A Study on Thermal Behavior of Solid Tumor by using Functionally Graded Approach

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Abstract
The aim of this work concerned with the temperature prediction of the solid tumor due to the nonuniform heat source inside the sphere. The sphere is considered to be graded along the radial direction with an exponentially varying distribution. The problem is solved by using the implicit finite difference scheme.
Keywords: *Bioheat transfer; Functionally graded tissue; Spherical solid tumor; Implicit finite difference scheme.*

I.INTRODUCTION

Functional gradation is one characteristics feature of the living tissue. Human tissue exhibit gradients across a spatial volume in which each identifiable layer has specific functions to perform so that the whole tissue can behave normally. Such a gradient is termed a functional gradient. Gradation of the properties is one of remarkable lessons reflected in modern engineering science.

Numerous examples found in nature maximizing the endurance and reliability of biological system. In biological system the properties vary over a short distance. Functionally graded materials have attracted broad interest since the concept was introduced and their application have been considered in such as diverse areas as bio-engineering, aerospace, mechanical and structural engineering, dentistry, orthopedics and electronics.

Pennes model [1] is widely used due to its simplicity in bioheat transfer phenomenon. Liu et al. [2] discussed the influences of physiological parameters on the non-Fourier thermal responses in tumor and normal tissue during magnetic hyperthermia. Das et al. [3] estimated the presence of a tumor and its various attributes inside a 2-D breast tissue.

In order to explore the DPL behavior of heat transfer in tissue, Liu et al.[4] estimated the relaxation time by using non-Fourier heat conduction. Phadnis et al. [5] investigated the impact of gold nanoshells on thermal response of the laser irradiated biological tissue phantom embedded with gold nanoshells.

Hassanpour et al. [6] evaluated the role of small vessels in heat transfer mechanisms of a tissue-like medium during an interstitial hyperthermia treatment. LeBrun et al. [7] studied microCT image based simulation to design heating protocols in magnetic nanoparticle hyperthermia for cancer treatment and estimated the time of it take to completely damage the tumor after a single heating session and to evaluate the extent of collateral thermal damage.

In thermoelaticity, lots of work is done on functionally graded materials (FGM). Dai and Rao [8] studied dynamic thermoelastic behavior of a double layered cylinder with an functionally graded material layer under mechanical and thermal loading. Dai et al. [9] obtained an analytical solution for time dependent behavior of a hollow sphere made of functionally graded piezoelectric material. Abbas et al. [10] solved a problem on two-temperature generalized thermoelasticity for a FGM. Pawar et al. [11] discussed the thermal behavior of functionally graded solid sphere with nonuniform heat generation. Recently, Kumar et al. [12] investigated the thermoelastic functionally graded beam in a modified couple stress theory subjected to a dual-phase-lag model.

In present work, an attempt is made to solve the bioheat transfer equation by assuming the solid tumor as a functionally graded material.

II.FORMULATION OF THE PROBLEM

Living tissues are highly inhomogeneous. A solid sphere of radius r is considered for the study of thermal behavior of tumor tissue, nonuniform metabolic heat generation rate $Q(r)$ is considered as the exponential function of the radius in the form,

$$Q(r) = Q_0 e^{\eta(r/a)} \tag{1}$$

where Q_0 is the magnitude of the heat generation at $r = 0$, r is the radial distance, measured from the center of solid sphere, a is the radius of the sphere and η is a source parameter.

One dimensional energy equation (Pennes' bioheat equation) is

$$\rho(r)c(r)\frac{\partial T}{\partial t} = -\frac{\partial q(r)}{\partial r} - \frac{2}{r}q(r) + \omega_b \rho_b c_b (T_a - T) + Q_{ex} + Q(r) \tag{2}$$

where ω_b, ρ_b, c_b, T_a, Q_{ex} are the perfusion rate, density, specific heat, temperature of blood and external heat source, respectively.

$$q = -k(r)\frac{\partial T}{\partial r} \tag{3}$$

From equation (2) and equation (3),

$$\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\left(\frac{2}{r} + \frac{1}{k(r)}\frac{dk(r)}{dr}\right) + \frac{\omega_b \rho_b c_b (T_a - T) + Q_m + Q(r)}{k(r)} = \frac{\rho(r)c(r)}{k(r)}\frac{\partial T}{\partial t} \quad t \geq 0. \tag{4}$$

Initial and boundary conditions are

$$\begin{aligned} T(r,t) &= T_0 \ at\ t = 0, \\ T(r,t) &= T_w \ at\ r = a, t > 0 \\ \frac{\partial T}{\partial r} &= 0 \ at\ r = 0. \end{aligned} \tag{5}$$

III. SOLUTION OF THE PROBLEM

Introducing the dimensionless parameters

$$\begin{aligned} R &= \frac{r}{a}, \\ \theta &= \frac{T - T_0}{T_w - T_0}, \\ \theta_a &= \frac{T_a - T_0}{T_w - T_0}, \\ \tau &= \frac{t}{t_0}, \\ t_0 &= \frac{a^2 \rho_m c_m}{k_m}. \end{aligned} \tag{6}$$

The properties of the functionally graded sphere are assumed as

$$\begin{aligned} k(R) &= k_m e^{pR}, \\ \rho(R) &= \rho_m e^{\nu R}, \\ c_p(R) &= c_m e^{\omega R}. \end{aligned} \tag{7}$$

The dimensionless nonhomogeneity parameters p, ν, and ω are expressed as

$$\begin{aligned} p &= \ln \frac{k_c}{k_m}, \\ \nu &= \ln \frac{\rho_c}{\rho_m}, \\ \omega &= \ln \frac{c_c}{c_m}, \end{aligned} \tag{8}$$

where k, ρ and c are thermal conductivity, density and specific heat of the sphere respectively. The subscripts c and m stands for the properties at center and outer surface of sphere, respectively. These properties grading along the radius of the sphere.

