Normalization of Fuzzy BG – Ideals in BG – Algebra

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ABSTRACT

In this paper, we define a normal fuzzy BG – Sub algebra, Normal fuzzy BG – ideal and discussed some of their properties.
Keywords
Normal fuzzy BG – sub algebra, Normal fuzzy BG-ideal.

1. INTRODUCTION

The concept of fuzzy set was initiated by Zadeh L A [9]. Imai Y and Iseki K [3] introduced two classes of abstract algebras BCK - algebras and BCI – algebras. Neggers J and Kim H S [7] introduced a new notion called B - algebra. Kim C B and Kim H S [5] introduced a new notion called BG - algebra which is a generalization of B - algebra. Muthuraj R, Sridharan M and Sitharselvam P M [6] introduced the notion of Fuzzy BG - ideals in BG - algebra. Priya T and Ramachandran T [8] discussed the Normalization of Fuzzy PS - ideals and Fuzzy PS - Sub algebras of PS - algebras. Ahn S S and Lee D [1] introduced a new concept Fuzzy sub-algebras of BG algebras. Akram M and Dar K H [2] discussed the notion of Fuzzy d- algebras. Jun Y B, Roh E H and Kim S H [4] introduced a new notion on BH–algebras. In this paper, we introduce the concept of normal fuzzy BG – sub algebras of BG – algebras and normal fuzzy BG – ideals of BG – algebras and establish some of its properties.

2. PRELIMINARIES

In this section, we give some basic definitions and preliminaries of BG – algebra and introduce normal fuzzy BG – ideals.

Definition 2.1: (Kim C B and Kim H S [5])

A non-empty set X with a constant 0 and a binary operation '*' is called a BG – Algebra, if it satisfies the following axioms

(i) $x * x = 0$
(ii) $x * 0 = x$
(iii) $(x * y) * (0 * y) = x, \ \forall\, x, y \in X$

Definition 2.2: (Kim C B and Kim H S [5])

Let S be non-empty subset of a BG-Algebra X, then S is called a BG –sub-algebra of X , if $x * y \in S \ \forall\, x, y \in S$

Definition 2.3: (Kim C B and Kim H S [5])

Let X be a BG – Algebra and I be a subset of X, then I is called a BG-Ideal of X , if is satisfies following conditions,

(i) $0 \in I$

(ii) $x * y \in I$ and $y \in I \Longrightarrow x \in I$
(iii) $x \in I$ and $y \in I \Longrightarrow x * y \in I,\ I \times I \subseteq I$

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Define a fuzzy set α in X by $\alpha(0) = 1$, $\alpha(1) = 0.6$, $\alpha(2) = 0.3$. Then α is a fuzzy BG – ideal of X.

Definition 2.4: (Muthuraj R et.al. [6])

A fuzzy set α in X is called fuzzy BG-Ideal of X, if it satisfies the following inequalities,

(i) $\alpha(0) \geq \alpha(x)$,
(ii) $\alpha(x) \geq \min\{\alpha(x * y), \alpha(y)\}$,
(iii) $\alpha(x * y) \geq \min\{\alpha(x), \alpha(y)\}\ \forall x, y \in X$.

Definition 2.5: (Muthuraj R et.al. [6])

A fuzzy BG-Ideal α of X is said to be normal if there exists $x \in X$ such that $\alpha(x) = 1$.

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Then $(X, *, 0)$ is a BG – algebra. Define a fuzzy set α in X by $\alpha(0) = 1$, $\alpha(1) = 0.7$, $\alpha(2) = 0.4$, $\alpha(3) = 0.2$. Then α is a normal fuzzy BG – ideal of X.

Lemma 2.6:

A fuzzy BG-Ideal α of X is normal, if and only if $\alpha(0) = 1$.

Theorem 2.7:

For any fuzzy BG-Ideal α of X, we can generate a normal fuzzy BG-Ideal of X which contains α.

Proof:

Let α be a fuzzy BG-ideal of X.

Define a fuzzy subset α^+ of X as

$$\alpha^+(x) = \alpha(x) + \alpha^c(0)\ \forall x \in X$$

Let $x, y \in X$

(i) $\alpha^+(0) = \alpha(0) + \alpha^c(0) \geq \alpha(x) + \alpha^c(0)$
$= \alpha^+(x)$.

(ii) $\alpha^+(x) = \alpha(x) + \alpha^c(0)$

$$\geq \min\{\alpha(x * y), \alpha(y)\} + \alpha^c(0)$$

$\geq \min\{\alpha(x * y) + \alpha^c(0), \alpha(y) + \alpha^c(0)\}$

$$= \min\{\alpha^+(x * y), \alpha^+(y)\}$$

$\Longrightarrow \alpha^+(x) \geq \min\{\alpha^+(x * y), \alpha^+(y)\}$.

(iii) $\alpha^+(x * y) = \alpha(x * y) + \alpha^c(0)$

$$\geq \min\{\alpha(x), \alpha(y)\} + \alpha^c(0)$$

$$= \min\{\alpha(x) + \alpha^c(0), \alpha(y) + \alpha^c(0)\}$$

$$= \min\{\alpha^+(x), \alpha^+(y)\}$$

$\Longrightarrow \alpha^+(x * y) \geq \min\{\alpha^+(x), \alpha^+(y)\}$.

