On a unified integral formula involving multivariable I-functions and classes of polynomials

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ABSTRACT
In this paper we evaluate a unified and general finite integral involves the product of the multivariable I-functions defined by Prathima and Nambisan [1] and the general classes of multivariable polynomials with general arguments. On account of the most general nature of the functions and the classes of the polynomials and their general arguments occuring in our main integral, several new and known integrals follow as its simple specials cases. We shall study the particular case concerning the multivariable H-function defined by Srivastava et al [6,7],the Srivastava-Daoust polynomial [4] and the I-function of two variables defined by Rathie et al [2].

Keyworlds : Multivariable I-function, general class of multivariable polynomials, multivariable H-function, I-function of two variables

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1. Introduction

The multivariable I-function defined by Prathima and Nambisan [1] is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,N:M_1,N_1;\cdots;M_r,N_r}_{P,Q:P_1,Q_1;\cdots;P_r,Q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,P}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,Q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,N_1},(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{N_1+1,P_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,N_r},(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{N_r+1,P_r} \\ (\mathrm{d}_j^{(1)},\delta_j^{(1)};1)_{1,M_1},(d_j^{(1)},\delta_j^{(1)};D_1)_{M_1+1,Q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};1)_{1,M_r},(d_j^{(r)},\delta_j^{(r)};D_r)_{M_r+1,Q_r} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

where $\phi(s_1,\cdots,s_r), \theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\phi_i(s_i) = \frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.4}$$

For more details, see Prathima and Nambisan [1].

We can obtain the series representation and behaviour for small values for the function $\bar{I}(z_1,\cdots,z_r)$ defined and represented by (1.16). The series representation may be given as follows :

which is valid under the following conditions :

$$\delta_i^{(h)}[d_i^{(j)}+r] \neq \delta_i^{(j)}[d_i^{(h)}+\mu] \text{ for } j\neq h, j,h=1,\cdots,M_i, r,\mu=0,1,2,\cdots$$

$$U_i=\sum_{j=1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{P_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=M_i+1}^{Q_i}D_j^{(i)}\delta_j^{(i)}\leqslant 0, i=1,\cdots,r \quad \text{and } z_i\neq 0$$

and if all the poles of (1.4) are simple ,then the integral (1.2) can be evaluated with the help of the Residue theorem to give

$$\bar{I}(z_1,\cdots,z_r)=\sum_{G_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi\frac{\prod_{i=1}^{r}\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\delta_{G^{(i)}}^{(i)}\prod_{i=1}^{r}g_i!} \tag{1.5}$$

where ϕ_1 and ϕ_i are defined by

$$\phi=\frac{\prod_{j=1}^{N}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{Q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}\eta_{G_i,g_i}\right)} \tag{1.6}$$

and

$$\phi_i=\frac{\prod_{j=1}^{N_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=N_i+1}^{P_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=M_i+1}^{Q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}\eta_{G_i,g_i}\right)}, i=1,\cdots,r \tag{1.7}$$

where $\eta_{G_i,g_i}=\dfrac{d_{G^{(i)}}^{(i)}+g_i}{\delta_{G^{(i)}}^{(i)}}, i=1,\cdots,r$

The generalized polynomials of multivariables defined by Srivastava [3], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v]=\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}$$

$$A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \tag{1.8}$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex.

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]=\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.9}$$

The coefficients are $B[E;R_1,\ldots,R_u]$ arbitrary constants, real or complex.

We will note $a_v=\dfrac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\dfrac{(-N_v)_{\mathfrak{M}_vK_v}}{K_v!}A[N_1,K_1;\cdots;N_v,K_v]$ and

