# Analysis of Two-Dissimilar Component System with Uncertain Availability of Repairman 

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#### Abstract

The system consists of a two-dissimilar components working in parallel, say A and B. Both the components are operative initially at time $t=0$. A single repair facility is available for the repair. Upon failure of a component the repair facility, if not busy, is available with some fixed probability p. If repair facility is not available at the time of a failure of a component, it is called for repair. The repair facility appearance time distribution is exponential. When repair facility is busy in repair of the failed component, the other failed component waits for its repair. After repair, the components become as good as new. The repair time of both the components are arbitrary functions of time. Failure time distributions are assumed to be exponential.


Key words: Reliability, Mean time to system failure, Availability, Exponential distribution.

## Introduction

Two-unit standby redundant systems have been extensively studied by several authors in the past. Said and Sherbeny (2010) analyzed a two-unit cold standby system with two stage repair and waiting time. In this paper a two dis-similar component system is considered. The system operates even if a single component operates. A single repair facility is available with some fixed probability for the repair of failed components.

## System Assumption and Description

The system consists of a two-dissimilar components working in parallel, say A and B. Both the components are operative initially at time $t=0$. A single repair facility is available for the repair. Upon failure of a component the repair facility, if not busy, is available with some fixed probability p. If repair facility is not available at the time of a failure of a component, it is called for repair. The repair facility appearance time distribution is exponential. When repair facility is busy in repair of the failed component, the other failed component waits for its repair. After repair, the components become as good as new. The repair time of both the components are arbitrary functions of time. Failure time distributions are assumed to be exponential. Several measures of system effectiveness such as MTSF, A, B etc. are obtained by using regenerative point technique.

## Notations and States of the System

| E | $\equiv$ set of regenerative states $\left\{\mathrm{S}_{0}, \mathrm{~S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{4}, \mathrm{~S}_{6}\right\}$ |
| ---: | :--- |
| $\overline{\mathrm{E}}$ | $\equiv \quad$ set of non-regenerative states $\left\{\mathrm{S}_{3}, \mathrm{~S}_{4}\right\}$ |
| $\alpha$ | $\equiv \quad$ failure rate of component A. |
| $\beta$ | $\equiv \quad$ failure rate of component B. |
| $\theta$ | $\equiv \quad$ rate of appearance of repair facility. |
| $\mathrm{f}()$. | $\equiv \quad$ Pdf of repair rate of component A. |
| $\mathrm{g}()$. | $\equiv \quad$ Pdf of repair rate of component B. |

$p \quad=(1-q)=$ probability that the repairman is available for repairs.
The system may be in one of the following states:

| $\mathrm{S}_{0}\left(\mathrm{~A}_{\mathrm{N}} \mathrm{B}_{\mathrm{N}}\right)$ | $\equiv$ | The components A and B both are in normal operative mode. |
| :--- | :--- | :--- |
| $\mathrm{S}_{1}\left(\mathrm{~A}_{\mathrm{F}} \mathrm{B}_{\mathrm{N}}\right)$ | $\equiv$ | Failed component A is waiting for repair and component B is operative. |
| $\mathrm{S}_{3}\left(\mathrm{~A}_{\mathrm{R}} \mathrm{B}_{\mathrm{N}}\right)$ | $\equiv$ | Component A is under repair and B is operative. |
| $\mathrm{S}_{4}\left(\mathrm{~A}_{\mathrm{N}} \mathrm{B}_{\mathrm{R}}\right)$ | $\equiv$ | Component A is in operative mode and component B is under repair. |
| $\mathrm{S}_{5}\left(\mathrm{~A}_{\mathrm{WR}} \mathrm{B}_{\mathrm{R}}\right)$ | $\equiv$ | Component A waits for repair and component B is under repair. |

$\mathrm{S}_{6}\left(\mathrm{~A}_{\mathrm{R}} \mathrm{B}_{\mathrm{WR}}\right) \quad \equiv \quad$ Component A is under repair and component B waits for its repair.


Figure 1 Analysis of two dissimilar component with uncertain availability of repairman+

## Transition Probabilities and Sojourn Times

Let $T_{0}(=0), T_{1}, T_{2} \ldots$ be the epochs at which the system enters the state $S_{i} \square \square$ Eand
Let $X_{n}$ denotes the state entered at epoch $T_{n+1}$, i.e. just after the transition at $T_{n}$. Then
$\left\{X_{n}, T_{n}\right\}$ constitutes a Markov-renewal process with the state space $E$ and

$$
\mathrm{Q}_{\mathrm{ij}}(\mathrm{t})=\operatorname{Pr}\left[\mathrm{X}_{\mathrm{n}+1}=\mathrm{S}_{\mathrm{j}}\left|\mathrm{~T}_{\mathrm{n}+1}-\mathrm{T}_{\mathrm{n}} \leq \mathrm{t}\right| \mathrm{X}_{\mathrm{n}}=\mathrm{S}_{\mathrm{i}}\right]
$$

The transition probability matrix of the embedded Markov chain is:

$$
\mathrm{P}=\left(\mathrm{p}_{\mathrm{ij}}\right)=\underset{\mathrm{t} \rightarrow \infty}{\left\{\mathrm{Q}_{\mathrm{ij}}(\mathrm{t})\right\}=\left\{\mathrm{Q}_{\mathrm{ij}}(\infty)\right\} . . . . ~}
$$

