Unsteady MHD Free Convection Flow of a Viscous Dissipative Kuvshinski Fluid Pastan Infinite Vertical Porous Plate in the Presence of Thermal Radiation and Chemical Effects

R.Sakthikala(1), S.Prahala(2)

(1)*Assistant Professor, Department of Mathematics, PSGR Krishnammal College for women, India*
(2) *Research scholar, Department of Mathematics, PSGR Krishnammal College for women, India*

Abstract: *In this paper, a two dimensional unsteady free convection flow of viscous, incompressible, conducting, radiating and chemically reacting kuvshinski fluid through porous medium over a vertical moving plate is considered which is gray, absorbing emitting but non scattering medium and the Rosseland approximation is used to describe the radiative heat flux energy equation. The governing equations are solved for the velocity profile, temperature and concentration by using perturbation technique. The effects of various physical parameter like Schmidt number, Prandtl number, and chemical reaction are studied numerically with the help of graphs.*

Keywords: *Kuvshinski fluid, MHD, porous medium, thermal radiation, chemical reaction*.

I.INTRODUCTION

The MHD is attracting the attention of many authors due to its applications in geophysics, radio propagation through the ionosphere, power and cooling systems, cooling of nuclear reactor, magneto-hydrodynamic power generation systems etc. the applications of hydro magnetic incompressible viscous flow in science and engineering. This frequently occurs in petro-chemical industry heat exchanger design, forest fire dynamics, and chemical vapour deposition on surfaces. Moreover, considerable interest has been evinced in the effects of thermal diffusion, viscous dissipation, radiation and chemical reaction on a well-known non Newtonian fluid namely kuvshinski fluid interaction on unsteady MHD flow over a vertical moving porous plate. The problem of unsteady free convection with heat transfer from an isothermal vertical flat plate to a non-Newtonian fluid saturated porous medium was examined numerically by Nassar [1]. An investigation on natural convection flow over a fluid saturated porous medium enclosed by non-isothermal walls with heat generations done by Anwar and Wilson [2]. Amin [3] studied about the combined effect of magnetic field and viscous dissipation on a power law fluid over plate with variable surface heat flux embedded in aporous medium. Chamka [4] addressed a heat and mass transfer problem of a non-Newtonian fluid flow over a permeable wedge in porous medium with variable wall temperature and concentration and source or sink. In his investigation chamka and Humoud [5] discussed mixed convection heat and mass transfer of a non-Newtonian fluids from a permeable surface embedded in a porous medium. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effect of magnetic field on the temperature distribution and heat transfer, when the fluid is not only an electrical conductor but also it is capable of emitting and absorbing thermal radiation. Muthucumarswamy and kumar [6] studied heat mass transfer effects on moving vertical plate in the presence of thermal radiation. Mzumder and Deka [7] discussed about MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. A chemical reaction can be codified as either a homogenous or heterogeneous process. This depends on whether it occurs on an interface or a single phase volume reaction. A reaction is said to be of first order if it rate is directly propositional to the concentration itself. An unsteady flow of a viscous, incompressible electrically conducting, laminar free convection boundary layer flow of a moving infinite vertical plate in a radiative and chemically reactive medium in the presence of a transverse magnetic field was investigated by Reddy. T. S et al [8]. Raju, Ananda and Varma [9] studied about MHDfree convective, dissipative boundary layer flow past a vertical porous surface in the presence of thermal radiation, chemical reaction and constant suction, under the influence of uniform magnetic field which is applied normal to the surface. Chamkha [10] et al, consider free convective boundary layer flow over an exponentially accelerated infinite vertical plate embedded in a porous medium in presence of thermal diffusion, radiation and temperature dependent heat source or sink. The fluid considered is a gray, absorbing/emitting radiation but no-scattering medium.