Also

$$\alpha^+(0) = \alpha(0) + \alpha^c(0)$$

$$= \alpha(0) + 1 - \alpha(0) = 1$$

$\Longrightarrow \alpha^+(0) = 1$.

$\therefore \alpha^+$ is a normal fuzzy BG ideal of X.

Clearly,

$$\alpha \subset \alpha^+$$

Thus α^+ is a normal fuzzy BG – ideal of X which contains α.

Hence the proof.

Corollary 2.8:

Let α^+ be a fuzzy set in X defined by $\alpha^+ = \alpha(x) + \alpha^c(0)\ \forall x \epsilon X$. If there is an element $x \epsilon X$ such that $\alpha^+(x) = 0$ then $\alpha(x) = 0$.

Corollary 2.9:

(i) If α itself is normal then $(\alpha^+)^+ = \alpha$

(ii) If α is a fuzzy BG - ideal of X then $(\alpha^+)^+ = \alpha^+$

Theorem 2.10:

Let α be fuzzy BG-ideal of X. If α contains a normal BG-ideal of X generated by any other fuzzy BG-ideal of X then α is normal.

Proof:

Let γ be a fuzzy BG-ideal of X. By theorem 2.7, γ^+ is a normal fuzzy BG-ideal of X.

$\therefore \gamma^{+}(0)=1$ (by lemma 2.6)

Let α be a fuzzy BG-ideal of X such that $\gamma^{+} \subset \alpha$

$$\Longrightarrow \alpha(x) \geq \gamma^{+}(x) \forall x \in X$$

Put $x = 0$,

$$\Longrightarrow \alpha(0) \geq \gamma^{+}(0) = 1$$

$$\Longrightarrow \alpha(0) \geq 1$$

Hence α is normal.

Lemma 2.11:

Define a set $X_{\alpha} = \{x \in X: \alpha(x) = \alpha(0)\}$ and let α and γ be normal fuzzy BG-ideal of X. If $\alpha \subset \gamma$, then $X_{\alpha} \subset X_{\gamma}$.

Proof:

Let $x \in X_{\alpha}$

Then

$$\gamma(x) \geq \alpha(x) = \alpha(0) = 1 = \gamma(0)$$

$$\Longrightarrow x \in X_{\gamma}$$

$$\Longrightarrow X_{\alpha} \subset X_{\gamma}$$

Hence the proof.

Theorem 2.12:

Let α be a fuzzy BG-ideal of X. Let $f: [0, \alpha(0)] \rightarrow [0,1]$ be an increasing function. Define a fuzzy set $\alpha^{f}: X \rightarrow [0,1]$ by $\alpha^{f} = f\big(\alpha(x)\big)\ \forall x \in X$.

Then,

(i) α^{f} is a fuzzy BG-ideal of X.
(ii) If $f\big(\alpha(0)\big) = 1$, then α^{f} is normal.
(iii) If $f(t) \geq t\ \forall\ t \in [0, \alpha(0)]$, then $\alpha \subset \alpha^{f}$

Proof:

(i) $\alpha^{f}(0) = f\big(\alpha(0)\big)$

$$\geq f\big(\alpha(x)\big)$$

$$= \alpha^{f}(x)$$

$$\Longrightarrow \alpha^{f}(0) \geq \alpha^{f}(x)$$

Also

$$\alpha^{f}(x) = f\big(\alpha(x)\big)$$

$$\geq f\{\min\{\alpha(x * y), \alpha(y)\}\}$$

$$= \min\{f\{\alpha(x*y)\}, f\{\alpha(y)\}\}$$

$$= \min\{\alpha^f(x*y), \alpha^f(y)\}$$

$$\Longrightarrow \alpha^f(x) \geq \min\{\alpha^f(x*y), \alpha^f(y)\}$$

And

$$\alpha^f(x*y) = f\big(\alpha(x*y)\big)$$

$$\geq f\{\min\{\alpha(x), \alpha(y)\}$$

$$= \min\{f\big(\alpha(x), f\big(\alpha(y)\big)\}$$

$$= \min\{\alpha^f(x), \alpha^f(y)\}$$

$\Longrightarrow \alpha^f(x*y) \geq \min\{\alpha^f(x), \alpha^f(y)\}$

$\Longrightarrow \alpha^f$ is a fuzzy BG-ideal.

(ii) If $f\big(\alpha(0)\big) = 1 \Longrightarrow \alpha^f(0) = 1$

$\Longrightarrow \alpha^f$ is normal

(iii) Let $f(t) \geq t \forall t \in [0, \alpha(0)]$

Then

$$\alpha^f(x) = f\big(\alpha(x)\big)$$

$$\geq \alpha(x) \forall x \in X$$

Hence

$$\alpha \subseteq \alpha^f$$

Hence the proof.

