Normalization of Fuzzy B – Ideals in B – Algebra

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Abstract

In this paper, we define a normal fuzzy B – sub algebra, normal fuzzy B – ideal and discuss some of their properties.

Keywords

B–algebra, B–Ideal, Fuzzy B–Ideal, Normal fuzzy B – sub algebra, Normal fuzzy B-ideal.

1. INTRODUCTION

After the introduction of fuzzy subsets by Zadeh L A [7], several researchers explored on the generalization of the notion of fuzzy subset. Biswas R [1], introduced the Fuzzy Subgroups and Anti Fuzzy Subgroups. Cho J R and Kim H S [2] discussed relations between B-algebras and other topics, especially quasi-groups. Park H K and Kim H S [3] introduced the notion of Quadratic B-algebras. Sun Shin Ahn and Keumseong Bang [5] discussed the fuzzy sub-algebra in B-algebra. Yamini C and Kailasavalli S [6] introduced the notion of Fuzzy B-ideals. Priya T and Ramachandran T [4] discussed the Normalization of Fuzzy PS-ideals and Fuzzy PS-Sub algebras of PS-algebras. In this paper, we introduce the concept of normal fuzzy B – sub algebras of B – algebras and normal fuzzy B – ideals of B – algebras and establish some of its properties in detail.

2. PRELIMINARIES

In this section, we give some basic definitions and preliminaries of B – algebra and introduce the normal fuzzy B – ideals.

Definition 2.1: (Jung R Cho and Kim H S [2])

A B-algebra is a non-empty set X with a constant 0 and a binary operation '*' satisfying the following axioms:

(i) $x * x = 0$
(ii) $x * 0 = x$
(iii) $(x * y) * z = x * (z * (0 * y)), for\ all\ x, y, z \in X$

For brevity, we also call X a B-algebra. In X we can define a binary relation "≤" by $x \leq y$ if and only if $x * y = 0$.

Definition 2.2: (Jung R Cho and Kim H S [2])

A non-empty subset M of a B-algebra X is called a sub-algebra of X if $x * y \in M$ for any $x, y \in M$.

Definition 2.3: (Park H K and Kim H S [3])

A non-empty subset N of a B-algebra X is called a B-ideal of X if it satisfies for $x, y, z \in X$

(i) $0 \in N$
(ii) $(x * y) \in N$ and $(z * x) \in N$ implies $(y * z) \in N$

Definition 2.4: (Yamini C and Kailasavalli S [6])

Let $(X, *, 0)$ be a B-algebra, a fuzzy set β in X is called a fuzzy B-ideal of X if it satisfies the following axioms

(i) $\beta(0) \geq \beta(x)$
(ii) $\beta(y * z) \geq \beta(x * y) \wedge \beta(z * x), for\ all\ x, y, z \in X$

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

The fuzzy set β given by $\beta(0) = 0.8$, $\beta(1) = 0.5, \beta(2) = 0.2$ is a fuzzy B-ideal.

Definition 2.5: (Yamini C and Kailasavalli S [6])

A fuzzy BG-Ideal β of X is said to be normal if there exists $x \in X$ such that $\beta(x) = 1$.

Lemma 2.6:

A fuzzy B-Ideal β of X is normal if and only if $\beta(0) = 1$.

Theorem 2.7:

For any fuzzy B-Ideal β of X, we can generate a normal fuzzy B-Ideal of X which contains β.

Proof:

Let β be a fuzzy B-ideal of X.

Define a fuzzy subset β^+ of X as

$$\beta^+(x) = \beta(x) + \beta^c(0)\ \forall\ x \in X$$

Let $x, y, z \in X$

(i) $\beta^+(0) \quad = \beta(0) + \beta^c(0)$
$\geq \beta(x) + \beta^c(0)$
$= \beta^+(x)$

(ii) $\beta^+(y * z) \ = \beta(y * z) + \beta^c(0)$

$\geq \{\beta(x * y) \wedge \beta(z * x)\} + \beta^c(0)$

$= \{\beta(x * y) + \beta^c(0)\} \wedge \{\beta(z * x) + \beta^c(0)\}$

$= \beta^+(x * y) \wedge \beta^+(z * x)$

$\Longrightarrow \beta^+(x) \geq \beta^+(x * y) \wedge \beta^+(z * x)$

and

$$\beta^+(0) = \beta(0) + \beta^c(0)$$

$$= \beta(0) + 1 - \beta(0) = 1$$

$$\Longrightarrow \beta^+(0) = 1$$

$\therefore \beta^+$ is a normal fuzzy B ideal of X.