II.MATHEMATICAL FORMULATION

We consider an unsteady two dimensional free convective flow of a viscous incompressible electrically conducting, radiating and chemically reacting kuvshinski fluid through porous medium occupying a semi-infinite region of the space bounded by an infinite vertical plate in the presence of magnetic field B_0 applied in the direction of the flow. On this plate, an arbitrary point has been chosen as the origin of a coordinate system with x*- axis along the plate in the upward direction and the y*-axis normal to the plate. Initially for time $t^* \leq 0$, the plate and the fluids are at the same constant temperature T_∞*in a stationary condition with concentration level C_∞* at all points. The level of foreign mass is assumed to below, so that the Dofour effect is neglected. The fluid is assumed to be gray emitting and absorbing radiation but non-scattering medium. The radioactive heat flux in the x*-direction is negligible in comparison to their in y* - direction. All the fluid properties are assumed to be constant except the influence of the density variation with temperature in body force term. Under the above assumptions, the fully developed flow field is governed by the following set of equations

Continuity equation

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad (1)$$

Momentum equation

$$\left(1+\lambda\frac{\partial}{\partial t^*}\right)\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = g\beta(T^* - T_\infty{}^*) + g\beta(C^* - C_\infty{}^*) - \frac{v}{K^*}\left(1+\lambda\frac{\partial}{\partial t^*}\right)u^* - \frac{\sigma {B_0}^2}{\rho}u^* - \frac{\vartheta}{k}u^* (2)$$

Energy equation

$$\left(1+\lambda\frac{\partial}{\partial t^*}\right)\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{v}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} - \frac{Q_0}{\rho C_p}(T^* - T_\infty{}^*) \ (3)$$

Diffusion equation

$$\left(1+\lambda\frac{\partial}{\partial t^*}\right)\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - Kr^*(C^* - C_\infty{}^*) + D_1\frac{\partial^2 T^*}{\partial y^{*2}} (4)$$

The initial and boundary conditions are:

$u^*=0, \quad T^* = T_\infty{}^*, C^* = C_\infty{}^*$ at $\quad y^* = 0$

$$u^* \to 0, T^* \to T_\infty^*, C^* \to C_\infty^* \quad as\ y^* \to \infty \qquad (5)$$

The equation (1) gives

$$v^* = -v_0 \qquad (6)$$

Where v_0 is the constant suction velocity using the Rosseland approximation for optically thick fluids and the Taylor's series about T_∞ neglecting the second and higher order terms we have equation (7)

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial {T^*}^4}{\partial y^*} \qquad (7)$$

Where σ*is the Stefan-bottzmann constant and is k* the mean absorption coefficient

In view of (6) and (7) the equations (2)-(4) reduce to

$$\left(1+\lambda\frac{\partial}{\partial t^*}\right)\frac{\partial u^*}{\partial t^*}-v_0\frac{\partial u^*}{\partial y^*}=g\beta(T^*-T_\infty{}^*)+g\beta(C^*-C_\infty{}^*)+v\frac{\partial^2u^*}{\partial y^{*2}}-\frac{v}{K^*}\left(1+\frac{\partial}{\partial t^*}\right)u^*-\frac{\sigma B_0{}^2}{\rho}u-\frac{\vartheta}{k}u^* \quad (8)$$

$$\left(1+\lambda\frac{\partial}{\partial t^*}\right)\frac{\partial T^*}{\partial t^*}-v_0\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho C_p}\frac{\partial^2T^*}{\partial y^{*2}}+\frac{v}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2+\frac{16\sigma^*T_\infty{}^{*3}}{3\rho C_p k^*}\frac{\partial^2T^*}{\partial y^{*2}}-\frac{Q_0}{\rho C_p}(T^*-T_\infty{}^*)(9)$$

$$\left(1+\lambda\frac{\partial}{\partial t^*}\right)\frac{\partial C^*}{\partial t^*}-v_0\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2C^*}{\partial y^{*2}}-Kr^*(C^*-C_\infty{}^*)+D_1\frac{\partial^2T^*}{\partial y^{*2}}(10)$$

On introducing the following non-dimensional quantities

$$u=\frac{u^*}{v_0},\qquad y=\frac{y^*v_0}{v},\theta=\frac{T^*-T_\infty{}^*}{T_w{}^*-T_\infty{}^*},\varphi=\frac{C^*-C_\infty{}^*}{C_w{}^*-C_\infty{}^*},M=\frac{\sigma B_0{}^2}{\rho},E=\frac{v_0{}^2}{C_p(T_w{}^*-T_\infty{}^*)},$$

$$k=\frac{v_0{}^2}{v^2}k^*,Gr=\frac{vg\beta(T_w{}^*-T_\infty{}^*)}{v_0{}^3},Gm=\frac{vg\beta(C_w{}^*-C_\infty{}^*)}{v_0{}^3},k_p=\frac{kv_0{}^2}{v^2},Pr=\frac{\rho vC_p}{k},$$

$$N=\frac{k^*k}{4\sigma^*T_\infty{}^{*3}},Kr=\frac{Kr^*v}{v_0{}^2},\alpha_1=\left(1+\frac{v\lambda}{K^*}\right),\alpha_2=\lambda\frac{v_0{}^2}{v},t=\frac{t^*v_0{}^2}{v},F=\frac{Q_0v}{\rho C_pv_0{}^2} \quad (11)$$

