

Placement of Staff in LIC using Fuzzy Assignment Problem

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Abstract: The decision of placement of a right person for a right job is difficult because of uncertainty and imprecise information. However fuzzy assignment problem can certainly solve this purpose. In this paper, fuzzy assignment problem is solved for the placement of four candidates for four different designations in Life Insurance Corporation (LIC). The problem is converted into crisp assignment problem by magnitude ranking method and then it is solved by Hungarian, MOA and Direct method. Results are compared for effective application of placement of right candidate for right job.

Keywords: Fuzzy Assignment Problem, Magnitude Ranking Method, Hungarian Method, MOA Method, Direct Method

I. INTRODUCTION

An assignment problem which is special type of linear programming is a well-studied optimization problem in management science. The main objective of this is to assign given number of persons to equal number of jobs on one to one basis in such way so as to minimize total cost of performing that task or to maximize the total profit of allocation. The assignment problem arises because of the varying capacity of person or machine to perform the given task or job [1]. The special structure of assignment problem allows us to use more convenient method of solution in comparison to simplex method [13].

Application of classical AP in solving real life problem has some limitations. Fuzzy assignment problem can certainly minimize these limitations. In this paper, fuzzy assignment problem for the placement of four different candidates for four different designations in Life Insurance Corporation sector is solved. Firstly the problem is converted into crisp one, using magnitude ranking technique [12] and then it is solved by three methods namely, Hungarian method, MOA method and a new methodology. Comparison of these three methods is also done in this paper.

Fuzzy Set: The fuzzy set is represented by a characteristic function, defined as follows:

$$\mu_A : X \rightarrow [0,1]$$
$$\mu_A(X) = \begin{cases} 1, & \text{if } X \text{ is totally in } A \\ 0, & \text{if } X \text{ is not in } A \\ (0,1), & \text{if } X \text{ is partially in } A \end{cases}$$

Mathematical Formulation of Assignment problem: Mathematically, the assignment problem is stated as,

Minimize total cost:

$$z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}, \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, n$$

Subject to condition:

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{th person is assigned } j\text{th job} \\ 0 & \text{if not} \end{cases}$$

$$\sum_{j=1}^n x_{ij} = 1, \quad (\text{one job is done by the } i\text{th person } i = 1, 2, \dots, n)$$

and

$$\sum_{i=1}^n x_{ij} = 1, \text{ (Only one person should be}$$

assigned the jth job $j = 1, 2, \dots, n$)

Where x_{ij} denotes that jth job is to be assigned to the ith person.

Here c_{ij} represents the cost of assignment of person i to the job j .

Magnitude Ranking Method: Triangular fuzzy number is defuzzified by method of magnitude of ranking because of its simplicity and accuracy [12]. For an arbitrary triangular fuzzy number $(a_1, a_2, a_3) = (a_0, a_*, a^*)$ magnitude of triangular fuzzy number is given by equation (1)

$$Mag(a_1, a_2, a_3) = \frac{1}{2} \left(\int_0^1 (a^* + 3a_0 - a_*) f(r) dr \right) \dots \dots (1)$$

Where $f(r)$ is taken as r for convenience which is non-negative and increasing function on $[0, 1]$.

II. LITERATURE REVIEW

In the year 1955, most widely used Hungarian method for solving assignment problem was developed by [5]. Fuzzy set concept was introduced by Zadeh in 1965 [16] in order to handle vagueness or imprecision or ambiguity. In 1985, the fuzzy assignment problem is proposed by Chen [3]. Shaikh Tajuddin Nizami et al. in 2011 applied a new method for finding the cost of assignment problem using genetic algorithm of artificial intelligence which is very easy and shorter method as compared to Hungarian method [8]. Another method of creating ones in assignment matrix called as MOA method is given in comparative analysis of assignment problem by [14] in year 2012. In the same year, a new method for solving fuzzy assignment problems is proposed by [9]. Again in the same year, method for solving Hungarian assignment problems using triangular and trapezoidal fuzzy number was used by Kadhivel K et al. which used Robust Ranking method for defuzzification of fuzzy assignment problem [4]. In cost minimization assignment problem using fuzzy quantifier by G. Nirmala and R. Anju, fuzzy quantifier, ranking method and ASM method were applied for finding an optimal solution to the assignment problem in 2014 [7]. In the year 2015, on solving fuzzy solid assignment problems by D. Anuradha presented transformation of fuzzy solid assignment problem by Robust ranking method and its solution by plane point method [2].

