez, Rey>0,$

c) $0<Re(\alpha+(2m+k)\beta)+\sum_{i=1}^{r}\alpha_i\min_{1\leqslant j\leqslant M^{(i)}}Re\left(\frac{B_j^{(i)}}{\delta_j^{(i)}}\right)+\sum_{i=1}^{s}\eta_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left(\frac{b_j'^{(i)}}{\beta_j'^{(i)}}\right)<$

$<Re(\rho+(2m+k)\gamma)+\sum_{i=1}^{r}\beta_i\min_{1\leqslant j\leqslant M^{(i)}}Re\left(\frac{B_j^{(i)}}{\delta_j^{(i)}}\right)+\sum_{i=1}^{s}\epsilon_i\min_{1\leqslant j\leqslant m^{(i)}}Re\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)+2$

d) $|argz_k|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.3) ; $i=1,\cdots,r$

e) $|argZ_k|<\frac{1}{2}\Omega_i'\pi$, where Ω_i is defined by (1.17) ; $i=1,\cdots,s$

f) The series occuring on the right-hand side of (3.1) is absolutely and uniformly convergent.

Proof

Expressing the spheroidal function involved in the integrand in its expression form with the help of (1.25) and the Bessel serie, the I-function of r variables in series with the help of equation (1.6), the general class of polynomial of several variables $S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}$ with the help of equation (1.22)and the I-function of s variables in Mellin-Barnes contour integral with the help of equation (1.9), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) and then evaluating the resulting integral with the help of equation (2.1) and expressing the Gauss hypergeometric function ${}_2F_1$ in serie, use the relations $\Gamma(a)(a)_n=\Gamma(a+n)$ and the relation $a=\frac{\Gamma(a+1)}{\Gamma(a)}$ with $Re(a)>0$. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

4. Multivariable H-function

If $U=V=A=B=U_1=V_1=A_1=B_1=0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [5]. Our integral contain two multivariable H-functions.

In this section, we note

$G_0(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})=G_{A=B=A_1=B_1=0}(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$

We have :

$$\int_0^{+\infty}\frac{x^{\alpha-1}}{\sqrt{(x^2+z^2)(x^2y^2+z^2)}\left(\sqrt{x^2+z^2}+\sqrt{(x^2y^2+z^2)}\right)^{\mu}}\psi_{\alpha n}(c^{\sigma},X_{\beta,\gamma})S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix}\text{y}_1X_{\gamma_1,\mu_1}\\ \cdots \\ \text{y}_tX_{\gamma_t,\mu_t}\end{pmatrix}$$

$$H_{P_r,Q_r;W_1}^{0,N_r;X_1}\left(\begin{array}{c} z_1X_{\alpha_1,\beta_1} \\ \cdot\ \cdot\ \cdot \\ z_rX_{\alpha_r,\beta_r} \end{array}\right) H_{p_s,q_s;W}^{0,n_s;X}\left(\begin{array}{c} Z_1X_{\eta_1,\epsilon_1} \\ \cdot\ \cdot\ \cdot \\ Z_sX_{\eta_s,\epsilon_s} \end{array}\right) \mathrm{d}x = \frac{i^n\sqrt{2\pi}}{V_{\alpha n}(c^\sigma)} 2^{-\mu} z^{\alpha-\mu-1} \sum_{k=0,or1}^{\infty_*} \sum_{m=0}^{\infty}$$

$$\sum_{G_1,\cdots,G_r=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} a_1 \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!} G_0(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})} y_1^{K_1}\cdots y_t^{K_t} z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}} c^{\sigma(2m+k)} 2^{-(\gamma(2m+k)+\sum_{i=1}^t K_i\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i)}$$

$$\frac{1}{n'!}(1-y^2)^{n'} z^{(\beta-\gamma)(2m+k)+\sum_{i=1}^t K_i(\gamma_i-\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-\beta_i)} H_{p_s+4,q_s+3;W}^{0,n_s+4;X}\left(\begin{array}{c} Z_1 z^{\eta_1-\epsilon_1}2^{-\epsilon_1} \\ \cdot\ \cdot\ \cdot \\ \cdots \\ Z_s z^{\eta_s-\epsilon_s}2^{-\epsilon_s} \end{array}\right|$$

$$(1\text{-}n'\text{-}\tfrac{1}{2}(\alpha+\beta(2m+k)+\textstyle\sum_{i=1}^t K_i\gamma_i+\sum_{i=1}^r \eta_{G_i,g_i}\alpha_i;\frac{\eta_1}{2},\cdots,\frac{\eta_s}{2}),$$
$$\cdot\ \cdot\ \cdot$$
$$\cdot\ \cdot\ \cdot$$

$$(\tfrac{1}{2}(\alpha-\mu+(\beta-\gamma)(2m+k)+\textstyle\sum_{i=1}^t K_i(\gamma_i-\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i-\beta_i));\frac{\epsilon_1-\eta_1}{2},\cdots,\frac{\epsilon_s-\eta_s}{2}),$$
$$\cdot\ \cdot\ \cdot$$
$$(\text{-}\tfrac{1}{2}(\mu+\gamma(2m+k)+\textstyle\sum_{i=1}^t K_i\mu_i+\sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\frac{\epsilon_1}{2},\cdots,\frac{\epsilon_s}{2}),$$

$$(\text{-}n'+\tfrac{1}{2}(1-\mu-\delta RA-\textstyle\sum_{i=1}^t K_i\mu_i-\sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s),$$
$$\cdot\ \cdot\ \cdot$$
$$(+\tfrac{1}{2}(1-\mu-\delta RA-\textstyle\sum_{i=1}^t K_i\mu_i-\sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s),$$

$$\left.\begin{array}{c} (\text{-}\mu-\delta RA-\sum_{i=1}^t K_i\mu_i-\sum_{i=1}^r \eta_{G_i,g_i}\beta_i;\epsilon_1,\cdots,\epsilon_s),\mathfrak{A}:A' \\ \cdot\ \cdot\ \cdot \\ (\text{-}n'\text{-}\mu-\delta RA-\sum_{i=1}^t K_i\mu_i-\sum_{i=1}^r \eta_{G_i,g_i}\beta_i;\epsilon_1,\cdots,\epsilon_s),\mathfrak{B}:B' \end{array}\right) \quad (4.1)$$

under the same notations and validity conditions that (3.1) with $A = B = A_1 = B_1 = 0$.

5. Conclusion

In this paper we have evaluated a unified generalized infinite integral involving the multivariable I-functions, a class of polynomials of several variables and the spheroidal function and general arguments.The integral established in this paper is of very general nature as it contains Multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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