In view of (11) the equations (8) – (10), reduce to the following non-dimensional form

$$\alpha_1\frac{\partial u}{\partial t}+\alpha_2\frac{\partial^2u}{\partial t^2}=\frac{\partial^2u}{\partial y^2}+\frac{\partial u}{\partial y}-\left(M+\frac{1}{k_p}\right)u+Gr\theta+Gm\varphi \quad (12)$$

$$Pr\frac{\partial\theta}{\partial t}+Pr\alpha_2\frac{\partial^2\theta}{\partial t^2}=\left(1+\frac{4}{3N}\right)\frac{\partial^2\theta}{\partial y^2}+Pr\frac{\partial\theta}{\partial y}+PrE\left(\frac{\partial u}{\partial y}\right)^2-F\theta \quad (13)$$

$$Sc\frac{\partial\varphi}{\partial t}+Sc\alpha_2\frac{\partial^2\varphi}{\partial t^2}=\frac{\partial^2\varphi}{\partial y^2}+Sc\frac{\partial\varphi}{\partial y}-KrSc\varphi+ScS_0\frac{\partial^2\theta}{\partial y^2}(14)$$

Where Gr is the thermal Grashof number, Gm is the modified Grashof number, Pr is the fluid Prantle number, Sc is the Schmidt number S_0 is the Soret number, Kr is the chemical reaction Parameter and F is the heat source parameter

The corresponding boundary conditions reduce to

$$u=0,\theta=1,\varphi=1\ \ at\ \ y=0$$

$$u\to0,\theta\to0,\varphi\to0,\quad as\ y\to\infty \quad (15)$$

III.METHOD OF SOLUTION

The governing equations (12), (13) and (14) of the flow, temperature and concentration are coupled non- linear differential equations. Assuming E to be small, we write

$$u=u_0+Eu_1e^{-nt}$$

$$\theta=\theta_0+E\theta_1e^{-nt}$$

$$\varphi=\varphi_0+E\varphi_1e^{-nt} \quad (16)$$

Substituting equations (16) into (12), (13) and (14) and equating the powers of E, we obtain equations to the zeroth order as

$$u_0{}^{11} + u_0{}^{1} - \left(M + \frac{1}{k_p} + \frac{1}{k}\right) u_0 = -Gr\theta_0 - Gm\varphi_0 \quad (17)$$

$$N_1\theta_0{}^{11} + Pr\theta_0{}^{1} = F\theta_0 \quad (18)$$

$$\varphi_0{}^{11} + Sc\varphi_0{}^{1} - KrSc\varphi_0 = -ScS_0\theta_0{}^{11} \quad (19)$$

The first order equations are

$$u_1{}^{11} + u_1{}^{1} - M_2u_1 = -Gr\theta_1 - Gm\varphi_1 \quad (20)$$

$$N_1\theta_1{}^{11} + Pr\theta_1{}^{1} + N_2\theta_1 = -Pru_0{}^{1^2}e^{nt} \quad (21)$$

$$\varphi_1{}^{11} + Sc\varphi_1{}^{1} + L_1\varphi_1 = -ScS_0\theta_1{}^{11} \quad (22)$$

The corresponding boundary conditions (15) now become

$$u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0, \varphi_0 = 1, \varphi_1 = 0 \quad at \quad y = 0$$

$$u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, \varphi_0 \to 0, \varphi_1 \to 0 \quad as \quad y \to \infty \quad (23)$$