$$b_u = \frac{(-E)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{1.10}$$

The multivariable I-function defined by Prathima and Nambisan [1] is an extension of the multivariable H-function defined by Srivastava et al [6,7]. It is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_s) = I^{0,N':M'_1,N'_1;\cdots;M'_s,N'_s}_{P',Q':P'_1,Q'_1;\cdots;P'_s,Q'_s}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}'_j;\alpha'^{(1)}_j,\cdots,\alpha'^{(s)}_j;A'_j)_{1,P'}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}'_j;\beta'^{(1)}_j,\cdots,\beta'^{(s)}_j;B'_j)_{1,Q'}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}'^{(1)}_j,\gamma'^{(1)}_j;C'^{(1)}_j)_{1,N'_1},(c'^{(1)}_j,\gamma'^{(1)}_j;C'^{(1)}_j)_{N'_1+1,P'_1};\cdots;(c'^{(s)}_j,\gamma'^{(s)}_j;C'^{(s)}_j)_{1,N'_s},(c'^{(s)}_j,\gamma'^{(s)}_j;C'^{(s)}_j)_{N'_s+1,P'_s} \\ \\ (\mathrm{d}'^{(1)}_j,\delta'^{(1)}_j;D'^{(1)}_j)_{1,M'_1},(d'^{(1)}_j,\delta'^{(1)}_j;D'^{(1)}_j)_{M'_1+1,Q'_1};\cdots;(d'^{(s)}_j,\delta'^{(s)}_j;D'^{(s)}_j)_{1,M'_s},(d'^{(s)}_j,\delta'^{(s)}_j;D'^{(s)}_j)_{M'_s+1,Q'_s} \end{array}\right) \tag{1.11}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\phi(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

where $\phi(t_1,\cdots,t_s),\theta_i(t_i), i=1,\cdots,s$ are given by :

$$\phi(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{N'}\Gamma^{A'_j}\left(1-a'_j+\sum_{i=1}^{s}\alpha'^{(i)}_jt_j\right)}{\prod_{j=N'+1}^{P'}\Gamma^{A'_j}\left(a'_j-\sum_{i=1}^{s}\alpha'^{(i)}_jt_j\right)\prod_{j=1}^{Q'}\Gamma^{B'_j}\left(1-b'_j+\sum_{i=1}^{s}\beta'^{(i)}_jt_j\right)} \tag{1.13}$$

$$\phi_i(t_i) = \frac{\prod_{j=1}^{N'_i}\Gamma^{C'^{(i)}_j}\left(1-c'^{(i)}_j+\gamma'^{(i)}_jt_i\right)\prod_{j=1}^{M'_i}\Gamma^{D'^{(i)}_j}\left(d'^{(i)}_j-\delta'^{(i)}_jt_i\right)}{\prod_{j=N'_i+1}^{P'_i}\Gamma^{C'^{(i)}_j}\left(c'^{(i)}_j-\gamma'^{(i)}_jt_i\right)\prod_{j=M'_i+1}^{Q'_i}\Gamma^{D'^{(i)}_j}\left(1-d'^{(i)}_j+\delta'^{(i)}_jt_i\right)} \tag{1.14}$$

For more details, see Prathima and Nambisan [1].

Following the result of Braaksma, the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^{P'}A'_j\alpha'^{(i)}_j - \sum_{j=1}^{Q'}B'_j\beta'^{(i)}_j + \sum_{j=1}^{P'_i}C'^{(i)}_j\gamma'^{(i)}_j - \sum_{j=1}^{Q'_i}D'^{(i)}_j\delta'^{(i)}_j \leqslant 0, i=1,\cdots,s \tag{1.15}$$

The integral (2.1) converges absolutely if

where$|arg(z_k)| < \frac{1}{2}\Delta_k\pi, k=1,\cdots,s$

$$\Delta_k = -\sum_{j=N'+1}^{P'}A'_j\alpha'^{(k)}_j - \sum_{j=1}^{Q'}B'_j\beta'^{(k)}_j + \sum_{j=1}^{M'_k}D'^{(k)}_j\delta'^{(k)}_j - \sum_{j=M'_k+1}^{Q'_k}D'^{(k)}_j\delta'^{(k)}_j + \sum_{j=1}^{N'_k}C'^{(k)}_j\gamma'^{(k)}_j - \sum_{j=N'_k+1}^{P'_k}C'^{(k)}_j\gamma'^{(k)}_j > 0 \tag{1.16}$$

For convenience, we will use the following notations in this paper.