By simple probabilistic considerations, the non-zero elements of $\mathrm{Q}=\left\{\mathrm{Q}_{\mathrm{ij}}(\mathrm{t})\right\}$ can be obtained as follows: For the system to reach state $S_{1}$ from $S_{0}$ on or before time $t$, we suppose that the system transits from $S_{0}$ to $S_{1}$ during ( $u$, $\mathrm{u}+\mathrm{du}$ ); $\mathrm{u} \leq \mathrm{t}$ while it does not transits to any of the state's $\mathrm{S}_{2}, \mathrm{~S}_{3}$ and $\mathrm{S}_{4}$ up to the time u . The probability of this event is:

$$
q \alpha e^{-\alpha} u d u \cdot(p+q) e-\beta u=q \alpha e^{-(\alpha+\beta)} u d u
$$

Since $u$ varies from 0 to 1 , therefore

$$
\mathrm{Q}_{01}(\mathrm{t})=\int_{0}^{\mathrm{t}} \mathrm{q} \alpha \mathrm{e}^{-(\alpha+\beta) \mathrm{u}} \mathrm{du}
$$

Taking limit $t$ tends to infinity, we have

$$
\mathrm{p}_{\mathrm{ij}}=\lim _{\mathrm{t} \rightarrow \infty} \mathrm{Q}_{\mathrm{ij}}(\mathrm{t}) .
$$

The non-zero elements of $\mathrm{p}_{\mathrm{ij}}$ are given below:

$$
\mathrm{p}_{01}=\mathrm{q} \alpha /(\alpha+\beta), \mathrm{p}_{02}=\mathrm{q} \beta /(\alpha+\beta)
$$

$$
\begin{array}{ll}
p_{03}=p \alpha /(\alpha+\beta), & p_{04}=p \beta /(\alpha+\beta), \\
p_{13}=\theta /(\theta+\beta), & p_{16}=\beta /(\theta+\beta), \\
p_{24}=\theta /(\theta+\alpha), & p_{25}=\alpha /(\theta+\alpha), \\
p_{30}=F(\beta), & p_{34}^{(6)}=[1-F(\beta)]=p_{36} \\
p_{40}=G(\alpha), & p_{43}^{(5)}=[1-G(\alpha)]=p_{45}
\end{array}
$$

We observe the following relationships among the above study state probabilities:

$$
\begin{aligned}
& \mathrm{p}_{01}+\mathrm{p}_{02}+\mathrm{p}_{03}+\mathrm{p}_{04}=1 \\
& \mathrm{p}_{13}+\mathrm{p}_{16}=1 \\
& \mathrm{p}_{24}+\mathrm{p}_{25}=1 \\
& \mathrm{p}_{30}+\mathrm{p}_{36}=\mathrm{p}_{36}+\mathrm{p}_{34}^{6}=1 \\
& \mathrm{p}_{40}+\mathrm{p}_{45}=\mathrm{p}_{40}+\mathrm{p}_{43}^{(5)}=1
\end{aligned}
$$

Mean sojourn time $\mu_{\mathrm{i}}$ in state $\mathrm{S}_{\mathrm{i}}$ is defined as the time that the system continues in state $\mathrm{S}_{\mathrm{i}}$ before transiting to any state. If $T$ denotes the sojourn timein $S_{i}$ then $\mu_{\mathrm{i}}$ in $\mathrm{S}_{\mathrm{i}}$ is:- $\mu_{\mathrm{i}}=\mathrm{E}(\mathrm{T})=\int_{0}^{\infty} \mathrm{P}(\mathrm{T}>\mathrm{t}) \mathrm{dt}$.
Using this we can obtain the following expressions,

$$
\begin{aligned}
& \mu_{0}=\int_{0}^{\infty} \mathrm{e}^{-(\alpha+\beta) t} \mathrm{dt}=1 /(\alpha+\beta), \\
& \mu_{1}=\int_{0}^{\infty} \mathrm{e}^{-(\theta+\beta) \mathrm{t}} \mathrm{dt}=1 /(\theta+\beta), \\
& \mu_{2}=\int_{0}^{\infty} \mathrm{e}^{-(\theta+\alpha) \mathrm{t}} \mathrm{dt}=1 /(\theta+\alpha), \\
& \mu_{3}=\int_{0}^{\infty} \mathrm{F}(\mathrm{t}) \mathrm{dt} \\
& \mu_{4}=\int_{0}^{\infty} \mathrm{G}(\mathrm{t}) \mathrm{dt}
\end{aligned}
$$