Where $M_1 = M + \frac{1}{k_p} + \frac{1}{k}, N_1 = 1 + \frac{4}{3N}$

Here a prime denotes the differentiation with respect to y. solving equations (19) and (20) under the corresponding boundary conditions (23) we obtain

$$u_0 = (k_4 - k_6)e^{-m_5y} + k_6e^{-k_1y} - k_4e^{-m_3y} \quad (24)$$

$$u_1 = k_{37}e^{-m_1y} - (k_{22} - k_{30})e^{-m_7y} + (k_{23} - k_{31})e^{-m_7y} + (k_{24} - k_{32})e^{-2k_1y} + (k_{25} - k_{34})e^{-2m_3y} + (k_{26} - k_{34})e^{-(k_1+m_5)y} - (k_{27} - k_{35})e^{-(k_1+m_3)y} - (k_{28} - k_{36})e^{-(m_3+m_5)y} - k_{29}e^{-m_9y} \quad (25)$$

$$\theta_0 = e^{-k_1y} \quad (26)$$

$$\theta_1 = k_{13}e^{-m_7y} - k_7e^{-2m_5y} - k_8e^{-2k_1y} - k_9e^{-2m_3y} - k_{10}e^{-(k_1+m_5)y} + k_{11}e^{-(k_1+m_3)y} + k_{12}e^{-(m_3-m_5)y} \quad (27)$$

$$\varphi_0 = (1 + k_2)e^{-m_3y} - k_2e^{-k_1y} \quad (28)$$

$$\varphi_1 = k_{21}e^{-m_9y} - k_{14}e^{-m_7y} + k_{15}e^{-2m_5y} + k_{16}e^{-2k_1y} + k_{17}e^{-2m_3y} + k_{18}e^{-(k_1+m_5)y} - k_{19}e^{-(k_1+m_3)y} - k_{20}e^{-(m_3-m_5)y} \quad (29)$$

IV.RESULT AND DISCUSSION

In order to get physical insight into problem, the numerical computations have been carried out and the influence of various physical parameters viz., magnetic parameter M, heat source parameter F, thermal Grashof number Gr, mass Grashof number Gm, Schmidt number Sc, Soret number So, Prandtl number Pr, chemical reaction parameter Kr, thermal radiation parameter N, are studied numerically by choosing arbitrary values. The effect of Schmidt number Sc on velocity is presented in figure 1. Due to the concentration buoyancy effect there is a decrease in fluid velocity. That is as the Schmidt number increases, the velocity decreases. Here the velocity attains its maximum near the plate and it reaches the free stream near the vicinity of either sides of the plate.

The effect of soret number on the velocity fluid is present in figure 2, it is observed that velocity increases with increase in soret number. Figure 3 depicts an increase in magnetic field parameter results decreases the velocity. It is due to the application of magnetic field that acts as Lorentz's force which retards the flow. The effects of thermal and solute buoyancy on velocity is presented in figures 4 and 5 in which it is noticed that velocity increases in both the cases as both the parameters namely Grashof number and modified grashof number increases. In figure 6, the result show that the effect of increasing values of prandtl number decreases the velocity. From figures 7 and 8 it is observed that an increase in the thermal radiation and chemical reaction parameters results in the decrease of velocity.

From the numerical study it is noticed that through figure 9, the velocity increases with the increasing values of porosity parameter. The effect of heat source parameter is represented in figure 10, when the parameter value increases the velocity value decreases. Figure 11 and 12 depicts the temperature profile against y for different values of radiation parameter and prandtl numbers. It is noticed that temperature decreases with the increasing in both radiation and prandtl number parameters whereas from figure 13 the effect of heat source parameter F on temperature shows that the increase in heat source decreases the temperature. Effect of Schmidt number on concentration is displayed in figure 14. From this figure it is noticed that concentration decreases with an increase in Schmidt number. Similar effect is noticed in the presence of chemical reaction parameter, which is shown in figure 15. But, figure 16 and 17 witnesses the reverse action on concentration in the presence of soret number and heat source parameter.