A new approach of solving single objective unbalanced assignment problem in 2016 is discussed by Ventepaka Yadaiah and V. V. Haragopal where a Lexi search algorithm is used to assign all the jobs to machines optimally [15]. In the same year, S. Vimala and S. Krishna Prabha presented assignment problems with fuzzy costs using ones assignment method where fuzzy assignment problem is solved by ones assignment method after its transformation by ranking method [10]. In the year 2017, method for solving fuzzy assignment problem using magnitude ranking technique is provided which converts fuzzy assignment problem into crisp one and solved it by Hungarian method [12]. In the same year, Dr. S. Muruganandam and K. Hema used Fourier elimination method for solving fuzzy assignment problem [6].

III. METHODOLOGY

In this section, conversion of fuzzy assignment problem into crisp one using magnitude ranking technique is given which is then solved by Hungarian method, MOA method and direct method.

A. Fuzzy assignment problem

In this paper, four candidates E1, E2, E3, E4 and four jobs namely Direct Sales Executive (DSE), Clerk (C), Development Officer (DO) and Assistant Administrative Officer (AAO) in Life Insurance Corporation are taken for placement by fuzzy assignment problem. As every person has varying capability and skills, the task here is to place a right person for a right job in order to improve the performance of the organization based on the marks scored by them. So here we have maximal assignment problem where each entry in the matrix is score gained by the employee for different jobs and our objective is to find out an optimal assignment for placement of right candidate at right place.

Fuzzy assignment problem for above placement is as shown in equation (2).

$$\begin{bmatrix} (10,13,16) & (7,10,13) & (8,11,14) & (7,10,13) \\ (9,12,15) & (10,13,16) & (8,11,14) & (9,12,15) \\ (5,8,11) & (7,10,13) & (7,10,13) & (5,8,11) \\ (8,11,14) & (6,9,12) & (8,11,14) & (5,8,11) \end{bmatrix} \dots\dots(2) \quad \text{The problem can be stated as Linear}$$

Programming Problem as follows:

Max

$$\begin{aligned} &Mag(10,13,16)x_{11} + Mag(7,10,13)x_{12} \\ &+ Mag(8,11,14)x_{13} + Mag(7,10,13)x_{14} \\ &+ Mag(9,12,15)x_{21} + Mag(10,13,16)x_{22} \\ &+ Mag(8,11,14)x_{23} + Mag(9,12,15)x_{24} \\ &+ Mag(5,8,11)x_{31} + Mag(7,10,13)x_{32} \\ &+ Mag(7,10,13)x_{33} + Mag(5,8,11)x_{34} \\ &+ Mag(8,11,14)x_{41} + Mag(6,9,12)x_{42} \\ &+ Mag(8,11,14)x_{43} + Mag(5,8,11)x_{44} \end{aligned}$$