Clearly $\beta \subset \beta^+$

Thus β^+ is a normal fuzzy B – ideal of X which contains β.

Hence the proof.

Theorem 2.8:

Let β be fuzzy B-ideal of X. If β contains a normal B-ideal of X generated by any other fuzzy B-ideal of X then β is normal.

Proof:

Let θ be a fuzzy B-ideal of X. By theorem 2.7, θ^+ is a normal fuzzy B-ideal of X.

$$\therefore \theta^+(0) = 1$$

Let β be a fuzzy B-ideal of X such that $\theta^+ \subset \beta$

$$\Longrightarrow \beta(x) \geq \theta^+(x)\ \forall\ x \in X$$

Put $x = 0$,

$$\Longrightarrow \beta(0) \geq \theta^+(0) = 1$$

$$\Longrightarrow \beta(0) \geq 1$$

Hence β is normal.

Lemma 2.9:

Define a set $X_\theta = \{x \in X: \beta(x) = \beta(0)\}$ and let β and θ be normal fuzzy B-ideals of X. If $\beta \subset \theta$, then $X_\beta \subset X_\theta$.

Proof:

Let $x \in X_\beta$

Then

$$\theta(x) \geq \beta(x) = \beta(0) = 1 = \theta(0)$$

$$\Longrightarrow x \in X_\theta$$

$$\Longrightarrow X_\beta \subset X_\theta$$

Hence the proof.

Theorem 2.10:

Let β be a fuzzy B-ideal of X. Let $f: [0, \beta(0)] \to [0,1]$ be an increasing function. Define a fuzzy set $\beta^f: X \to [0,1]$ by $\beta^f = f\big(\beta(x)\big)\ \forall\ x \in X$.

Then,

(i) β^f is a fuzzy B-ideal of X.
(ii) If $f\big(\beta(0)\big) = 1$, then β^f is normal.
(iii) If $f(s) \geq s \; \forall s \in [0, \alpha(0)]$, then $\beta \subset \beta^f$

Proof:

(i) $\beta^f(0) \quad = f\big(\beta(0)\big)$

$\geq f\big(\beta(x)\big)$

$= \beta^f(x)$

$\Longrightarrow \beta^f(0) \geq \beta^f(x)$

Also

$$\beta^f(y * z) \quad = f\big(\beta(y * z)\big)$$

$$\geq f\big(\beta(x * y) \wedge \beta(z * x)\big)$$

$$= f\big(\beta(x * y)\big) \wedge f\big(\beta(z * x)\big)$$

$$= \beta^f(x * y) \wedge \beta^f(z * x)$$

$$\Longrightarrow \beta^f(x * y) \geq \beta^f(x * y) \wedge \beta^f(z * x)$$

$\Longrightarrow \beta^f$ is a fuzzy B-ideal.

(ii) If $f\big(\beta(0)\big) = 1$

$$\Longrightarrow \beta^f(0) = 1$$

$\Longrightarrow \beta^f$ is normal

(iii) Let $f(s) \geq s \; \forall \; s \in [0, \beta(0)]$

Then

$\beta^f(x) = f\big(\beta(x)\big)$

$\geq \beta(x), \; \forall \; x \in X$

Hence

$$\beta \subset \beta^f$$

Hence the proof.

3. NORMALIZATION OF FUZZY B - SUBALGEBRAS

Definition 3.1:

Let β be a fuzzy set in B - Algebra. Then β is called a fuzzy sub-algebra of X if $\beta(x * y) \geq \beta(x) \wedge \beta(y) \; \forall \; x, y \in X$.

Definition 3.2:

A fuzzy B-sub-algebraβ of X is said to be normal if there exist $x \in X$ such that $\beta(x) = 1$.