Using relation (6), (7) in equation (4), leads to

$$\frac{\partial^2\theta}{\partial R^2}+\frac{\partial\theta}{\partial R}(\frac{2}{R}+p)+p_m(\theta_a-\theta)e^{-pR}++\Phi_1 e^{-pR}+\Phi_2 e^{(\eta-p)R}=e^{(\nu+\omega-p)R}\frac{\partial\theta}{\partial\tau} \tag{9}$$

where

$$\Phi_1 = \frac{Q_m a^2}{k_m(T_w-T_0)}$$

$$\Phi_2 = \frac{Q_0 a^2}{k_m(T_w-T_0)}$$

$$p_m = \frac{\omega_b\rho_b c_b a^2}{k_m}$$

Dimensionless form of initial and boundary conditions are

$$\theta(R,\tau)=0 \tag{10}$$

$$\theta(R,\tau)=1,\ at\ R=1 \tag{11}$$

$$\frac{\partial\theta}{\partial R}=0\ at\ R=0 \tag{12}$$

A fully implicit finite difference scheme is used to evaluate the dimensionless temperature governed by equations (9)-(12). Following [13], a backward-difference representation is used for the time derivative, and central difference formulas are used for other derivatives. The finite difference form of the equations (9)-(12) in the matrix form is

$$\begin{bmatrix} a_0 & c_0 & 0 & 0 & 0 & \dots & 0 & 0 \\ b_1 & a_1 & c_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & b_2 & a_2 & c_2 & 0 & \dots & 0 & 0 \\ 0 & 0 & . & . & . & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & a_{m-2} & c_{m-2} \\ 0 & 0 & 0 & 0 & 0 & \dots & b_{m-1} & a_{m-1} \end{bmatrix} \begin{bmatrix} \theta_0^{n+1} \\ \theta_1^{n+1} \\ \theta_2^{n+1} \\ \vdots \\ \theta_{m-2}^{n+1} \\ \theta_{m-1}^{n+1} \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_{m-2} \\ f_{m-1} \end{bmatrix} \tag{13}$$

where

$$\begin{aligned} a_0 &= -\frac{6}{(\Delta R)^2}-p_m-\frac{1}{\Delta\tau} \\ c_0 &= \frac{6}{(\Delta R)^2} \\ f_0 &= -p_m\theta_a-\Phi_1-\Phi_2-\frac{\theta_0^n}{\Delta\tau} \end{aligned} \tag{14}$$

For $i=1,\dots m-2$

$$\begin{aligned} b_i &= \frac{1}{(\Delta R)^2}-\frac{1}{i(\Delta R)^2}-\frac{p}{2\Delta R} \\ a_i &= -\frac{2}{(\Delta R)^2}-p_m e^{-p(i\Delta R)}-\frac{1}{\Delta\tau}e^{(\nu+\omega-p)i\Delta R} \\ c_i &= \frac{1}{(\Delta R)^2}+\frac{1}{(i\Delta R)\Delta R}+\frac{p}{2\Delta R} \\ f_i &= -\frac{1}{\Delta\tau}e^{(\nu+\omega-p)i\Delta R}\theta_i^n-\Phi_1 e^{-pi\Delta R}-\Phi_2 e^{(\eta-p)i\Delta R}-p_m\theta_a e^{-p(i\Delta R)} \end{aligned} \tag{15}$$

$$b_{m-1} = \frac{1}{(\Delta R)^2} - \frac{1}{(m-1)(\Delta R)^2} - \frac{p}{2\Delta R}$$

$$a_{m-1} = -\frac{2}{(\Delta R)^2} - p_m e^{-p(m-1)\Delta R} - \frac{1}{\Delta\tau} e^{(\nu+\omega-p)(m-1)\Delta R}$$

$$f_{m-1} = -\left(\frac{1}{(\Delta R)^2} + \frac{1}{(m-1)(\Delta R)^2} + \frac{p}{2\Delta R}\right) - p_m \theta_a e^{-p(m-1)\Delta R} - \Phi_1 e^{-p(m-1)\Delta R} - \Phi_2 e^{(\eta-p)(m-1)\Delta R} - \frac{1}{\Delta\tau} e^{(\nu+\omega-p)(m-1)\Delta R}\theta_{m-1}^n$$

The above equations represents a system of m equations in m unknowns. The resulted matrix is tridiagonal matrix.

IV.CONCLUSIONS

Living tissues are highly non homogeneous. In this work, an attempt is made to solve bioheat transfer equation by considering solid tumor tissue as functionally graded material with a nonuniform heat source inside the sphere. The problem is solved by using the implicit finite difference scheme.

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