On a unified integral formula involving multivariable A-functions and classes of polynomials

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ABSTRACT
In this paper we evaluate a unified and general finite integral involves the product of the multivariable A-functions defined by Gautam et al [1] and the general classes of multivariable polynomials with general arguments. On account of the most general nature of the functions and the classes of the polynomials and their general arguments occuring in our main integral, several new and known integrals follow as its simple specials cases. We shall study the particular case concerning the multivariable H-function defined by Srivastava et al [6] and the Srivastava-Daoust polynomial [3].

Keyworlds : Multivariable A-function, general class of multivariable polynomials, multivariable H-function

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1. Introduction

The serie representation of the multivariable A-function is given by Gautam [1] as

$$A[u_1,\cdots,u_r] = A^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(r)},\beta^{(r)})}_{A,C:(M',N');\cdots;(M^{(r)},N^{(r)})}\left(\begin{array}{c} \mathrm{u}_1 \\ \cdot \\ \cdot \\ \mathrm{u}_r \end{array}\middle| \begin{array}{c} [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(r)}]_{1,A} : \\ \cdots \\ [(\mathrm{f}_j);\xi',\cdots,\xi^{(r)}]_{1,C} : \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(r)},\eta^{(r)})_{1,M^{(r)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(r)},\epsilon^{(r)})_{1,N^{(r)}} \end{array}\right) = \sum_{M_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi\frac{\prod_{i=1}^{r}\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon^{i}_{m_i}g_i!} \tag{1.1}$$

where

$$\phi = \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-g_j+\sum_{i=1}^{r}\gamma_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\lambda'+1}^{A}\Gamma\left(g_j-\sum_{i=1}^{r}\gamma_j^{(i)}U_i\right)\prod_{j=1}^{C}\Gamma\left(1-f_j+\sum_{i=1}^{r}\xi_j^{(i)}\eta_{G_i,g_i}\right)} \tag{1.2}$$

$$\phi_i = \frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)}, i=1,\cdots,r \tag{1.3}$$

and $\eta_{G_i,g_i} = \dfrac{p_{m_i}^{(i)}+g_i}{\epsilon_{m_i}^{(i)}}, i=1,\cdots,r$ (1.4)

which is valid under the following conditions : $\epsilon_{m_i}^{(i)}[p_j^{(i)}+p_i'] \neq \epsilon_j^{(i)}[p_{m_i}+g_i]$ (1.5)

and

$$u_i \neq 0, \sum_{j=1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \cdots, r \tag{1.6}$$

Here $\lambda, A, C, \alpha_i, \beta_i, M_i, N_i \in \mathbb{N}^*; i = 1, \cdots, r ; f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

The generalized polynomials of multivariables defined by Srivastava [2], is given in the following manner :

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{\mathfrak{M}_v K_v}}{K_v!}$$

$$A[N_1, K_1; \cdots ; N_v, K_v] y_1^{K_1} \cdots y_v^{K_v} \tag{1.7}$$

where $\mathfrak{M}_1, \cdots, \mathfrak{M}_v$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots ; N_v, K_v]$ are arbitrary constants, real or complex.

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L} (-L)_{h_1R_1+\cdots+h_uR_u} B(E; R_1,\cdots,R_u) \frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.8}$$

The coefficients are $B[E; R_1, \ldots, R_u]$ arbitrary constants, real or complex.

We will note $a_v = \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{\mathfrak{M}_v K_v}}{K_v!} A[N_1, K_1; \cdots ; N_v, K_v]$ and

$$b_u = \frac{(-E)_{h_1R_1+\cdots+h_uR_u} B(E; R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{1.9}$$

Consider the second multivariable A-function.