In terms of Laplace-Stieltje's transform of $\mathrm{Q}_{\mathrm{i}}(\mathrm{t})$, we define $\mathrm{m}_{\mathrm{ij}}$ as follows:-

$$
\begin{gathered}
\mathrm{m}_{\mathrm{ij}}=-\tilde{\mathrm{Q}}_{\mathrm{ij}}(0)=\lim _{\mathrm{s} \rightarrow 0}-\frac{\mathrm{d}}{\mathrm{ds}} \mathrm{Q}_{\mathrm{ij}}(\mathrm{~s}) \\
\mathrm{Q}_{01}(\mathrm{~s})=\int_{0}^{\infty} \mathrm{q} \alpha \mathrm{e}^{-(\mathrm{s}+\alpha+\beta) \mathrm{u}} \mathrm{du}=\mathrm{q} \alpha /(\mathrm{s}+\alpha+\beta) \\
\mathrm{Q}_{02}(\mathrm{~s})=\int_{0}^{\infty} \mathrm{q} \beta \mathrm{e}^{-(\mathrm{s}+\alpha+\beta) \mathrm{u}} \mathrm{du}=\mathrm{q} \beta /(\mathrm{s}+\alpha+\beta) \\
\mathrm{Q}_{03}(\mathrm{~s})=\int_{0}^{\infty} \mathrm{p}_{0} \mathrm{e}^{-(\mathrm{s}+\alpha+\beta) \mathrm{u}} \mathrm{du}=\mathrm{p} \alpha /(\mathrm{s}+\alpha+\beta),
\end{gathered}
$$

$$
\begin{aligned}
& \mathrm{Q}_{04}(\mathrm{~s})=\int_{0}^{\infty} \mathrm{p} \beta \mathrm{e}^{-(\mathrm{s}+\alpha+\beta) u} \mathrm{du}=\mathrm{p} \beta /(\mathrm{s}+\alpha+\beta), \\
& \mathrm{Q}_{13}(\mathrm{~s})=\int_{0}^{\infty} \theta \mathrm{e}^{-(\mathrm{s}+\theta+\beta) \mathrm{u}} \mathrm{du}=\theta /(\mathrm{s}+\theta+\beta), \\
& \mathrm{Q}_{16}(\mathrm{~s})=\int_{0}^{\infty} \beta \mathrm{e}^{-(\mathrm{s}+\theta+\beta) \mathrm{u}} \mathrm{du}=\beta /(\mathrm{s}+\theta+\beta), \\
& \mathrm{Q}_{24}(\mathrm{~s})=\int_{0}^{\infty} \theta \mathrm{e}^{-(\mathrm{s}+\theta+\alpha) \mathrm{u}} \mathrm{du}=\theta /(\mathrm{s}+\theta+\alpha), \\
& \mathrm{Q}_{30}(\mathrm{~s})=\int_{0}^{\infty} \mathrm{e}^{-(\beta+\mathrm{s}) \mathrm{u}} \mathrm{dF}(\mathrm{u})=\beta F^{*}(\beta+\mathrm{s}), \\
& \mathrm{Q}_{40}(\mathrm{~s})=\int_{0}^{\infty} \mathrm{e}^{-(\alpha+s) u} \mathrm{dG}(\mathrm{u})=\alpha \mathrm{G}^{*}(\alpha+\mathrm{s}), \\
& \mathrm{Q}_{34}^{(6)}(\mathrm{s})=\int_{0}^{\infty} \mathrm{e}^{-s \mathrm{st}}\left(1-\mathrm{e}^{-\beta t}\right) \mathrm{dF}(\mathrm{t}), \mathrm{f}^{*}(\mathrm{~s})-\mathrm{f}^{*}(\mathrm{~s}+\beta) \\
& \mathrm{Q}_{43}^{(5)}(\mathrm{s})=\int_{0}^{\infty} \mathrm{e}^{-\mathrm{st}}\left(1-\mathrm{e}^{\alpha \mathrm{t}}\right) \mathrm{dG}(\mathrm{t}), \mathrm{g}^{*}(\mathrm{~s})-\mathrm{g}^{*}(\mathrm{~s}+\alpha)
\end{aligned}
$$

We have,

$$
\begin{gathered}
\mathrm{m}_{01}=-\mathrm{Q}_{01}^{\prime}(\mathrm{s})=\mathrm{q} \alpha /(\alpha+\beta)^{2}, \\
\mathrm{~m}_{02}=-\mathrm{Q}_{02}^{\prime}(0)=\mathrm{q} \beta /(\alpha+\beta)^{2}, \\
\mathrm{~m}_{03}=-\mathrm{Q}_{03}^{\prime}(0)=\mathrm{p} \alpha /(\alpha+\beta)^{2}, \\
\mathrm{~m}_{04}=-\mathrm{Q}_{04}^{\prime}(0)=\mathrm{p} \beta /(\alpha+\beta)^{2}, \\
\mathrm{~m}_{13}=-\mathrm{Q}_{13}^{\prime}(0)=\theta /(\theta+\beta)^{2}, \\
\mathrm{~m}_{16}=-\mathrm{Q}_{16}^{\prime}(0)=\beta /(\theta+\beta)^{2} \\
\mathrm{~m}_{24}=-\mathrm{Q}_{24}^{\prime}(0)=\theta /(\theta+\alpha)^{2}, \\
\mathrm{~m}_{25}=-\mathrm{Q}_{25}^{\prime}(0)=\theta /(\theta+\alpha)^{2}, \\
\mathrm{~m}_{34}^{(6)}=-\mathrm{Q}_{34}^{\prime(6)}(0)=\int_{0}^{\infty} \mathrm{te}^{-\beta \mathrm{t}} \mathrm{dF}(\mathrm{t})-\beta \int_{0}^{\infty} \mathrm{te}^{-\beta t} \mathrm{E}(\mathrm{t}) \mathrm{dt} \\
\mathrm{~m}_{43}^{(5)}=-\mathrm{Q}_{43}^{\prime(5)}(0)=\int_{0}^{\infty} \mathrm{te}^{-\alpha \mathrm{t}} \mathrm{dG}(\mathrm{t})-\alpha \int_{0}^{\infty} \mathrm{te}^{-\alpha \mathrm{t}} \mathrm{G}(\mathrm{t}) \mathrm{dt} \\
\mathrm{~m}_{30}=-\mathrm{Q}_{30}^{\prime}(\mathrm{O})=\int_{0}^{\infty} \mathrm{te}^{-\beta t} \mathrm{dF}(\mathrm{t}),
\end{gathered}
$$