3. NORMALIZATION OF FUZZY BG – SUB-ALGEBRAS

Definition 3.1:

Let α be a fuzzy set in BG - Algebra. Then α is called a fuzzy sub-algebra of X if $\alpha(x*y) \geq \min\{\alpha(x), \alpha(y)\}, \forall x, y \in X$.

Definition 3.2:

A fuzzy BG-sub-algebra α of X is said to be normal, if there exist $x \in X$ such that $\alpha(x) = 1$.

Lemma 3.3:

A fuzzy BG – sub-algebraα of X is normal, if and only if $\alpha(0) = 1$.

Theorem 3.4:

For any fuzzy sub-algebraα of X, we can generate a normal fuzzy subalgebra X which contains α.

Proof:

Let α be a fuzzy BG-sub-algebra of X.

Define a fuzzy subset α^+ of X as $\alpha^+(x) = \alpha(x) + \alpha^c(0) \forall x \in X$

Let $x, y \in X$

$$\alpha^{+}(x * y) = \alpha(x * y) + \alpha^{c}(0)$$

$$\geq \min\{\alpha(x), \alpha(y)\} + \alpha^{c}(0)$$

$$= \min\{\alpha(x) + \alpha^{c}(0), \alpha(y) + \alpha^{c}(0)\}$$

$$= \min\{\alpha^{+}(x), \alpha^{+}(y)\}$$

$$\Rightarrow \alpha^{c}(x * y) \geq \min\{\alpha^{+}(x), \alpha^{+}(y)\}$$

Also

$$\alpha^{+}(0) = \alpha(0) + \alpha^{c}(0)$$

$$= \alpha(0) + 1 - \alpha(0)$$

$$= 1$$

$\therefore \alpha^{+}$is normal fuzzy BG-algebra of X.

Clearly $\alpha \subset \alpha^{+}$

Thus $\alpha^{+}$is a normal fuzzy BG-algebra of X which contains α.

Corollary 3.5:

(i) If α itself is normal then $\alpha = \alpha^{+}$
(ii) If α is a fuzzy BG sub- algebra of X then $(\alpha^{+})^{+} = \alpha^{+}$.

Theorem 3.6:

Let α be fuzzy BG-subalgebra of X. If α contains a normal fuzzy BG-subalgebra of X generated by any other fuzzy BG-sub-algebra of X then α is normal.

Proof:

Let γ be a fuzzy BG-sub-algebra of X. By theorem 3.4, γ^{+} is a normal fuzzy BG-sub-algebra of X.
$\therefore \gamma^{+}(0)$=1 (by lemma 3.3)

Let α be a fuzzy BG-algebra of X such that $\gamma^{+} \subset \alpha$

$$\Rightarrow \alpha(x) \geq \gamma^{+}(x) \forall x \in X$$

Put $x = 0$,

$$\Rightarrow \alpha(0) \geq \gamma^{+}(0) = 1$$

$$\Rightarrow \alpha(0) \geq 1$$

Hence α is normal.

Theorem 3.7:

Let α and γ be normal fuzzy BG-sub-algebra of X. If $\alpha \subset \gamma$, then $X_{\alpha} \subset X_{\gamma}$.

Theorem 3.8:

Let α be a fuzzy BG-sub-algebra of X. Let $f: [0, \alpha(0)] \rightarrow [0,1]$ be an increasing function. Define a fuzzy set $\alpha^{f}: X \rightarrow [0,1]$ by $\alpha^{f} = f\big(\alpha(x)\big) \; \forall x \in X$.

Then

(i) α^f is a fuzzy BG-sub-algebra of X.
(ii) If $f(\alpha(0)) = 1$, then α^f is normal.
(iii) If $f(t) \geq t, \forall t \in [0, \alpha(0)]$, then $\alpha \subset \alpha^f$

Proof:

$$\alpha^f(x * y) = f(\alpha(x * y))$$

$$\geq f\{\min\{\alpha(x), \alpha(y)\}$$

$$= \min\{f(\alpha(x), f(\alpha(y))\}$$

$$= \min\{\alpha^f(x), \alpha^f(y)\}$$

$$\Longrightarrow \alpha^f(x * y) \geq \min\{\alpha^f(x), \alpha^f(y)\}$$

$\Longrightarrow \alpha^f$ is a fuzzy BG-ideal.

(iv) If $f(\alpha(0)) = 1 \Longrightarrow \alpha^f(0) = 1$

$\Longrightarrow \alpha^f$ is normal

(v) Let $f(t) \geq t \ \forall t \in [0, \alpha(0)]$

Then

$$\alpha^f(x) = f(\alpha(x))$$

$$\geq \alpha(x) \ \forall x \in X$$

Hence

$$\alpha \subseteq \alpha^f$$

Hence the proof.

4. CONCLUSION

Hence we have discussed the Normalization of Fuzzy BG – ideals and Fuzzy BG – sub algebra over BG – algebra. It adds an another dimension to the defined BG-algebras. This concept can further be generalized to n-fold fuzzy BG-ideals in n-fold fuzzy BG- algebra using fuzzy translation and fuzzy multiplication.

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