Subject to

$$\sum_{i,j=1}^4 x_{ij} = 1$$

where $x_{ij} \in [0,1]$

B. Magnitude Ranking Method

We use magnitude ranking method proposed by [12] for converting above fuzzy assignment problem into crisp assignment problem. We calculate first magnitude of (10, 13, 16) from (1), we get,

$$Mag(10,13,16) = \frac{1}{2} \int_0^1 (16 + 3(10) - 13)r \, dr = 8.25$$

$$Mag(7,10,13) = \frac{1}{2} \int_0^1 (13 + 3(7) - 10)r \, dr = 6$$

Similarly,

$$\begin{aligned} &Mag(8,11,14) = 6.75, \quad Mag(7,10,13) = 6, \\ &Mag(9,12,15) = 7.5, \quad Mag(10,13,16) = 8.25, \\ &Mag(8,11,14) = 6.75, \quad Mag(9,12,15) = 7.5, \\ &Mag(5,8,11) = 4.5, \quad Mag(7,10,13) = 6, \\ &Mag(7,10,13) = 6, \quad Mag(5,8,11) = 4.5, \\ &Mag(8,11,14) = 6.75, \quad Mag(6,9,12) = 5.25, \\ &Mag(8,11,14) = 6.75, \quad Mag(5,8,11) = 4.5 \end{aligned}$$

The crisp assignment problem of above fuzzy assignment problem (2) by using (1) is as follows:

$$\begin{bmatrix} 8.25 & 6 & 6.75 & 6 \\ 7.5 & 8.25 & 6.75 & 7.5 \\ 4.5 & 6 & 6 & 4.5 \\ 6.75 & 5.25 & 6.75 & 4.5 \end{bmatrix} \dots\dots(3)$$

C. Solution of FAP by Hungarian Method

Step 1: As the given problem is maximal assignment problem, we first convert it into minimization problem by subtracting all the elements of the matrix from the highest element of the matrix (or simply by placing minus sign before each element of the matrix).

$$\begin{bmatrix} 0 & 2.25 & 1.50 & 2.25 \\ 0.75 & 0 & 1.50 & 0.75 \\ 3.75 & 2.25 & 2.25 & 3.75 \\ 1.50 & 3 & 1.50 & 3.75 \end{bmatrix} \dots\dots\dots(4)$$

Step 2: Subtract minimum element of each row from all the elements of that row.

$$\begin{bmatrix} 0 & 2.25 & 1.50 & 2.25 \\ 0.75 & 0 & 1.50 & 0.75 \\ 1.5 & 0 & 0 & 1.5 \\ 0 & 1.5 & 0 & 2.25 \end{bmatrix}$$

Step 3: Next subtract the smallest element of each column from every element of that column.

$$\begin{bmatrix} 0 & 2.25 & 1.50 & 1.50 \\ 0.75 & 0 & 1.50 & 0 \\ 1.5 & 0 & 0 & 0.75 \\ 0 & 1.5 & 0 & 1.5 \end{bmatrix}$$

Step 4: Now we check whether zero assignment is possible or not. It is possible in above table. So starting with row 1 examine one by one row containing exactly one zero and mark that cell of zero. Cross all other zeros in the column in which the assignment has been made. When all rows are examined, follow the identical procedure for the columns. When the assignment is made in column, cross all zeros in the row in which the assignment is made. Continue this until all zeros are either assigned or crossed-out. We get,

$$\begin{bmatrix} [0] & 2.25 & 1.50 & 1.50 \\ 0.75 & 0 & 1.50 & [0] \\ 1.5 & [0] & 0 & 0.75 \\ 0 & 1.5 & [0] & 1.5 \end{bmatrix}$$

So fuzzy optimal assignment by Hungarian Method is E1-DSE, E2-AAO, E3-C, E4-DO.

$$\begin{aligned} \text{Fuzzy optimal value} &= (10,13,16)+(9,12,15) \\ &+ (7,10,13)+(8,11,14)=(34,46,58) \end{aligned}$$

i.e. $Mag(34,46,58) = 28.5$

D. Solution of FAP by MOA Method

This section presents MOA method i.e. Matrix one’s assignment method given by [14] whose algorithm is stated as follows:

Step1: In a minimization (maximization) problem, find the minimum (maximum) element of each row (say a_i) and write it on the right hand side of the matrix. Then divide each element of the i th row of the matrix by a_i which will result in creation of at least one ones in each rows. In terms of ones for each row and column do assignment, otherwise go to step 2.

