Kaluza-Klein Bulk Viscous String Cosmological Model in Barber's Second Self-Creation Theory of Gravitation

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Abstract: *In this paper, we have investigated a Kaluza-Klein bulk viscous string cosmological model in Barber's (Gen. Relativ. Gravit. 14:117, 1982) second self-creation theory of gravitation. The source for energy momentum tensor is bulk viscous fluid containing one dimensional cosmic string. A cosmological model, which we obtained, represents a bulk viscous string universe in self-creation theory of gravitation. The model is obtained by using some of the possible physical conditions. Several physical and kinematical properties of the model are also discussed.*

Keywords: *Kaluza-Klein universe, Self-creation cosmology, cosmic string, Bulk viscosity.*

I. INTRODUCTION

Barber has proposed two theories well known as self creation theories. First theory is the modification of Brans Dicke theory [1] in which Barber has included continuous creation of matter. The universe is seen to be created out of self–contained gravitational, scalar and matter fields. However, the solution of the one body problem reveals unsatisfactory characteristics of the first theory. Further, it also violated the principle of equivalence. Brans has also pointed out that Barber's first theory of gravitation is in disagreement with experiment as well as inconsistent in general, due to non obeying principle of equivalence. Barber's second self creation theory [2] is the modification of general theory of relativity to variable G-theory. This second theory retains the attractive features of the first theory and overcome previous objections. This second theory is an adoption of General relativity to include continuous creation and is within the observational limits. In this theory, the scalar field does not directly gravitate, but simply divide the matter tensor acting as a reciprocal of G i.e. gravitational constant. This scalar field is postulated to couple with the trace of energy-momentum tensor. This theory predicts local effects within observational limits. The field equations in Barber's second self-creation theory are

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1} T_{ij}, \tag{1}$$

and

$$\phi^{k}_{;k} = \frac{8\pi}{3} \mu T, \tag{2}$$

Where μ is a coupling constant to be determined from the experiment ($|\mu| \le 0.1$). In the limit as $\mu \to 0$, this theory approaches to the Einstein theory of General Relativity in every respect. T_{ij} is the energy-momentum tensor, $G = \phi^{-1}$. T is the trace of energy-momentum tensor and semicolon denotes covariant differentiation.

Reddy and Naidu [3], Katore *et al*. [4], Pradhan and Vishwakarma [5], Panigrahi and Sahoo [6], Singh and Kumar [7], are some of the authors who have studied various aspects of cosmological models in Barber's second self-creation theory. Reddy and Venkateswarlu [8] presented Bianchi type-VI_0 cosmological model in Barber's second self-creation theory of gravitation. Shri Ram and Singh [9] have discussed the spatially homogeneous and isotropic Robertson-Walker and Bianchi type-II models of the universe in Barber's self-creation theory in presence of perfect fluid by using gamma law equation of state. Reddy [10] and Reddy *et al.* [11] have studied Bianchi type space-times solution in Barber's second self-creation theory of gravitation. Katore *et al*. [12] have studied accelerating and decelerating hyper surface homogeneous cosmological models in Barber's second self-creation theory. Mahanta [13] have studied dark energy (DE) models with variable EoS parameter in self-creation theory of gravitation. Jain and Jain [14] discussed Bianchi type-I magnetized radiating cosmological model in self creation theory of gravitation. Recently, Shen M. [15] and Rao *et al*. [16] also have studied some cosmological models in Barber's second self creation theory of gravitation.

In order to understand evolution of the universe, many authors constructed cosmological models containing a viscous fluid. The presence of viscosity in the fluid introduces many interesting features in the dynamics of homogeneous cosmological model. Bulk viscosity plays an important role in cosmology in getting the accelerated expansion of the universe. The possibilities of bulk viscosity leading to inflationary like solution in General Relativity. Another feature of bulk viscosity is that it acts like a negative energy field in an expanding universe. Many authors have studied cosmological models with the presence of bulk viscous fluid and cosmic strings coupled with scalar fields which play a vital role in the discussion of large scale structure and behaviour of the early universe. Khadekar *et al*. [17] discussed bulk viscosity in Freedman universe with a varying speed of light described by modified equation of state. The five-dimensional Kaluza-Klein universe with bulk viscosity and cosmic strings in Brans Dicke theory has been studied by Naidu *et al*. [18]. Reddy *et al*. [19] have studied five-dimensional Kaluza-Klein universe with bulk viscosity and cosmic strings in f(R, T) theory of gravity. Further Reddy *et al.* [20] discussed LRS Bianchi type-II universe in the presence of a bulk viscous fluid with one-dimensional cosmic strings in the scale-covariant theory of gravity proposed by Canuto *et al*. [21].
Inspired by the above investigations and discussions, in this paper, we study Kaluza-Klein bulk viscous string cosmological model in Barber's second self creation theory.