$$A(z_1', \cdots, z_s') = A_{p',q':p_1',q_1';\cdots;p_r',q_r'}^{m',n':m_1',n_1';\cdots;m_r',n_r'} \left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a'}_j; A_j'^{(1)}, \cdots, A_j'^{(s)})_{1,p'} : \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b'}_j; B_j'^{(1)}, \cdots, B_j'^{(s)})_{1,q'} : \end{array} \right.$$

$$\left. \begin{array}{c} (\mathrm{c'}_j^{(1)}, C_j'^{(1)})_{1,p_1'}; \cdots ; (c_j'^{(s)}, C_j'^{(s)})_{1,p_s'} \\ \\ (\mathrm{d'}_j^{(1)}, D_j'^{(1)})_{1,q_1'}; \cdots ; (d_j'^{(s)}, D_j'^{(s)})_{1,q_s'} \end{array} \right) \tag{1.10}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1'} \cdots \int_{L_s'} \phi'(t_1, \cdots, t_s) \prod_{i=1}^{s} \theta_i'(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.11}$$

where $\phi'(t_1, \cdots, t_s), \theta_i'(t_i), i = 1, \cdots, s$ are given by :

$$\phi'(t_1, \cdots, t_s) = \frac{\prod_{j=1}^{m'} \Gamma(b_j' - \sum_{i=1}^{s} B_j'^{(i)} t_i) \prod_{j=1}^{n'} \Gamma(1 - a_j' + \sum_{i=1}^{s} A_j'^{(i)} t_j)}{\prod_{j=n'+1}^{p'} \Gamma(a_j' - \sum_{i=1}^{s} A_j'^{(i)} t_j) \prod_{j=m'+1}^{q'} \Gamma\left(1 - b_j' + \sum_{i=1}^{s} B_j'^{(i)} t_j\right)} \tag{1.12}$$

$$\theta'_i(t_i) = \frac{\prod_{j=1}^{n'_i}\Gamma(1-c'^{(i)}_j+C'^{(i)}_j t_i)\prod_{j=1}^{m'_i}\Gamma(d'^{(i)}_j-D'^{(i)}_j t_i)}{\prod_{j=n'_i+1}^{p'_i}\Gamma(c'^{(i)}_j-C'^{(i)}_j t_i)\prod_{j=m'_i+1}^{q'_i}\Gamma(1-d'^{(i)}_j+D'^{(i)}_j t_i)} \tag{1.13}$$

Here $m',n',p',m'_i,n'_i,p'_i,c'_i \in \mathbb{N}^*; i=1,\cdots,r\,; a'_j,b'_j,c'^{(i)}_j,d'^{(i)}_j,A'^{(i)}_j,B'^{(i)}_j,C'^{(i)}_j,D'^{(i)}_j \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega'_i)z'_k| < \frac{1}{2}\eta'_k\pi, \xi'^{*}=0, \eta'_i>0 \tag{1.14}$$

$$\Omega'_i = \prod_{j=1}^{p'}\{A'^{(i)}_j\}^{A'^{(i)}_j}\prod_{j=1}^{q'}\{B'^{(i)}_j\}^{-B'^{(i)}_j}\prod_{j=1}^{q'_i}\{D'^{(i)}_j\}^{D'^{(i)}_j}\prod_{j=1}^{p'_i}\{C'^{(i)}_j\}^{-C'^{(i)}_j}; i=1,\cdots,s \tag{1.15}$$

$$\xi'^{*}_i = Im\Big(\sum_{j=1}^{p'}A'^{(i)}_j-\sum_{j=1}^{q'}B'^{(i)}_j+\sum_{j=1}^{q'_i}D'^{(i)}_j-\sum_{j=1}^{p'_i}C'^{(i)}_j\Big); i=1,\cdots,s \tag{1.16}$$

$$\eta'_i = Re\left(\sum_{j=1}^{n'}A'^{(i)}_j-\sum_{j=n'+1}^{p'}A'^{(i)}_j+\sum_{j=1}^{m'}B'^{(i)}_j-\sum_{j=m'+1}^{q'}B'^{(i)}_j+\sum_{j=1}^{m'_i}D'^{(i)}_j-\sum_{j=m'_i+1}^{q'_i}D'^{(i)}_j+\sum_{j=1}^{n'_i}C'^{(i)}_j-\sum_{j=n'_i+1}^{p'_i}C'^{(i)}_j\right)$$

$$i=1,\cdots,s \tag{1.17}$$

In this paper, we shall note.