$m_{40}=-Q_{40}^{\prime}(0)=\int_{0}^{\infty} t^{-\alpha t} d G(t)$.

It can be easily seen that,
$\mathrm{m}_{01}+\mathrm{m}_{02}+\mathrm{m}_{03}+\mathrm{m}_{04}=\mu_{0}$,
$\mathrm{m}_{16}+\mathrm{m}_{13}=\mu_{1}$,
$\mathrm{m}_{24}+\mathrm{m}_{25}=\mu_{2}$,
$\mathrm{m}_{30}+\mathrm{m}_{34}^{(6)}=\mu_{3}$,
$\mathrm{m}_{40}+\mathrm{m}_{43}^{(5)}=\mu_{4}$.

## Reliability and Mean Time to System Failure (MTSF)

Let the random variable $T_{i}$ be the time to system failure (TSF) when the system starts its operation from $\mathrm{S}_{\mathrm{i}} \square \square \mathrm{E}$, then the reliability of the systm is given by,
$R_{i}(t)=P\left(T_{i}>t\right)$.
In order to determine $R_{i}(t)$, we regard the failed states $S_{5}, S_{6}$ of the system as absorbing states. By simple probabilistic reasoning, we observe that $R_{0}(t)$ is the sum of following contingencies:
(i) System remains up in state $\mathrm{S}_{0}$ without making any transition to any other state up to time t . The probability of this contingency is,

$$
\mathrm{e}^{-(\alpha+\beta) \mathrm{t}}=\mathrm{Z}_{0}(\mathrm{t})
$$

(ii) System first enters the regenerative state $S_{1}$ during $(\mathrm{u}, \mathrm{u}+\mathrm{du}), \mathrm{u} \leq \mathrm{t}$ and then starting from $\mathrm{S}_{1}$, it remains up without any break down for the time duration $(\mathrm{t}-\mathrm{v})$. The probability of this contingency is,

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{01}(\mathrm{u}) \mathrm{duR}_{1}(\mathrm{t}-\mathrm{u})=\mathrm{q}_{01}(\mathrm{t}) \odot \mathrm{R}_{1}(\mathrm{t})
$$

(iii) System first enters the regenerative state $\mathrm{S}_{2}$ during $(\mathrm{u}, \mathrm{u}+\mathrm{du}), \mathrm{u} \leq \mathrm{t}$ and then starting from $\mathrm{S}_{2}$, it remains up without any breakdown for the time duration $(t-v)$. The probability of this contingency is,

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{\mathrm{o} 2}(\mathrm{u}) \mathrm{duR}_{2}(\mathrm{t}-\mathrm{u})=\mathrm{q}_{\mathrm{o} 2}(\mathrm{t}) \odot \mathrm{R}_{2}(\mathrm{t})
$$

(iv) System first enters the regenerative state $S_{3}$ during $(u, u+d u), u \leq t$ and then starting from $S_{3}$, it remains up without any breakdown for the time duration $(\mathrm{t}-\mathrm{v})$. The probability of this contingency is,

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{\mathrm{o3}}(\mathrm{u}) \mathrm{duR}_{3}(\mathrm{t}-\mathrm{u})=\mathrm{q}_{\mathrm{o3}}(\mathrm{t}) \odot \mathrm{R}_{3}(\mathrm{t})
$$

(v) System first enters the regenerative state $S_{4}$ during ( $\mathrm{u}, \mathrm{u}+\mathrm{du}$ ); $\mathrm{u} \square \square \mathrm{t}$ and then starting from S , it remains up without any breakdown for the time duration $(t-v)$. The probability of this contingency is,

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{04}(\mathrm{u}) \mathrm{duR}_{4}(\mathrm{t}-\mathrm{u})=\mathrm{q}_{04}(\mathrm{t}) \odot \mathrm{R}_{4}(\mathrm{t})
$$

Thus,

$$
R_{0}(t)=Z_{1}(t)+q_{13}(t) \odot R_{3}(t)
$$

Similarly,

$$
\begin{aligned}
& \mathrm{R}_{1}(\mathrm{t})=\mathrm{Z}_{1}(\mathrm{t})+\mathrm{q}_{13}(\mathrm{t}) \odot \mathrm{R}_{3}(\mathrm{t}) \\
& \mathrm{R}_{2}(\mathrm{t})=\mathrm{Z}_{2}(\mathrm{t})+\mathrm{q}_{24}(\mathrm{t}) \odot \mathrm{R}_{4}(\mathrm{t}) \\
& \mathrm{R}_{3}(\mathrm{t})=\mathrm{Z}_{3}(\mathrm{t})+\mathrm{q}_{30}(\mathrm{t}) \odot \mathrm{R}_{0}(\mathrm{t}) \\
& \mathrm{R}_{4}(\mathrm{t})=\mathrm{Z}_{4}(\mathrm{t})+\mathrm{q}_{40}(\mathrm{t}) \odot \mathrm{R}_{0}(\mathrm{t}),
\end{aligned}
$$