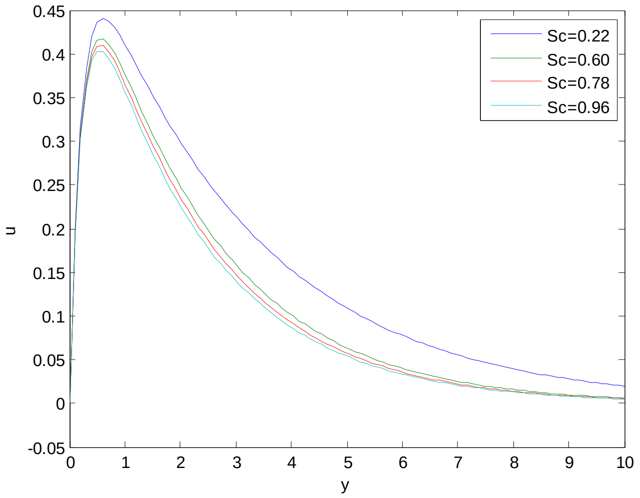

Fig 1: Effect of Sc on velocity

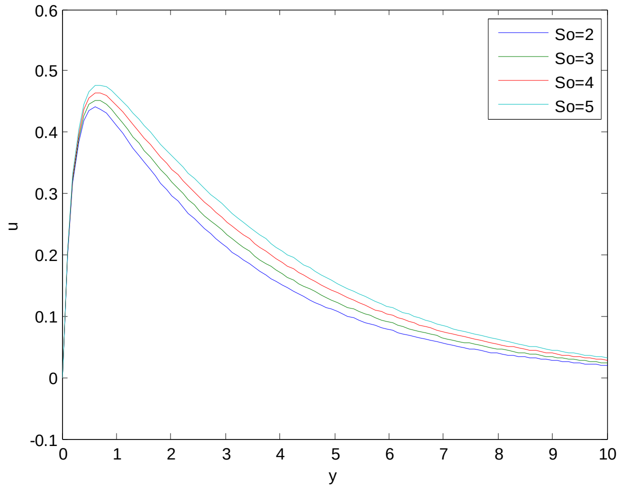

Fig 2: Effect of So on velocity

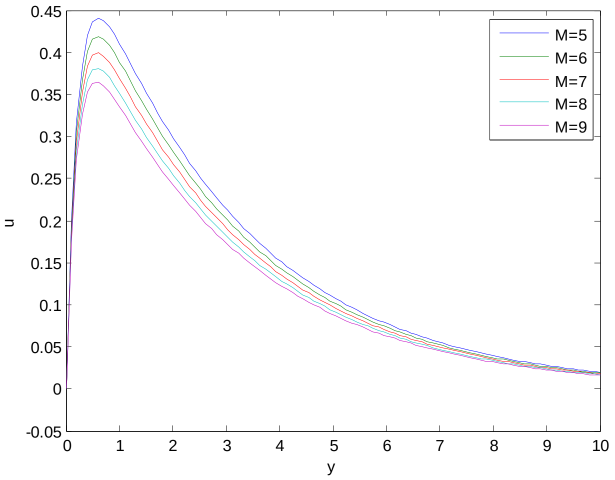

Fig 3: Effect of M on velocity

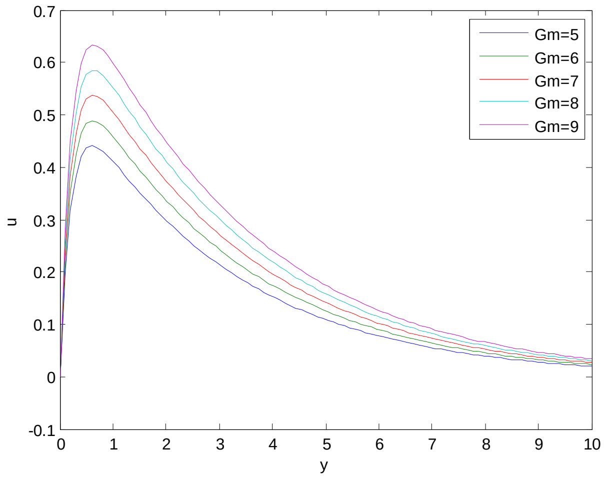

Fig 4: Effect of Gm on velocity

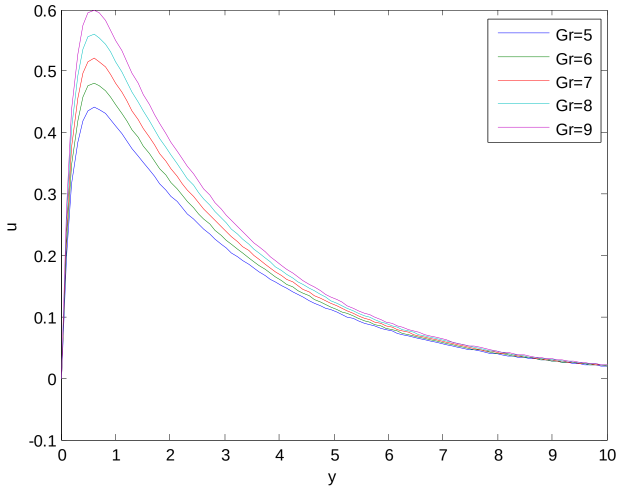