$$X = m_1,n_1;\cdots;m_s,n_s \quad : Y = p_1,q_1;\cdots;p_s,q_s \tag{1.18}$$

$$\mathbb{A} = (a'_j;A'^{(1)}_j,\cdots,A'^{(s)}_j)_{1,p} : \; (c'^{(1)}_j,C'^{(1)}_j)_{1,p_1};\cdots;(c'^{(s)}_j,C'^{(s)}_j)_{1,p_s} \tag{1.19}$$

$$\mathbb{B} = (b'_j;B'^{(1)}_j,\cdots,B'^{(s)}_j)_{1,q} : \quad (d'^{(1)}_j,D'^{(1)}_j)_{1,q_1};\cdots;(d'^{(s)}_j,D'^{(s)}_j)_{1,q_s} \tag{1.20}$$

2. Main integral

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda}\; S_L^{h_1,\cdots,h_u}\begin{pmatrix} z''_1x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1}\\ \cdot\\ \cdot\\ \cdot\\ z''_ux^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u}\end{pmatrix}$$

$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}_{N_1,\cdots,N_v}\begin{pmatrix} z'''_1x^{e'_1}(a-x)^{f'_1}(1+bx^l)^{-g'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'''_vx^{e_v}(a-x)^{f_v}(1+bx^l)^{-g'_v}\end{pmatrix} A\begin{pmatrix} z_1x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rx^{\mu_r}(a-x)^{\upsilon_r}(1+bx^l)^{-\eta_r}\end{pmatrix}$$

$$A\begin{pmatrix} z'_1x^{\mu'_1}(a-x)^{\upsilon'_1}(1+bx^l)^{-\eta'_1}\\ \cdot\\ \cdot\\ \cdot\\ z'_sx^{\mu'_s}(a-x)^{\upsilon'_s}(1+bx^l)^{-\eta'_s}\end{pmatrix}\mathrm{d}x = a^{\rho+\sigma}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}$$

$$\sum_{g_1,\cdots,g_r=0}^{\infty}\sum_{M_1=0}^{\alpha^{(1)}}\cdots\sum_{M_r=0}^{\alpha^{(r)}}\sum_{w=0}^{\infty}\prod_{i=1}^{v} z_i'''^{K_i}\prod_{k=1}^{u} z''^{R_k} a_v b_u \frac{(-b)^w}{w!}\frac{\phi\prod_{i=1}^{r}\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r} g_i}}{\prod_{i=1}^{r}\epsilon_{m_i}^{i} g_i!}$$

$$a^{\sum_{i=1}^{v}(e_i+f_i)K_i+\sum_{i=1}^{u}(e_i'+f_i')R_i+\sum_{i=1}^{r}(\mu_i+\upsilon_i)\eta_{G_i,g_i}+lw} A_{p+3,q+2;Y}^{m,n+3;X}\left(\begin{array}{c} \mathrm{z}_1' a^{\mu_1'+\upsilon_1'} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s' x^{\mu_s'+\upsilon_s'} \end{array}\right|$$

$$(1\text{-}\rho-\textstyle\sum_{i=1}^{v} e_iK_i-\sum_{i=1}^{u} e_i'R_i-\sum_{i=1}^{r}\mu_i\eta_{G_i,g_i}-lw;\mu_1',\cdots,\mu_s'),$$
$$\cdot$$
$$\cdot$$
$$(1\text{-}\lambda-\textstyle\sum_{i=1}^{v} g_iK_i-\sum_{i=1}^{u} g_i'R_i-\sum_{i=1}^{r}\eta_i\eta_{G_i,g_i};\eta_1',\cdots,\eta_s'),$$

$$(\text{-}\sigma-\textstyle\sum_{i=1}^{v} f_iK_i-\sum_{i=1}^{u} f_i'R_i-\sum_{i=1}^{r}\upsilon_i\eta_{G_i,g_i};\upsilon_1',\cdots,\upsilon_s'),$$
$$\cdot$$
$$\cdot$$
$$(\text{-}\rho-\sigma-\textstyle\sum_{i=1}^{v}(e_i+f_i)K_i-\sum_{i=1}^{u}(e_i'+f_i')R_i-\sum_{i=1}^{r}(\upsilon_i+\mu_i)\eta_{G_i,g_i}-lw;\upsilon_1'+\mu_1',\cdots,\upsilon_s'+\mu_s'),$$

$$\left.\begin{array}{c}(1\text{-}\lambda-\sum_{i=1}^{v} g_iK_i-\sum_{i=1}^{u} g_i'R_i-\sum_{i=1}^{r}\eta_i\eta_{G_i,g_i}-w;\eta_1',\cdots,\eta_s'),\mathbb{A} \\ \cdot \\ \cdot \\ \mathbb{B}\end{array}\right) \tag{2.1}$$