Lemma 3.3:

A fuzzy B – sub-algebra β of X is normal if and if only $\beta(0) = 1$.

Theorem 3.4:

For any fuzzy subalgebra β of X, we can generate a normal fuzzy sub-algebra X which contains β.

Proof:

Let β be a fuzzy B-sub-algebra of X.

Define a fuzzy subset β^+ of X as

$$\beta^+(x) = \beta(x) + \beta^c(0)\ \forall\ x \in X$$

Let $x, y \in X$

$$\beta^+(x * y) = \beta(x * y) + \beta^c(0)$$

$$\geq \{\beta(x) \wedge \beta(y)\} + \beta^c(0)$$

$$= \{\beta(x) + \beta^c(0)\} \wedge \{\beta(y) + \beta^c(0)\}$$

$$= \beta^+(x) \wedge \beta^+(y)$$

$$\Longrightarrow \beta^c(x * y) \geq \beta^+(x) \wedge \beta^+(y)$$

Also

$$\beta^+(0) = \beta(0) + \beta^c(0)$$

$$= \beta(0) + 1 - \beta(0)$$

$$= 1$$

$\therefore\ \beta^+$is normal fuzzy B-algebra of X.

Clearly $\beta \subset \beta^+$

Thus $\beta^+$is a normal fuzzy B-algebra of X which contains β.

Hence the proof.

Theorem 3.5:

Let β be fuzzy B – sub-algebra of X. If β contains a normal fuzzy B - sub algebra of X generated by any other fuzzy B – sub-algebra of X then β is normal.

Proof:

Let θ be a fuzzy B – sub-algebra of X. By theorem 3.4, θ^+ is a normal fuzzy B – sub algebra of X.

$$\therefore\ \theta^+(0) = 1$$

Let β be a fuzzy B-algebra of X such that $\theta^+ \subset \beta$

$$\Longrightarrow \beta(x) \geq \theta^+(x)\ \forall\ x \in X$$

Put $x = 0$,

$$\Longrightarrow \beta(0) \geq \theta^+(0) = 1$$

$$\Longrightarrow \beta(0) \geq 1$$

Thus β is normal.

Hence the proof.

Theorem 3.6:

Let β and θ be normal fuzzy B- sub algebra of X. If $\beta \subset \theta$ then $X_\beta \subset X_\theta$.

Theorem 3.7:

Let β be a fuzzy B – sub algebra of X. Let $f:[0, \beta(0)] \rightarrow [0,1]$ be an increasing function. Define a fuzzy set $\beta^f : X \rightarrow [0,1]$ by $\beta^f = f(\beta(x))\ \forall\ x \in X$.

Then,

(i) β^f is a fuzzy B – subalgebra of X.
(ii) If $f(\beta(0)) = 1$, then β^f is normal.
(iii) If $f(s) \geq s\ \ \forall s \in [0, \beta(0)]$, then $\beta \subset \beta^f$.

Proof:

(i) $$\beta^f(x * y) = f(\beta(x * y))$$

$$\geq f\{\beta(x) \wedge \beta(y)\}$$

$$= f(\beta(x)) \wedge f(\beta(y))$$

$$= \beta^f(x) \wedge \beta^f(y)$$

$$\Longrightarrow \beta^f(x * y) \geq \beta^f(x) \wedge \beta^f(y)$$

$\Longrightarrow \beta^f$ is a fuzzy B-ideal.

(ii) If $f(\beta(0)) = 1$

$$\Longrightarrow \beta^f(0) = 1$$

$\Longrightarrow \beta^f$ is normal

(iii) Let $f(s) \geq s\ \ \forall\ s \in [0, \beta(0)]$

Then

$$\beta^f(x) = f(\beta(x))$$

$$\geq \beta(x)\ \forall\ x \in X$$

Hence $\beta \subseteq \beta^f$.

4. CONCLUSION

This paper defines Normalization of Fuzzy B – ideals and Fuzzy B – sub algebra of B – algebra. It adds an another dimension to the definition of B- algebras. This concept can further be generalized to n-fold fuzzy B-ideals and n-fold fuzzy B- sub algebras using fuzzy translations and fuzzy multiplications.

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