Where,

$$
\begin{array}{ll}
\mathrm{Z}_{0}(\mathrm{t})=\mathrm{e}^{-(\alpha+\beta) \mathrm{t}}, & \mathrm{Z}_{1}(\mathrm{t})=\mathrm{e}^{-(\theta+\beta) \mathrm{t}} \\
\mathrm{Z}_{2}(\mathrm{t})=\mathrm{e}^{-(\theta+\alpha) \mathrm{t}}, & \mathrm{Z}_{3}(\mathrm{t})=\mathrm{e}^{-\beta \mathrm{t}} \overline{\mathrm{~F}}(\mathrm{t}) \\
\mathrm{Z}_{4}(\mathrm{t})=\mathrm{e}^{-\alpha \mathrm{t}} \overline{\mathrm{G}}(\mathrm{t}) .
\end{array}
$$

For brevity, we have omitted the argument's' from $\mathrm{q}_{\mathrm{ij}}^{*}(\mathrm{~s}), Z_{\mathrm{i}}^{*}(\mathrm{~s})$ and $\mathrm{R}_{\mathrm{i}}^{*}(\mathrm{~s})$. Solving the above equation for $R_{0}^{*}(s)$, we get

$$
\mathrm{R}_{0}^{*}(\mathrm{~s})=\frac{\mathrm{N}_{1}(\mathrm{~s})}{\mathrm{D}_{1}(\mathrm{~s})}
$$

Where, $\mathrm{N}_{1}(\mathrm{~s})=\left[\mathrm{Z}_{0}^{*}+\mathrm{q}_{01}^{*} \mathrm{Z}_{1}^{*}+\left(\mathrm{q}_{01}^{*} \mathrm{q}_{13}^{*}+\mathrm{q}_{\mathrm{o2}}^{*}\right)\left(\mathrm{Z}_{2}^{*}+\mathrm{q}_{24}^{*} Z_{4}^{*}\right)\right.$ $\left.+\left(\mathrm{q}_{01}^{*} \mathrm{q}_{13}^{*}+\mathrm{q}_{24}^{*}\right)\left(\mathrm{Z}_{3}^{*}+\mathrm{q}_{34}^{*} Z_{4}^{*}\right)\right]$
$D_{1}(s)=\left[1-q_{01}^{*} q_{13}^{*} q_{30}^{*}-q_{02}^{*} q_{24}^{*}-q_{03}^{*} q_{30}^{*}-q_{04}^{*} q_{40}^{*}\right]$.
Taking the inverse Laplace transform (ILT) of the above equation, we can get the reliability of the system when it starts from state $S_{0}$, the mean time to system failure (MTSF) can be obtained on using the formula

$$
\begin{aligned}
\mathrm{E}\left(\mathrm{~T}_{0}\right) & =\int_{0}^{\infty} \mathrm{R}_{0}(\mathrm{t}) \mathrm{dt} \\
& =\lim _{\mathrm{s} \rightarrow 0} \mathrm{R}_{0}^{*}(\mathrm{~s}) \\
& =\frac{\mathrm{N}_{1}(0)}{\mathrm{D}_{1}(0)} .
\end{aligned}
$$

To determine $N_{1}(0)$ and $D_{1}(0)$, we must first obtain $Z_{i}^{*}(\mathbf{s})$, using the result

$$
\lim _{\mathrm{s} \rightarrow 0} Z_{\mathrm{i}}^{*}(\mathrm{~s})=\int_{0}^{\infty} Z_{\mathrm{i}}(\mathrm{t}) \mathrm{dt}
$$

Therefore,

$$
\begin{array}{ll}
Z_{0}^{*}(0)=\mu_{0}, & Z_{1}^{*}(0)=\mu_{1} \\
Z_{2}^{*}(0)=\mu_{3}, & Z_{4}^{*}(0)=\mu_{4}
\end{array}
$$

Thus, using $q_{i j}^{*}(0)=p_{i j}$ and above $Z_{i}^{*}(s)$, we get,

$$
\begin{gathered}
\mathrm{N}_{1}(0)=\mu_{0}+\mathrm{p}_{01} \mu_{1}+\left(\mathrm{p}_{01} \mathrm{p}_{13}+\mathrm{p}_{02}\right)\left(\mu_{2}+\mathrm{p}_{24} \mu_{4}\right) \\
+\left(\mathrm{p}_{01} \mathrm{p}_{13}+\mathrm{p}_{24}\right)\left(\mu_{3}+\mathrm{p}_{34} \mu_{4}\right)
\end{gathered}
$$

And, $\mathrm{D}_{1}(0)=\left(1-\mathrm{p}_{01} \mathrm{p}_{13} \mathrm{p}_{30}-\mathrm{p}_{02} \mathrm{p}_{24}-\mathrm{p}_{03} \mathrm{p}_{30}-\mathrm{p}_{04} \mathrm{p}_{40}\right)$.