II. Metric and Field Equations

We consider a five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\chi^2, \qquad (3)$$

where A, B are the scale factors and χ is the fifth co-ordinate taken to be a space like.
The energy-momentum tensor for a bulk viscous fluid containing one-dimensional string is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p} g_{ij} - \lambda_s x_i x_j, \qquad (4)$$

and

$$\bar{p} = p - 3\xi H = \varepsilon\rho, \qquad (5)$$

where , we take $\varepsilon = \varepsilon_0 - \gamma, (0 \le \varepsilon_0 \le 1)$, $p = \varepsilon_0 \rho$ and $\rho = \eta\lambda_s$.
Here $\bar{p}$ is the total pressure which includes the proper pressure p, ρ is the rest energy density of the system, $\xi(t)$ is the coefficient of bulk viscosity, H is Hubble's parameter, λ_s is the string tension density and ε_0, γ, η are constants. Also $u^i = \delta^i_4$ is a four-velocity vector and x^i is a space-like vector, which represents anisotropic direction of the string. Here u^i and x^i satisfies the relation:

$$g_{ij}u^i u_j = g_{ij}x^i x_j = 1, \; u^i x_i = 0. \qquad (6)$$

We assume the string to be lying along the x- axis. One-dimensional strings are assumed to be loaded with particles. The particle energy density is

$$\rho_p = \rho - \lambda_s. \qquad (7)$$

In co-moving coordinate system, the components of energy-momentum tensor for bulk viscous fluid from equations (4) and (5) are obtained as:

$$T_0^0 = \rho, T_1^1 = T_2^2 = T_3^3 = -\bar{p}, \; T_4^4 = -(\bar{p} + \lambda_s), \qquad (8)$$

Hence

$$T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4 = (\rho - 4\bar{p} - \lambda_s)$$

Here we consider ρ, $\bar{p}$ and λ_s as function of time t only.
Therefore, in the co-moving coordinate system, the field equations (1)-(2) for the metric (3) with the help of equations (4)-(8) can be written as

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B} = 8\pi\phi^{-1}\rho, \qquad (9)$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -8\pi\phi^{-1}\bar{p}, \qquad (10)$$

$$3\frac{\ddot{A}}{A} + 3\left(\frac{\dot{A}}{A}\right) = -8\pi\phi^{-1}(\bar{p} + \lambda_s), \qquad (11)$$

$$\ddot{\phi} + \dot{\phi}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = -\frac{8\pi\mu}{3}(\rho - 4\bar{p} - \lambda_s) \qquad (12)$$

where the over dot indicates differentiation with respect to t.
For the model (3), the physical and kinematical variables which are important in cosmology are as follows:
The spatial volume for metric (3) is given by

$$V^3 = A^3 B = R^4(t), \qquad (13)$$

where $R(t)$ is the average scale factor of the universe. which is given by

$$R(t) = \left(A^3 B\right)^{1/4} . \qquad (14)$$

The scalar expansion is

$$\theta = \frac{1}{3}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \qquad (15)$$

The mean Hubble parameter H for the metric (3) is,

$$H = \frac{\dot{R}}{R} = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \qquad (16)$$

Where $H_x = H_y = H_z = \frac{\dot{A}}{A}$ and $H_\chi = \frac{\dot{B}}{B}$.

Also, the shear scalar in term of scalar expansion θ is,

$$\sigma^2 = \frac{2}{3}\theta^2 . \qquad (17)$$

III. SOLUTIONS OF THE FIELD EQUATIONS

Equations (9)-(12) are system of four independent equations in six unknowns *A*, *B*, $\varphi, \rho, \bar{p}$ and λ_s. Also, these equations are highly nonlinear in nature. Hence to find the complete determinate solutions, we take the help of following physical conditions:

(i) The shear scalar σ^2 is proportional to scalar expansion θ of the metric (3) so that we have (Collins et. al. [22])

$$A = B^n , \qquad (18)$$

where $n \neq 1$ is non zero constant .

(ii) We use the condition

$$T = \left(\rho - 4\bar{p} - \lambda_s\right) = 0, \qquad (19)$$

This, physically, corresponds to the trace free energy tensor of the matter source under consideration.
Now, using equation (18) and (19), the field equations (9)-(11) reduces to the equation

$$\frac{\ddot{B}}{B} + N\frac{\dot{B}^2}{B^2} = 0 \qquad (20)$$

where $N = \frac{6n^2}{3n+1}$

after integrating and simplifying above equation, we obtained an exact solutions given by

$$A = \left[\frac{(6n^2 + m)(at + b)}{m}\right]^{\frac{mn}{6n^2+m}} \qquad (21)$$

and

$$B = \left[\frac{(6n^2 + m)(at + b)}{m}\right]^{\frac{m}{6n^2+m}} \qquad (22)$$

where *a* and *b* are constants of integration and we take *(3n+1)* = *m* for the sake of simplicity.