$$A = (a'_j;\alpha'^{(1)}_j,\cdots,\alpha'^{(s)}_j;A'_j)_{1,P'}: \ (c'^{(1)}_j,\gamma'^{(1)}_j;C'^{(1)}_j)_{1,P'_1};\cdots;(c'^{(s)}_j,\gamma'^{(s)}_j;C'^{(s)}_j)_{1,P'_s} \tag{1.17}$$

$$B = (b'_j;\beta'^{(1)}_j,\cdots,\beta'^{(s)}_j;B'_j)_{1,Q'}:(d'^{(1)}_j,\delta'^{(1)}_j;D'^{(1)}_j)_{1,Q'_1};\cdots;(d'^{(s)}_j,\delta'^{(s)}_j;D'^{(s)}_j)_{1,Q'_s} \tag{1.18}$$

$$X = M_1', N_1'; \cdots ; M_s', N_s'; Y = P_1', Q_1'; \cdots ; P_s', Q_s' \tag{1.19}$$

2. Main integral

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda}\ S_L^{h_1,\cdots,h_u}\begin{pmatrix} z_1'' x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1} \\ \cdot \\ \cdot \\ \cdot \\ z_u'' x^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u} \end{pmatrix}$$

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} z_1''' x^{e_1'}(a-x)^{f_1'}(1+bx^l)^{-g_1'} \\ \cdot \\ \cdot \\ \cdot \\ z_v''' x^{e_v}(a-x)^{f_v}(1+bx^l)^{-g_v'} \end{pmatrix} \bar{I}\begin{pmatrix} z_1 x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r x^{\mu_r}(a-x)^{\upsilon_r}(1+bx^l)^{-\eta_r} \end{pmatrix}$$

$$I\begin{pmatrix} z_1' x^{\mu_1'}(a-x)^{\upsilon_1'}(1+bx^l)^{-\eta_1'} \\ \cdot \\ \cdot \\ \cdot \\ z_s' x^{\mu_s'}(a-x)^{\upsilon_s'}(1+bx^l)^{-\eta_s'} \end{pmatrix}\mathrm{d}x = a^{\rho+\sigma}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\ \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}$$

$$\sum_{G_i=1}^{M_i}\sum_{g_i=1}^{\infty}\phi\frac{\prod_{i=1}^r \phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \delta_{G^{(i)}}^{(i)}\prod_{i=1}^r g_i!}\sum_{w=0}^{\infty}\prod_{i=1}^{v} z_i'''^{K_i}\prod_{k=1}^{u} z''^{R_k} a_v b_u\ \frac{(-b)^w}{w!}$$

$$a^{\sum_{i=1}^v(e_i+f_i)K_i+\sum_{i=1}^u(e_i'+f_i')R_i+\sum_{i=1}^r(\mu_i+\upsilon_i)\eta_{G_i,g_i}+lw}I_{P'+3,Q'+2;Y}^{0,N'+3;X}\left(\begin{matrix} z_1' a^{\mu_1'+\upsilon_1'} \\ \cdot \\ \cdot \\ \cdot \\ z_s' x^{\mu_s'+\upsilon_s'} \end{matrix}\ \middle|\right.$$

$$(1\text{-}\rho-\textstyle\sum_{i=1}^v e_iK_i-\sum_{i=1}^u e_i'R_i-\sum_{i=1}^r \mu_i\eta_{G_i,g_i}-lw;\mu_1',\cdots,\mu_s';1),$$
$$\cdot$$
$$\cdot$$
$$(1\text{-}\lambda-\textstyle\sum_{i=1}^v g_iK_i-\sum_{i=1}^u g_i'R_i-\sum_{i=1}^r \eta_i\eta_{G_i,g_i};\eta_1',\cdots,\eta_s';1),$$