Step 2: Find the minimum (maximum) element of each column in the assignment matrix (say b_j) and write it below j th column of the matrix. Then divide each element of the j th column of the matrix by b_j which will result in creation of at least one ones in each column. In terms of ones make the assignment. If no feasible assignment can be achieved from step 1 and step 2 then go to step 3.

Note: In a maximization case, at the end of the step 2 we have fuzzy matrix whose all elements belong to $[0,1]$ and the greatest element is one.

Step 3: Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines is exactly equal to n , then the complete assignment is obtained else the complete assignment is not possible and then go to step 4.

Step 4: Select the smallest (largest) element (say d_{ij}) which do not lie on any of the lines in the above matrix and divide each element of the uncovered rows or columns by d_{ij} . This will result in creating some new ones to this row or column. If still a complete optimal assignment is not achieved in the new matrix, then use step 4 and step 3 iteratively. By repeating the same procedure the optimal assignment will be obtained. Priority plays an important role in this method, when we want to assign the ones.

Priority rule: For maximization (minimization) assignment problem assign the ones on the rows which have greatest (smallest) element on the right hand side, respectively.

We solve assignment problem (3) converted after ranking method by this MOA method as follows:

Step 1: Find the maximum element of each row and write it on the right hand side of the matrix.

$$\begin{bmatrix} 8.25 & 6 & 6.75 & 6 \\ 7.5 & 8.25 & 6.75 & 7.5 \\ 4.5 & 6 & 6 & 4.5 \\ 6.75 & 5.25 & 6.75 & 4.5 \end{bmatrix} \begin{matrix} 8.25 \\ 8.25 \\ 6 \\ 6.75 \end{matrix}$$

Now, divide each element of the i th row of the matrix by maximum number a_i , we get,

$$\begin{bmatrix} 1 & 0.73 & 0.82 & 0.73 \\ 0.91 & 1 & 0.82 & 0.91 \\ 0.75 & 1 & 1 & 0.75 \\ 1 & 0.78 & 1 & 0.67 \end{bmatrix} \begin{matrix} 8.25 \\ 8.25 \\ 6 \\ 6.75 \end{matrix}$$

Step 2: Find the maximum element of each column and write it below j th column of the matrix.

$$\begin{bmatrix} 1 & 0.73 & 0.82 & 0.73 \\ 0.91 & 1 & 0.82 & 0.91 \\ 0.75 & 1 & 1 & 0.75 \\ 1 & 0.78 & 1 & 0.67 \\ 1 & 1 & 1 & 0.91 \end{bmatrix} \begin{matrix} 8.25 \\ 8.25 \\ 6 \\ 6.75 \end{matrix}$$

Then divide each element of the j th column of the matrix by b_j , we get,

$$\begin{bmatrix} 1 & 0.73 & 0.82 & 0.80 \\ 0.91 & 1 & 0.82 & 1 \\ 0.75 & 1 & 1 & 0.82 \\ 1 & 0.78 & 1 & 0.74 \\ 1 & 1 & 1 & 0.91 \end{bmatrix} \begin{matrix} 8.25 \\ 8.25 \\ 6 \\ 6.75 \end{matrix}$$

Here the complete assignment is possible.

$$\begin{bmatrix} [1] & 0.73 & 0.82 & 0.80 \\ 0.91 & 1 & 0.82 & [1] \\ 0.75 & [1] & 1 & 0.82 \\ 1 & 0.78 & [1] & 0.74 \\ 1 & 1 & 1 & 0.91 \end{bmatrix} \begin{matrix} 8.25 \\ 8.25 \\ 6 \\ 6.75 \end{matrix}$$

So fuzzy optimal assignment by MOA Method is E1-DSE, E2-AAO, E3-C, and E4-DO.