## Availability Analysis

Let $A_{1}^{1}(t), A_{1}^{2}(t)$ and $A_{1}^{3}(t)$ be the probabilities that the system is up at epoch $t$ due to first component, due to second component and due to both the components in parallel respectively when initially system starts from $\mathrm{S}_{\mathrm{i}} \square \square \mathrm{E}$.

Using simple probabilistic laws it can be seen that $\mathrm{A}_{0}(\mathrm{t})$ is the sum of the following probabilities of mutually exclusive contingencies.
(i) The system does not transits to state $\mathrm{S}_{0}$ till time t . The probability of this event is

$$
\mathrm{e}^{-(\alpha+\beta) \mathrm{t}}=\mathrm{Z}_{0}(\mathrm{t}) .
$$

(ii) The system transits to $\mathrm{S}_{1}$ from $\mathrm{S}_{0}$ in $(\mathrm{u}, \mathrm{u}+\mathrm{du}) ; \mathrm{u} \leq \mathrm{t}$ and then starting from $\mathrm{S}_{1}$, it is observed to be up at epoch $t$, with probability $A_{1}(t-u)$. Therefore

$$
\int_{0}^{t} \mathrm{q}_{01}(\mathrm{u}) \mathrm{duA}_{1}(\mathrm{t}-\mathrm{u}) .
$$

(iii) The system transits to $\mathrm{S}_{2}$ from $\mathrm{S}_{0}$ in $(\mathrm{u}, \mathrm{u}+\mathrm{du}) ; \mathrm{u} \leq \mathrm{t}$ and then starting from $\mathrm{S}_{2}$, it is observed to be up at epoch $t$, with probability $\mathrm{A}_{2}(\mathrm{t}-\mathrm{u})$. Therefore,

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{\mathrm{o} 2}(\mathrm{u}) \mathrm{duA}_{2}(\mathrm{t}-\mathrm{u}) .
$$

(iv) The system transits to $\mathrm{S}_{3}$ from $\mathrm{S}_{0}$ in $(\mathrm{u}, \mathrm{u}+\mathrm{du}) ; \mathrm{u} \leq \mathrm{t}$ and then starting from $\mathrm{S}_{3}$, it is observed to be up at epoch $t$, with probability $A_{3}(t-u)$. Therefore

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{03}(\mathrm{u}) \mathrm{duA}_{3}(\mathrm{t}-\mathrm{u}) .
$$

(v) The system transits to $\mathrm{S}_{4}$ from $\mathrm{S}_{0}$ in $(\mathrm{u}, \mathrm{u}+\mathrm{du}) ; \mathrm{u} \leq \mathrm{t}$ and then starting from $\mathrm{S}_{4}$, it is observed to be up at epoch $t$, with probability $A_{4}(t-u)$. Therefore

$$
\int_{0}^{\mathrm{t}} \mathrm{q}_{04}(\mathrm{u}) \mathrm{duA}_{4}(\mathrm{t}-\mathrm{v})
$$

Taking Laplace Transform (L.T.) of the above relation and writing the resulting set of equation in the matrix form, we get

$$
\left[\begin{array}{l}
A_{0}^{1} \\
A_{1}^{1} \\
A_{2}^{1} \\
A_{3}^{1} \\
A_{4}^{1} \\
A_{5}^{1} \\
A_{6}^{1}
\end{array}\right]=\left[\begin{array}{ccccccc}
q_{0} & q_{01} & q_{02} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -q_{13}^{*} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -q_{24}^{*} & -q_{25}^{*} & 0 \\
-q_{30}^{*} & 0 & 0 & 0 & -q_{34}^{6^{*}} & 0 & 0 \\
-q_{40}^{*} & 0 & 0 & -q_{43}^{(5)} & 1 & 0 & 0 \\
0 & 0 & 0 & -q_{53}^{*} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -q_{64}^{*} & 0 & 0
\end{array}\right]\left[\begin{array}{c}
Z_{0}^{*} \\
0 \\
Z_{2}^{*} \\
0 \\
Z_{4}^{*} \\
0 \\
0
\end{array}\right] .
$$

For brevity, the argument's' has been omitted from $\mathrm{q}_{\mathrm{ij}}^{*}(\mathrm{~s}), \mathrm{A}_{\mathrm{i}}^{1^{*}}(\mathrm{~s})$ and $Z_{1}^{*}(\mathrm{~s})$. Solving the above matrix for $A_{0}^{\prime *}(\mathrm{~S})$, we get,
$A_{0}^{\prime *}(s)=N_{2}(s) / D_{2}(s)$,
Where $\mathrm{N}_{2}(\mathrm{~s})=\left(\mathrm{Z}_{0}^{*}+\mathrm{q}_{02}^{*} Z_{2}^{*}\right)\left(1-\mathrm{q}_{34}^{*} \mathrm{Z}_{43}^{*}\right)+\left[\mathrm{q}_{01}\left(\mathrm{q}_{13} \mathrm{q}_{34}+\mathrm{q}_{16}\right)\right.$
And,
$D_{2}(s)=\left(1-q_{34}^{*} q_{43}^{*}\right)\left(1-q_{01}^{*} q_{10}^{*}\right)-q_{01}^{*}\left(q_{13}^{*}+q_{24}^{*} q_{43}^{*}\right) q_{30}^{*}-q_{03}^{*} q_{30}^{*}$
$-q_{04}^{*} q_{53}^{*} q_{64}^{*} q_{30}^{*}-q_{01}^{*}\left(q_{13}^{*} q_{34}^{*}+q_{13}^{*}\right) q_{40}^{*}-q_{04}^{*} q_{64}^{*} q_{53}^{*}-q_{34}^{*}\left(q_{03}^{*}+q_{04}^{*} q_{53}^{*} q_{64}^{*}\right) q_{40}$.
Now to obtain the steady-state probabilities that the system will be operative due to first component, we proceed as follows -