$$(\text{-}\sigma-\textstyle\sum_{i=1}^v f_iK_i-\sum_{i=1}^u f_i'R_i-\sum_{i=1}^r \upsilon_i\eta_{G_i,g_i};\upsilon_1',\cdots,\upsilon_s';1),$$
$$\cdot$$
$$\cdot$$
$$(\text{-}\rho-\sigma-\textstyle\sum_{i=1}^v(e_i+f_i)K_i-\sum_{i=1}^u(e_i'+f_i')R_i-\sum_{i=1}^r(\upsilon_i+\mu_i)\eta_{G_i,g_i}-lw;\upsilon_1'+\mu_1',\cdots,\upsilon_s'+\mu_s';1),$$

$$\left.\begin{matrix}(1\text{-}\lambda-\sum_{i=1}^v g_iK_i-\sum_{i=1}^u g_i'R_i-\sum_{i=1}^r \eta_i\eta_{G_i,g_i}-w;\eta_1',\cdots,\eta_s';1),\mathbb{A} \\ \cdot \\ \cdot \\ \mathbb{B}\end{matrix}\right) \tag{2.1}$$

ϕ_1, ϕ_i are defined respectively by (1.6) and (1.7)

Provided that

$min\{l, e_i, f_i, g_i, e'_j, f'_j, g'_j, \mu_k, \upsilon_k, \eta_k, \mu'_{\mathbf{m}}, \upsilon'_{\mathbf{m}}, \eta'_{\mathbf{m}}\} > 0; i = 1, \cdots, u, j = 1, \cdots, v, k = 1, \cdots, r, \mathbf{m} = 1, \cdots, s$

$$Re(\lambda) \geqslant 0 \; U_i = \sum_{j=1}^{P'} A'_j \alpha'^{(i)}_j - \sum_{j=1}^{Q'} B'_j \beta'^{(i)}_j + \sum_{j=1}^{P'_i} C'^{(i)}_j \gamma'^{(i)}_j - \sum_{j=1}^{Q'_i} D'^{(i)}_j \delta'^{(i)}_j \leqslant 0, i = 1, \cdots, s$$

$$Re\left(\rho + \sum_{j=1}^{r} \mu_i \eta_{G_i, g_i}\right) + \sum_{j=1}^{s} \mu'_i \min_{1 \leqslant k \leqslant M'_i} Re\left(\frac{d'^{(j)}_k}{\delta'^{(j)}_k}\right) > 0$$

$$Re\left(1 + \sigma + \sum_{j=1}^{r} \upsilon_i \eta_{G_i, g_i}\right) + \sum_{j=1}^{s} \upsilon'_i \min_{1 \leqslant k \leqslant M'_i} Re\left(\frac{d'^{(j)}_k}{\delta'^{(j)}_k}\right) > 0$$

$|arg(z_k)| < \frac{1}{2}\Delta_k \pi, k = 1, \cdots, s$, Δ_k is defined by (1.16)

Proof

To prove (2.1), first expressing a class of multivariable polynomials $S^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v}_{N_1, \cdots, N_v}[.]$ defined by Srivastava [3] in serie with the help of (1.8), a class of multivariable polynomials $S^{h_1, \cdots, h_u}_L[.]$ defined by Srivastava et al [5] in serie with the help of (1.9), the I-functions of r-variables defined by Prathima and Nambisan [1] in serie with the help of (1.5) and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the I-function of s-variables in Mellin-contour integral and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now evaluating the inner x-integral thus obtained by the help of special case of result [6, page 61, eq.(5.2.1)]. Interpreting the Mellin-Barnes integral in multivariable I-function, we obtain the desired result.

3. Particular cases

a) If $A_i = B_j = A'_j = B'_j = C^{(i)}_j = D^{(i)}_j = C'^{(i)}_j = D'^{(i)}_j = 1$, the multivariable I-functions reduce to multivariable H-functions defined by Srivastava et al [6,7], we obtain the following result.