ϕ_1,ϕ_i are defined respectively by (1.2) and (1.3)

Provided that

$min\{l,e_i,f_i,g_i,e_j',f_j',g_j',\mu_k,\upsilon_k,\eta_k,\mu_{\mathbf{m}}',\upsilon_{\mathbf{m}}',\eta_{\mathbf{m}}'\}>0; i=1,\cdots,u, j=1,\cdots,v, k=1,\cdots,r,\mathbf{m}=1,\cdots,s$

$Re(\lambda)\geqslant 0\,; z_i\neq 0,\sum_{j=1}^{A}\gamma_j^{(i)}-\sum_{j=1}^{C}\xi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\eta_j^{(i)}-\sum_{j=1}^{N^{(i)}}\epsilon_j^{(i)}<0, i=1,\cdots,r$

$$Re\left(\rho+\sum_{j=1}^{r}\mu_i\eta_{G_i,g_i}\right)+\sum_{j=1}^{s}\mu_i'\min_{1\leqslant k\leqslant m_i'} Re\left(\frac{d_k'^{(j)}}{D_k'^{(j)}}\right)>0$$

and $Re\left(1+\sigma+\sum_{j=1}^{r}\upsilon_i\eta_{G_i,g_i}\right)+\sum_{j=1}^{s}\upsilon_i'\min_{1\leqslant k\leqslant m_i'} Re\left(\frac{d_k'^{(j)}}{D_k'^{(j)}}\right)>0$

$|arg(\Omega_i')z_k'|<\frac{1}{2}\eta_k'\pi,\xi'^{*}=0,\eta_i'>0$

Proof

To prove (2.1), first expressing a class of multivariable polynomials $S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}[.]$ defined by Srivastava [2] in serie with the help of (1.7), a class of multivariable polynomials $S_L^{h_1,\cdots,h_u}[.]$ defined by Srivastava et al [4] in serie with the help of (1.8), the A-functions of r-variables defined by Gautam and Asgar [1] in serie with the help of (1.1) and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the A-function of s-variables in Mellin-contour integral and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now evaluating the inner x-integral thus obtained by the help of special case of result [5, page 61, eq(5.2.1)]. Interpreting the Mellin-Barnes integral in multivariable A-function, we obtain the desired result.

3. Particular cases

a) If $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$,$m=0$ and $A_j'^{(i)},B_j'^{(i)},C_j'^{(i)},D_j'^{(i)}\in\mathbb{R}$ and $m'=0$, the multivariable A-functions reduce to multivariable H-functions defined by Srivastava et al [6], we obtain the following result.

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda}S_L^{h_1,\cdots,h_u}\begin{pmatrix} z_1''x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1}\\ \cdot\\ \cdot\\ \cdot\\ z_u''x^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u}\end{pmatrix}$$

$$S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} z_1'''x^{e_1'}(a-x)^{f_1'}(1+bx^l)^{-g_1'}\\ \cdot\\ \cdot\\ \cdot\\ z_v'''x^{e_v}(a-x)^{f_v}(1+bx^l)^{-g_v'}\end{pmatrix} H\begin{pmatrix} z_1x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1}\\ \cdot\\ \cdot\\ \cdot\\ z_rx^{\mu_r}(a-x)^{\upsilon_r}(1+bx^l)^{-\eta_r}\end{pmatrix}$$

$$H\begin{pmatrix} z_1'x^{\mu_1'}(a-x)^{\upsilon_1'}(1+bx^l)^{-\eta_1'}\\ \cdot\\ \cdot\\ \cdot\\ z_s'x^{\mu_s'}(a-x)^{\upsilon_s'}(1+bx^l)^{-\eta_s'}\end{pmatrix}\mathrm{d}x = a^{\rho+\sigma}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}$$