$$
Z_{i}^{*}(0)=\int Z_{i}(\mathrm{t}) \mathrm{dt}=\mu_{\mathrm{i}}(\mathrm{i}=0,2,4)
$$

and using the result $q_{i j}^{*}(0)=p_{i j}$, we have

$$
\begin{gathered}
\mathrm{D}_{2}(0)=\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right)\left(1-\mathrm{p}_{10} \mathrm{p}_{01}\right)-\mathrm{p}_{01}\left(\mathrm{p}_{13}+\mathrm{p}_{24} \mathrm{p}_{43}\right) \mathrm{p}_{30}-\mathrm{p}_{03} \mathrm{p}_{30} \\
-\mathrm{p}_{04} \mathrm{p}_{53} \mathrm{p}_{64} \mathrm{p}_{30}-\mathrm{p}_{01}\left(\mathrm{p}_{13} \mathrm{p}_{34}+\mathrm{p}_{13}\right) \mathrm{p}_{40}-\mathrm{p}_{04} \mathrm{p}_{64} \mathrm{p}_{53} \\
-\mathrm{p}_{34}\left(\mathrm{p}_{03}+\mathrm{p}_{04} \mathrm{p}_{53} \mathrm{p}_{64}\right) \mathrm{p}_{40} \\
=1-\mathrm{p}_{34} \mathrm{p}_{43}-\mathrm{p}_{01} \mathrm{p}_{10}+\mathrm{p}_{01} \mathrm{p}_{10} \mathrm{p}_{34} \mathrm{p}_{43}-\mathrm{p}_{01} \mathrm{p}_{13} \mathrm{p}_{30}-\mathrm{p}_{01} \mathrm{p}_{24} \mathrm{p}_{43} \mathrm{p}_{30} \\
-\mathrm{p}_{03} \mathrm{p}_{30}-\mathrm{p}_{044} \mathrm{p}_{53} \mathrm{p}_{64} \mathrm{p}_{30}-\mathrm{p}_{01} \mathrm{p}_{13} \mathrm{p}_{34} \mathrm{p}_{40}-\mathrm{p}_{01} \mathrm{p}_{13} \mathrm{p}_{40}
\end{gathered}
$$

$$
-\mathrm{p}_{04} \mathrm{p}_{64} \mathrm{p}_{53} \mathrm{p}_{34} \mathrm{p}_{03} \mathrm{p}_{40}-\mathrm{p}_{34} \mathrm{p}_{40} \mathrm{p}_{04} \mathrm{p}_{53} \mathrm{p}_{64}
$$

$$
\begin{gathered}
=1-\mathrm{p}_{01}-\mathrm{p}_{03}-\mathrm{p}_{04}-\mathrm{p}_{34} \mathrm{p}_{43}\left(1-\mathrm{p}_{01}-\mathrm{p}_{03}-\mathrm{p}_{04}\right)-\mathrm{p}_{02}\left(\mathrm{p}_{10}+\mathrm{p}_{13}\right) \\
\quad+\mathrm{p}_{02} \mathrm{p}_{34} \mathrm{p}_{43}\left(\mathrm{p}_{10}+\mathrm{p}_{13}\right) \\
=\mathrm{p}_{02}-\mathrm{p}_{34} \mathrm{p}_{43} \mathrm{p}_{02}-\mathrm{p}_{02}+\mathrm{p}_{02} \mathrm{p}_{34} \mathrm{p}_{43} \\
=0
\end{gathered}
$$

Therefore, the steady-state probability that the system will be operative due to first component is given by,

$$
\begin{aligned}
A_{0}^{\prime}= & \lim _{t \rightarrow \infty} A_{0}^{\prime}(t) \\
& =\lim _{s \rightarrow 0} s A_{0}^{*}(t)=\lim _{s \rightarrow 0} s \frac{N_{2}(s)}{D_{2}(s)} \\
& =\frac{N_{2}(0)}{D_{2}^{\prime}(0)}, \text { as } D_{2}(0)=0 .
\end{aligned}
$$