Fig 5: Effect of Gr on velocity

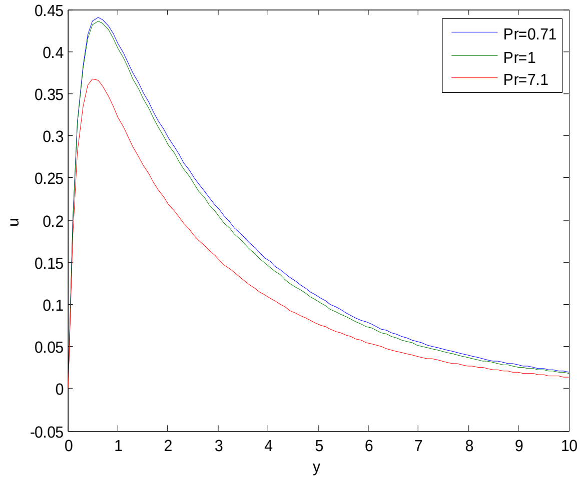

Fig 6: Effect of Pr on velocity

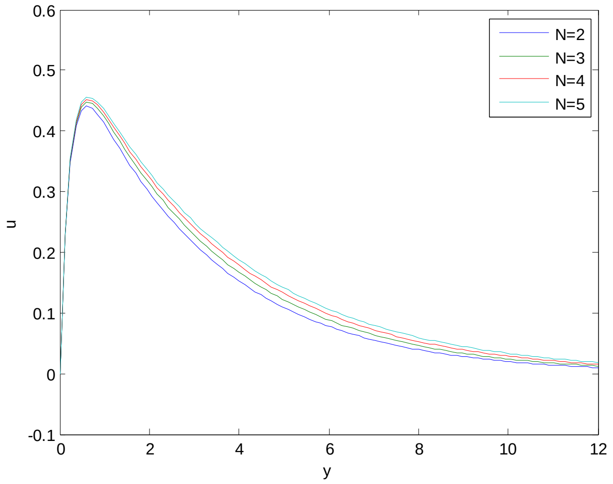

Fig 7: Effect of N on velocity

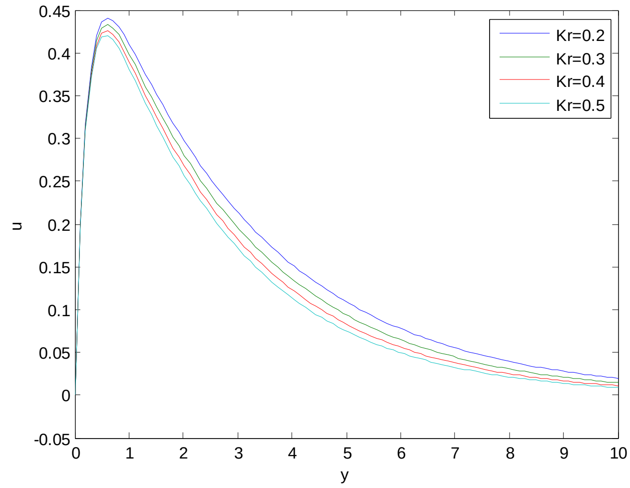

Fig 8: Effect of Kr on velocity

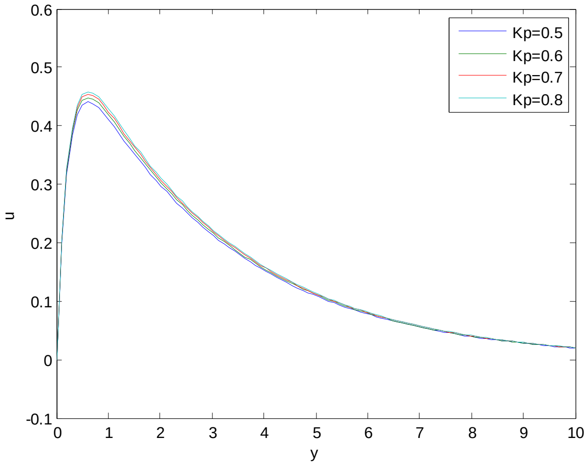