In view of equations (18) and (19), with proper choice of constants and co-ordinates (*a=1* and *b=0*) the Kaluza-Klein bulk viscous string cosmological model in Barber's second self creation theory can be written as

$$ds^2 = dt^2 - \left[\frac{(6n^2+m)t}{m}\right]^{\frac{2mn}{(6n^2+m)}}(dx^2 + dy^2 + dz^2) + \left[\frac{(6n^2+m)t}{m}\right]^{\frac{2m}{(6n^2+m)}} d\chi^2 \qquad (23)$$

Also, using equations (18) and (19), equation (12) leads to

$$\phi = \frac{(6n^2 + m)}{6n^2 - m^2 + m} t^{\frac{6n^2 - m^2 + m}{6n^2+m}} \qquad (24)$$

IV. PHYSICAL PROPERTIES OF THE MODEL

Equation (23) represents the Kaluza-Klein bulk viscous string cosmological model in Barber's second self creation theory with the following expression for physical and kinematical parameters which are significant in the physical discussion of the cosmological model.

$$V = A^3 B = \left[\frac{(6n^2 + m)}{m} t\right]^{\frac{m^2}{6n^2+m}} \qquad (25)$$

The expansion scalar is obtained as

$$\theta = \frac{1}{3}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{m^2}{3(6n^2 + m)}\frac{1}{t} \qquad (26)$$

Expansion scalar at an initial epoch diverges. As cosmic time t increases gradually, expansion scalar decreases and finally vanishes when t –› ∞. It shows that it possesses late time singularity. Further model has non-zero expansion rate i.e. universe start with an infinite rate of expansion. This behaves like big bang model of the universe.

The Hubble parameter is obtained as

$$H=\frac{1}{4}\left(3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=\frac{m^2}{4(6n^2+m)}\frac{1}{t} \tag{27}$$

The shear scalar is given as

$$\sigma^2=\frac{2}{3}\theta^2=\frac{2m^4}{27(6n^2+m)^2}\frac{1}{t^2} \tag{28}$$

Shear scalar has initial singularity at t = 0 and dies out for large value of t.

Using equations (18), (19) in equation (9), the energy density is obtained as

$$\rho=\frac{3m^2n(n+1)}{8\pi(6n^2+m)(6n^2-m^2+m)}\left(\frac{1}{t}\right)^{\frac{6n^2+m^2+m}{6n^2+m}} \tag{29}$$

The string tension density is given as

$$\lambda_s=\frac{3m^2n(n+1)}{\eta 8\pi(6n^2+m)(6n^2-m^2+m)}\left(\frac{1}{t}\right)^{\frac{6n^2+m^2+m}{6n^2+m}} \tag{30}$$

It can be noted that, string tension density vanishes at present epoch that is why string disappears from present universe.

The particle energy density is obtained as

$$\rho_p=\frac{(\eta-1)}{\eta}\frac{3m^2n(n+1)}{8\pi(6n^2+m)(6n^2-m^2+m)}\left(\frac{1}{t}\right)^{\frac{6n^2+m^2+m}{6n^2+m}} \tag{31}$$

It can be observed that, string particle energy density gets vanished as the cosmic time t increases.

The total pressure is obtained as

$$\bar{p}=\frac{3\varepsilon m^2n(n+1)}{8\pi(6n^2+m)(6n^2-m^2+m)}\left(\frac{1}{t}\right)^{\frac{6n^2+m^2+m}{6n^2+m}} \tag{32}$$

The coefficient of bulk viscosity is obtained as

$$\xi=\frac{(\varepsilon_0-\varepsilon)(6n^2+m)n(n+1)}{2\pi(6n^2+m)(6n^2-m^2+m)}\left(\frac{1}{t}\right)^{\frac{m^2}{6n^2+m}} \tag{33}$$

It is observed that as the cosmic time t increases the coefficient of bulk viscosity decreases which leads to inflationary model.

V. Conclusions

In this paper, we have investigated Kaluza-Klein cosmological model in Barber's second self creation theory, when the matter source is a bulk viscous fluid containing one-dimensional cosmic strings. To determine the solution of the field equations which are highly non-linear, we assume two physical conditions (i) trace free matter source, (ii) proportionality of scalar expansion and shear scalar of the space-time. The solution of the field equations, we obtained is an over determined problem but however at least they are mathematically justified since it satisfy the field equations and are consistent. Further, at an initial epoch cosmological parameters of the model are all diverges. With the increase in cosmic time t, these parameters vanish as t → ∞. It is observed that as the cosmic time t increases, bulk viscosity decreases which lead to inflationary model. The model obtained is expanding, shearing and remains anisotropic throughout the evolution of the early universe. It is interesting to note that the solutions we obtained resemble with the solutions obtained by Reddy and Santhi Kumar [23] in which they considered bulk viscosity as a matter source while in our paper we considered one dimensional cosmic string attached to bulk viscous fluid as a matter source in Barber's second self creation theory.

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