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda} S^{h_1, \cdots, h_u}_L \begin{pmatrix} \mathrm{z}''_1 x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}''_u x^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u} \end{pmatrix}$$

$$S^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v}_{N_1, \cdots, N_v} \begin{pmatrix} \mathrm{z}'''_1 x^{e'_1}(a-x)^{f'_1}(1+bx^l)^{-g'_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'''_v x^{e_v}(a-x)^{f_v}(1+bx^l)^{-g'_v} \end{pmatrix} H \begin{pmatrix} \mathrm{z}_1 x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\mu_r}(a-x)^{\upsilon_r}(1+bx^l)^{-\eta_r} \end{pmatrix}$$

$$H\left(\begin{array}{c} z'_1 x^{\mu'_1}(a-x)^{\upsilon'_1}(1+bx^l)^{-\eta'_1} \\ . \\ . \\ . \\ z'_s x^{\mu'_s}(a-x)^{\upsilon'_s}(1+bx^l)^{-\eta'_s} \end{array}\right) \mathrm{d}x = a^{\rho+\sigma} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L}$$

$$\sum_{G_i=1}^{M_i} \sum_{g_i=1}^{\infty} \phi \frac{\prod_{i=1}^r \phi_i z_i^{\eta_{G_i,g_i}} (-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \delta_{G^{(i)}}^{(i)} \prod_{i=1}^r g_i!} \sum_{w=0}^{\infty} \prod_{i=1}^{v} z_i'''^{K_i} \prod_{k=1}^{u} z''^{R_k} a_v b_u \frac{(-b)^w}{w!}$$

$$a^{\sum_{i=1}^v (e_i+f_i)K_i + \sum_{i=1}^u (e'_i+f'_i)R_i + \sum_{i=1}^r (\mu_i+\upsilon_i)\eta_{G_i,g_i} + lw} H^{0,N'+3;X}_{P'+3,Q'+2;Y}\left(\begin{array}{c} z'_1 a^{\mu'_1+\upsilon'_1} \\ . \\ . \\ . \\ z'_s x^{\mu'_s+\upsilon'_s} \end{array}\right|$$

$$(1\text{-}\rho - \textstyle\sum_{i=1}^v e_i K_i - \sum_{i=1}^u e'_i R_i - \sum_{i=1}^r \mu_i \eta_{G_i,g_i} - lw; \mu'_1, \cdots, \mu'_s),$$
$$.$$
$$.$$
$$(1\text{-}\lambda - \textstyle\sum_{i=1}^v g_i K_i - \sum_{i=1}^u g'_i R_i - \sum_{i=1}^r \eta_i \eta_{G_i,g_i}; \eta'_1, \cdots, \eta'_s),$$

$$(\text{-}\sigma - \textstyle\sum_{i=1}^v f_i K_i - \sum_{i=1}^u f'_i R_i - \sum_{i=1}^r \upsilon_i \eta_{G_i,g_i}; \upsilon'_1, \cdots, \upsilon'_s),$$
$$.$$
$$.$$
$$(\text{-}\rho - \sigma - \textstyle\sum_{i=1}^v (e_i+f_i)K_i - \sum_{i=1}^u (e'_i+f'_i)R_i - \sum_{i=1}^r (\upsilon_i+\mu_i)\eta_{G_i,g_i} - lw; \upsilon'_1+\mu'_1, \cdots, \upsilon'_s+\mu'_s),$$

$$\left.\begin{array}{c} (1\text{-}\lambda - \sum_{i=1}^v g_i K_i - \sum_{i=1}^u g'_i R_i - \sum_{i=1}^r \eta_i \eta_{G_i,g_i} - w; \eta'_1, \cdots, \eta'_s), \mathbb{A} \\ . \\ . \\ \mathbb{B} \end{array}\right) \tag{3.1}$$

under the same notations and conditions that (2.1) with $A_i = B_j = A'_j = B'_j = C_j^{(i)} = D_j^{(i)} = C_j'^{(i)} = D_j'^{(i)} = 1$,

b) If $B(L; R_1, \cdots, R_u) = \dfrac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}} \prod_{j=1}^{B'} (b'_j)_{R_1\phi'_j} \cdots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{R_1\psi'_j+\cdots+R_u\psi_j^{(u)}} \prod_{j=1}^{D'} (d'_j)_{R_1\delta'_j} \cdots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (3.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda}\; S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\left(\begin{array}{c} z_1''' x^{e'_1}(a-x)^{f'_1}(1+bx^l)^{-g'_1} \\ . \\ . \\ . \\ z_v''' x^{e_v}(a-x)^{f_v}(1+bx^l)^{-g'_v} \end{array}\right)$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c} z''_1 x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1} \\ \cdot \\ \cdot \\ \cdot \\ z''_u x^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u} \end{array}\right)$$