Fuzzy optimal value = (10,13,16)+(9,12,15)

$$+(7,10,13)+(8,11,14)=(34,46,58)$$

i.e. $Mag(34,46,58) = 28.5$

E. Solution of FAP by Direct method

Direct method of solving maximal assignment problem [11] whose algorithm is given as:

Step 1: Construct a balanced assignment problem where persons are taken along the rows and jobs along the columns. If the problem is unbalanced then turn it to balanced one by adding dummy row or column.

Step 2: Now create a new matrix by subtracting each row from maximum element of that row.

Step 3: Locate the zero position of (i,j)th entry for the column of the matrix. Make allocation where zero has unique position and delete the corresponding row and column. Continue the process till all the persons are assigned.

Step 4: If some rows have the same column then find the value of next successor of zero and make the allocation to the row where there is maximum value of successor. If tie is found for the maximum value then find value of next to next successor of zero and make the allocation to maximum value.

Step 5: In reduced matrix after allocation, each row must have at least one zero, if not then subtract the minimum element of each row from every element of that row.

Step 6: Repeat all the steps from step 3 to step 5 until all the rows are assigned and calculate the optimal solution.

We solve maximal assignment problem (3) by this method.

Step 1: Subtract each row from maximum element of that row.

	DSE(J1)	C(J2)	DO(J3)	AAO(J4)
E1	0	2.25	1.5	2.25
E2	0.75	0	1.5	0.75
E3	1.5	0	0	1.5
E4	0	1.5	0	2.25

Now locate the position of zeros.

Rows	Columns	Successor of zeros
E1	J1	1.5
E2	J2	0.75
E3	J2,J3	0
E4	J1,J3	0

Assign E1 to J1 and delete the corresponding row and column.

Step 2:

	C(J2)	DO(J3)	AAO(J4)
E2	0	1.5	0.75
E3	0	0	1.5
E4	1.5	0	2.25

Locate the position of zeros.

Rows	Columns	Successor of zeros
E2	J2	0.75
E3	J2,J3	0
E4	J3	1.5

Assign E4 to J3 and delete the corresponding row and column.

	C(J2)	AAO(J4)
E2	0	0.75
E3	0	1.5

Locate the position of zeros.

Rows	Columns	Successor of zeros
E2	J2	0.75
E3	J2	1.5

Assign E3 to J2 and E2 to J4.

So fuzzy optimal assignment by this direct method is E1-DSE, E2-AAO, E3-C, and E4-DO.

$$\text{Fuzzy optimal value} = (10,13,16)+(9,12,15) \\ + (7,10,13)+(8,11,14)=(34,46,58)$$

$$\text{i.e. } \text{Mag}(34,46,58) = 28.5$$

IV. RESULTS AND DISCUSSION

Fuzzy assignment problem of the assignment matrix for the placement of four candidates for four different posts is converted to crisp assignment problem by using magnitude ranking method. Further it is solved by Hungarian method, Matrix Ones assignment method and direct method. By Hungarian method the given maximal assignment problem is solved in 3 iterations where first candidate E1 is placed to post of Direct sales executive, second candidate is placed to post of Assistant Administrative Officer, E3 to post of Clerk and E4 to post of Development Officer with fuzzy optimal solution of 28.5.

Similarly by matrix one's assignment method and direct method, it is found that placements of the candidates are same as those in Hungarian method with same optimal solution. It seems that fuzzy assignment problem can be used efficiently for placement policy in Life Insurance Corporation to place a suitable person at suitable place.

V. CONCLUSION

In this paper, maximum fuzzy assignment problem for the placement of four candidates for four different posts is solved successfully. Magnitude ranking method is used to convert fuzzy assignment problem into crisp assignment problem and further three methods namely Hungarian method, Matrix Ones Assignment method and direct method are used to find out the optimal assignment and solution. It is observed that optimal solution by all three methods is same and also observed that each candidate is placed to right posts of LIC as per capacity of the candidate. Results are confirmed as per the opinion of the experts in this field. The solution of the fuzzy assignment problem is more relevant as it considers vague information and gives effective solution to place a suitable person at suitable place.

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