$$\sum_{g_1,\cdots,g_r=0}^{\infty}\sum_{M_1=0}^{\alpha^{(1)}}\cdots\sum_{M_r=0}^{\alpha^{(r)}}\sum_{w=0}^{\infty}\prod_{i=1}^{v}z_i'''^{K_i}\prod_{k=1}^{u}z''^{R_k}a_vb_u\,\frac{(-b)^w}{w!}\;\frac{\phi\prod_{i=1}^r\phi_iz_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^r g_i}}{\prod_{i=1}^r\epsilon^i_{m_i}g_i!}$$

$$a^{\sum_{i=1}^v(e_i+f_i)K_i+\sum_{i=1}^u(e_i'+f_i')R_i+\sum_{i=1}^r(\mu_i+\upsilon_i)\eta_{G_i,g_i}+lw}H_{p+3,q+2;Y}^{0,n+3;X}\left(\begin{matrix} z_1'a^{\mu_1'+\upsilon_1'}\\ \cdot\\ \cdot\\ \cdot\\ z_s'x^{\mu_s'+\upsilon_s'}\end{matrix}\right|$$

$(1\text{-}\rho-\sum_{i=1}^v e_iK_i-\sum_{i=1}^u e_i'R_i-\sum_{i=1}^r\mu_i\eta_{G_i,g_i}-lw;\mu_1',\cdots,\mu_s'),$

$\cdot$

$\cdot$

$(1\text{-}\lambda-\sum_{i=1}^v g_iK_i-\sum_{i=1}^u g_i'R_i-\sum_{i=1}^r\eta_i\eta_{G_i,g_i};\eta_1',\cdots,\eta_s'),$

$$(\text{-}\sigma - \sum_{i=1}^{v} f_iK_i - \sum_{i=1}^{u} f_i'R_i - \sum_{i=1}^{r} \upsilon_i\eta_{G_i,g_i}; \upsilon_1', \cdots, \upsilon_s'),$$
$$\vdots$$
$$(\text{-}\rho - \sigma - \sum_{i=1}^{v}(e_i + f_i)K_i - \sum_{i=1}^{u}(e_i' + f_i')R_i - \sum_{i=1}^{r}(\upsilon_i + \mu_i)\eta_{G_i,g_i} - lw; \upsilon_1' + \mu_1', \cdots, \upsilon_s' + \mu_s'),$$

$$\left.\begin{matrix}(1\text{-}\lambda - \sum_{i=1}^{v} g_iK_i - \sum_{i=1}^{u} g_i'R_i - \sum_{i=1}^{r} \eta_i\eta_{G_i,g_i} - w; \eta_1', \cdots, \eta_s'), \mathbb{A} \\ \vdots \\ \mathbb{B}\end{matrix}\right) \tag{3.1}$$

under the same notations and conditions that (2.1) with $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$,$m = 0$, ${A'}_j^{(i)}, {B'}_j^{(i)}, {C'}_j^{(i)}, {D'}_j^{(i)} \in \mathbb{R}$ and $m' = 0$.

b) If $B(L; R_1, \cdots, R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta_j'+\cdots+R_u\theta_j^{(u)}} \prod_{j=1}^{B'}(b_j')_{R_1\phi_j'} \cdots \prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{R_1\psi_j'+\cdots+R_u\psi_j^{(u)}} \prod_{j=1}^{D'}(d_j')_{R_1\delta_j'} \cdots \prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (3.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1, \cdots, z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [3]. We have

$$\int_0^a x^{\rho-1}(a-x)^{\sigma}(1+bx^l)^{-\lambda}\ S_{N_1,\cdots,N_v}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_v}\begin{pmatrix} z_1'''x^{e_1'}(a-x)^{f_1'}(1+bx^l)^{-g_1'} \\ \vdots \\ z_v'''x^{e_v}(a-x)^{f_v}(1+bx^l)^{-g_v'}\end{pmatrix}$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\begin{pmatrix} z_1''x^{e_1}(a-x)^{f_1}(1+bx^l)^{-g_1} \\ \vdots \\ z_u''x^{e_u}(a-x)^{f_u}(1+bx^l)^{-g_u}\end{pmatrix}$$