Similarly, the steady state probabilities that the system will be operative due to second component,

$$
\begin{aligned}
\mathrm{A}_{0}^{2}= & \lim _{\mathrm{t} \rightarrow \infty} \mathrm{~A}_{0}^{2}(\mathrm{t}) \\
& =\lim _{\mathrm{s} \rightarrow 0} s \mathrm{~A}_{0}^{2^{*}}(\mathrm{~s})=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~s} \frac{\mathrm{~N}_{3}(\mathrm{~s})}{\mathrm{D}_{2}^{\prime}(\mathrm{s})} \\
& =\frac{\mathrm{N}_{3}(0)}{\mathrm{D}_{2}^{\prime}(\mathrm{O})}, \text { as } \mathrm{D}_{2}(0)=0 \\
\mathrm{~A}_{0}^{3}= & \lim _{\mathrm{t} \rightarrow \infty} \mathrm{~A}_{0}^{3}(\mathrm{t}) \\
& =\lim _{\mathrm{s} \rightarrow 0} \mathrm{SA}_{0}^{3^{*}}(\mathrm{t})=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~S} \frac{\mathrm{~N}_{4}(\mathrm{~s})}{\mathrm{D}_{2}^{\prime}(\mathrm{s})} \\
& =\frac{\mathrm{N}_{4}(0)}{\mathrm{D}_{2}^{\prime}(0)}, \text { as } \mathrm{D}_{2}(0)=0,
\end{aligned}
$$

Where,

$$
\begin{gathered}
\mathrm{N}_{2}(0)=\left(\mu_{0}+\mathrm{p}_{02} \mu_{2}\right)\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right)+\left[\mathrm{p}_{01}\left(\mathrm{p}_{13} \mathrm{p}_{34}+\mathrm{p}_{16}\right)\right. \\
\left.+\mathrm{p}_{02}\left(\mathrm{p}_{24}+\mathrm{p}_{25} \mathrm{p}_{64} \mathrm{p}_{53}\right)+\mathrm{p}_{34}\left(\mathrm{p}_{03}+\mathrm{p}_{04}+\mathrm{p}_{43} \mathrm{p}_{53} \mathrm{p}_{64}\right)\right] \mu_{4} \\
\mathrm{~N}_{3}(\mathrm{O})=\mu_{0} \mathrm{p}_{01}\left(\mathrm{p}_{13}+\mathrm{p}_{43} \mathrm{p}_{64}\right)+\mathrm{p}_{02}\left(\mathrm{p}_{24} \mathrm{p}_{43}+\mathrm{p}_{25} \mathrm{p}_{34}\right) \\
\quad+\mathrm{p}_{03} \mathrm{p}_{30} \mu_{3}+\mu_{1} \mathrm{p}_{13}\left(\mathrm{p}_{04} \mathrm{p}_{40}+\mathrm{p}_{03} \mathrm{p}_{30}\right)
\end{gathered}
$$

And,

$$
\mathrm{N}_{4}(0)=\mathrm{p}_{01}\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right) \mu_{1}+\mathrm{p}_{02}\left(1-\mathrm{p}_{24} \mathrm{p}_{40}\right) \mu_{2}
$$

To obtain $D_{2}^{\prime}(0)$, we collect the coefficient of $-q_{i j}^{*}(0)\left(=m_{i j}\right)$ in $D_{2}^{\prime}(0)$ for various values of $i$ and $j$ as follows -

Coefficient of $\mathrm{m}_{01}=\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right) \mathrm{p}_{10}+\left(\mathrm{p}_{13}+\mathrm{p}_{43}\right) \mathrm{p}_{30}+\left(\mathrm{p}_{13} \mathrm{p}_{34}+\mathrm{p}_{24}\right) \mathrm{p}_{40}$

$$
\begin{gathered}
=\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right) \mathrm{p}_{10}+\left(\mathrm{p}_{13}+\mathrm{p}_{16}\right)\left(\mathrm{p}_{30}+\mathrm{p}_{40}-\mathrm{p}_{30} \mathrm{p}_{40}\right) \\
=\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right) \mathrm{p}_{10}+\left(\mathrm{p}_{13}+\mathrm{p}_{16}\right)\left(1-\mathrm{p}_{34} \mathrm{p}_{43}\right) \\
=1-\mathrm{p}_{34} \mathrm{p}_{43}
\end{gathered}
$$

Coefficient of $\mathrm{m}_{02}=\left(\mathrm{p}_{24} \mathrm{p}_{43}+\mathrm{p}_{25} \mathrm{p}_{64}\right) \mathrm{p}_{30}+\left(\mathrm{p}_{24}+\mathrm{p}_{25} \mathrm{p}_{64} \mathrm{p}_{34}\right) \mathrm{p}_{40}$

$$
=1-\mathrm{p}_{34} \mathrm{p}_{43}
$$

Coefficient of $\mathbf{m}_{03}=\mathbf{p}_{30}+\mathbf{P}_{34} \mathbf{P}_{40}$

$$
=1-\mathrm{p}_{34} \mathrm{p}_{43}
$$

## Conclusion

This paper describes an improvement over the Said and Sherbeny (2010).They analyzed a two-unit cold standby system with two stage repair and waiting time. In this paper we analyzed a two dis-similar component system. The system operates even if a single component operates. A single repair facility is available with some fixed probability for the repair of failed components. . Several measures of system effectiveness such as MTSF, $A, B$ etc. are obtained by using regenerative point technique which shows that the proposed model is better than Said and Sherbeny (2010).

## Acknowledgement

It is with the grace of almighty God that I have been able to complete this paper and again pray him that my efforts may not go in vain. I am also highly thankful to Dr. H. P. Singh and Dr. Rajesh Tailor for stimulating and co-operating me throughout the work.

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