$$I\left(\begin{array}{c} z_1 x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r x^{\mu_r}(a-x)^{\upsilon_r}(1+bx^l)^{-\eta_r} \end{array}\right) \bar{I}\left(\begin{array}{c} z'_1 x^{\mu'_1}(a-x)^{\upsilon'_1}(1+bx^l)^{-\eta'_1} \\ \cdot \\ \cdot \\ \cdot \\ z'_s x^{\mu'_s}(a-x)^{\upsilon'_s}(1+bx^l)^{-\eta'_s} \end{array}\right) \mathrm{d}x = a^{\rho+\sigma}$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_\mathfrak{v}]} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L} \sum_{G_i=1}^{M_i} \sum_{g_i=1}^{\infty} \phi \frac{\prod_{i=1}^r \phi_i z_i^{\eta_{G_i,g_i}} (-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r \delta_{G^{(i)}}^{(i)} \prod_{i=1}^r g_i!} \sum_{w=0}^{\infty} \prod_{i=1}^{v} z_i'''^{K_i} \prod_{k=1}^{u} z''^{R_k} a_v b'_u \frac{(-b)^w}{w!}$$

$$a^{\sum_{i=1}^v (e_i+f_i)K_i + \sum_{i=1}^u (e'_i+f'_i)R_i + \sum_{i=1}^r (\mu_i+\upsilon_i)\eta_{G_i,g_i} + lw}$$

$$I_{P'+3,Q'+2;Y}^{0,N'+3;X}\left(\begin{array}{c} z'_1 a^{\mu'_1+\upsilon'_1} \\ \cdot \\ \cdot \\ \cdot \\ z'_s x^{\mu'_s+\upsilon'_s} \end{array}\middle| \begin{array}{c} (1\text{-}\rho - \sum_{i=1}^v e_iK_i - \sum_{i=1}^u e'_iR_i - \sum_{i=1}^r \mu_i\eta_{G_i,g_i} - lw; \mu'_1,\cdots,\mu'_s;1), \\ \cdot \\ \cdot \\ (1\text{-}\lambda - \sum_{i=1}^v g_iK_i - \sum_{i=1}^u g'_iR_i - \sum_{i=1}^r \eta_i\eta_{G_i,g_i}; \eta'_1,\cdots,\eta'_s;1), \end{array}\right.$$

$$(\text{-}\sigma - \sum_{i=1}^v f_iK_i - \sum_{i=1}^u f'_iR_i - \sum_{i=1}^r \upsilon_i\eta_{G_i,g_i}; \upsilon'_1,\cdots,\upsilon'_s;1), \\ \cdot \\ \cdot$$

$$(\text{-}\rho - \sigma - \sum_{i=1}^v (e_i+f_i)K_i - \sum_{i=1}^u (e'_i+f'_i)R_i - \sum_{i=1}^r (\upsilon_i+\mu_i)\eta_{G_i,g_i} - lw; \upsilon'_1+\mu'_1,\cdots,\upsilon'_s+\mu'_s;1),$$

$$\left.\begin{array}{c} (1\text{-}\lambda - \sum_{i=1}^v g_iK_i - \sum_{i=1}^u g'_iR_i - \sum_{i=1}^r \eta_i\eta_{G_i,g_i} - w; \eta'_1,\cdots,\eta'_s;1), \mathbb{A} \\ \cdot \\ \cdot \\ \mathbb{B} \end{array}\right) \tag{3.3}$$

where $b'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B(L;R_1,\cdots,R_u)$ is defined by (3.2)

under the same notations and conditions that (2.1)

If $r = s = 2$, the multivariable I-functions reduce to I-function of two variables defined by Rathie et al [2] and we have

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda}\ S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} \mathrm{z}_1''x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_u''x^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u}\end{array}\right)$$