$$A\begin{pmatrix} z_1x^{\mu_1}(a-x)^{\upsilon_1}(1+bx^l)^{-\eta_1} \\ \vdots \\ z_rx^{\mu_r}(a-x)^{\upsilon_r}(1+bx^l)^{-\eta_r}\end{pmatrix} A\begin{pmatrix} z_1'x^{\mu_1'}(a-x)^{\upsilon_1'}(1+bx^l)^{-\eta_1'} \\ \vdots \\ z_s'x^{\mu_s'}(a-x)^{\upsilon_s'}(1+bx^l)^{-\eta_s'}\end{pmatrix}\mathrm{d}x = a^{\rho+\sigma}$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L} \sum_{g_1,\cdots,g_r=0}^{\infty} \sum_{M_1=0}^{\alpha^{(1)}} \cdots \sum_{M_r=0}^{\alpha^{(r)}} \sum_{w=0}^{\infty} \prod_{i=1}^{v} z_i'''^{K_i} \prod_{k=1}^{u} z''^{R_k} a_v b_u' \frac{(-b)^w}{w!}$$

$$\frac{\phi\prod_{i=1}^{r}\phi_i z_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{r}g_i}}{\prod_{i=1}^{r}\epsilon_{m_i}^{i}g_i!}\,a^{\sum_{i=1}^{v}(e_i+f_i)K_i+\sum_{i=1}^{u}(e_i'+f_i')R_i+\sum_{i=1}^{r}(\mu_i+\upsilon_i)\eta_{G_i,g_i}+lw}$$

$$A_{p+3,q+2;Y}^{m,n+3;X}\left(\begin{array}{c|c} z_1' a^{\mu_1'+\upsilon_1'} & (1\text{-}\rho-\sum_{i=1}^{v}e_iK_i-\sum_{i=1}^{u}e_i'R_i-\sum_{i=1}^{r}\mu_i\eta_{G_i,g_i}-lw;\mu_1',\cdots,\mu_s'), \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \\ z_s' x^{\mu_s'+\upsilon_s'} & (1\text{-}\lambda-\sum_{i=1}^{v}g_iK_i-\sum_{i=1}^{u}g_i'R_i-\sum_{i=1}^{r}\eta_i\eta_{G_i,g_i};\eta_1',\cdots,\eta_s'), \end{array}\right.$$

$$(\text{-}\sigma-\sum_{i=1}^{v}f_iK_i-\sum_{i=1}^{u}f_i'R_i-\sum_{i=1}^{r}\upsilon_i\eta_{G_i,g_i};\upsilon_1',\cdots,\upsilon_s'),$$

$$\cdot$$

$$\cdot$$

$$(\text{-}\rho-\sigma-\sum_{i=1}^{v}(e_i+f_i)K_i-\sum_{i=1}^{u}(e_i'+f_i')R_i-\sum_{i=1}^{r}(\upsilon_i+\mu_i)\eta_{G_i,g_i}-lw;\upsilon_1'+\mu_1',\cdots,\upsilon_s'+\mu_s'),$$

$$\left.\begin{array}{c}(1\text{-}\lambda-\sum_{i=1}^{v}g_iK_i-\sum_{i=1}^{u}g_i'R_i-\sum_{i=1}^{r}\eta_i\eta_{G_i,g_i}-w;\eta_1',\cdots,\eta_s'),\mathbb{A} \\ \cdot \\ \cdot \\ \mathbb{B}\end{array}\right) \tag{3.3}$$

where $b_u'=\frac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B(L;R_1,\cdots,R_u)$ is defined by (3.2)

under the same notations and conditions that (2.1)

4. Conclusion

The multivariable A-functions defined by Gautam [1] and the classes of multivariable polynomials are the quite basic in nature. Therefore on specializing the parameters of these functions and polynomials, we may obtain the unified integral concerning various special functions of several variables and one variable such as multivariable H-function, Fox's H-function , Meijer's Gfunction, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, in cetera.

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