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\left(\begin{array}{c} \mathrm{z}_1'''x^{e_1'}(a-x)^{f_1'}(1+bx^l)^{-g_1'}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_v'''x^{e_v}(a-x)^{f_v}(1+bx^l)^{-g_v'}\end{array}\right)\bar{I}\left(\begin{array}{c} \mathrm{z}_1x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_2x^{\mu_2}(a-x)^{\upsilon_2}(1+bx^l)^{-\eta_2}\end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1'x^{\mu_1'}(a-x)^{\upsilon_1'}(1+bx^l)^{-\eta_1'}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_2'x^{\mu_2'}(a-x)^{\upsilon_2'}(1+bx^l)^{-\eta_2'}\end{array}\right)\mathrm{d}x = a^{\rho+\sigma}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\ \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}$$

$$\sum_{h_1=1}^{M_1}\sum_{h_2=1}^{M_2}\sum_{g_1,g_2=1}^{\infty}\phi\frac{\prod_{i=1}^2\phi_i z_i^{\eta_{h_i,g_i}}(-)^{\sum_{i=1}^2 g_i}}{\prod_{i=1}^2\delta_{h^{(i)}}^{(i)}\prod_{i=1}^2 g_i!}\ \sum_{w=0}^{\infty}\prod_{i=1}^{v}z_i'''^{K_i}\prod_{k=1}^{u}z''^{R_k}a_v b_u\ \frac{(-b)^w}{w!}$$

$$a^{\sum_{i=1}^v(e_i+f_i)K_i+\sum_{i=1}^u(e_i'+f_i')R_i+\sum_{i=1}^2(\mu_i+\upsilon_i)\eta_{G_i,g_i}+lw}I_{P'+3,Q'+2;Y}^{0,N'+3;X}\left(\begin{array}{c} \mathrm{z}_1'a^{\mu_1'+\upsilon_1'}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_2'x^{\mu_2'+\upsilon_2'}\end{array}\right|$$

$$(1\text{-}\rho-\textstyle\sum_{i=1}^v e_iK_i-\sum_{i=1}^u e_i'R_i-\sum_{i=1}^2\mu_i\eta_{G_i,g_i}-lw;\mu_1',\mu_2';1),$$
$$\cdot$$
$$\cdot$$
$$(1\text{-}\lambda-\textstyle\sum_{i=1}^v g_iK_i-\sum_{i=1}^u g_i'R_i-\sum_{i=1}^2\eta_i\eta_{G_i,g_i};\eta_1',\eta_2';1),$$

$$(\text{-}\sigma-\textstyle\sum_{i=1}^v f_iK_i-\sum_{i=1}^u f_i'R_i-\sum_{i=1}^2\upsilon_i\eta_{G_i,g_i};\upsilon_1',\upsilon_2';1),$$
$$\cdot$$
$$\cdot$$
$$(\text{-}\rho-\sigma-\textstyle\sum_{i=1}^v(e_i+f_i)K_i-\sum_{i=1}^u(e_i'+f_i')R_i-\sum_{i=1}^2(\upsilon_i+\mu_i)\eta_{G_i,g_i}-lw;\upsilon_1'+\mu_1',\upsilon_2'+\mu_2';1),$$

$$\left.\begin{array}{c}(1\text{-}\lambda-\sum_{i=1}^v g_iK_i-\sum_{i=1}^u g_i'R_i-\sum_{i=1}^2\eta_i\eta_{G_i,g_i}-w;\eta_1',\eta_2';1),\mathbb{A}\\ \cdot\\ \cdot\\ \mathbb{B}\end{array}\right) \tag{3.4}$$

under the same notations and conditions that (2.1) with $r = s = 2$

4. Conclusion

The multivariable I-functions defined by Prathima and Nambisan [1] and the classes of multivariable polynomials are the quite basic in nature. Therefore on specializing the parameters of these functions and polynomials, we may obtain the unified integral concerning various special functions of several variables and one variable such as multivariable H-function, Fox's H-function , Meijer's Gfunction, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, in cetera.

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