Fig 9: Effect of Kp on velocity

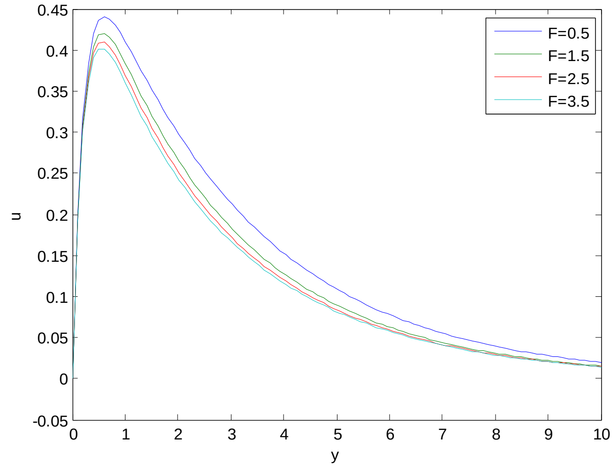

Fig 10: Effect of F on velocity

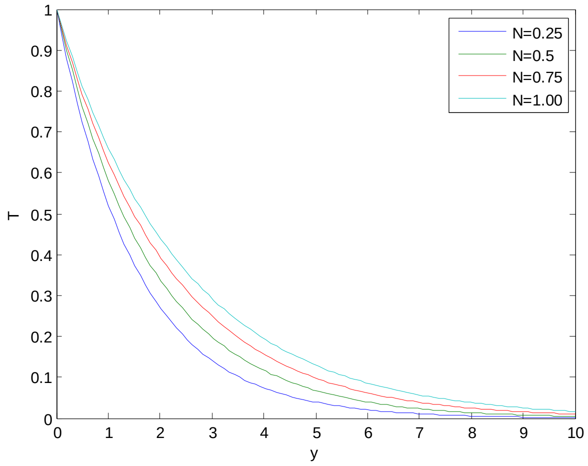

Fig 11: Effect of N on temperature

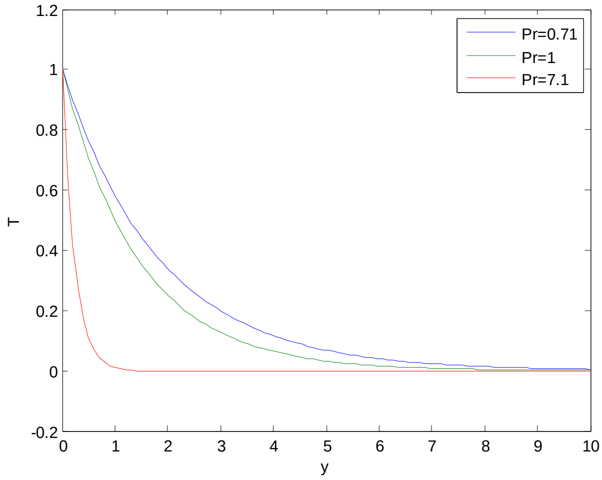

Fig 12: Effect of Pr on temperature

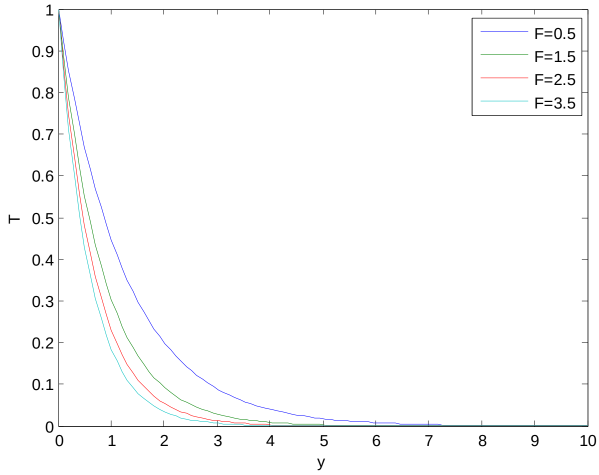

Fig 13: Effect of F on temperature

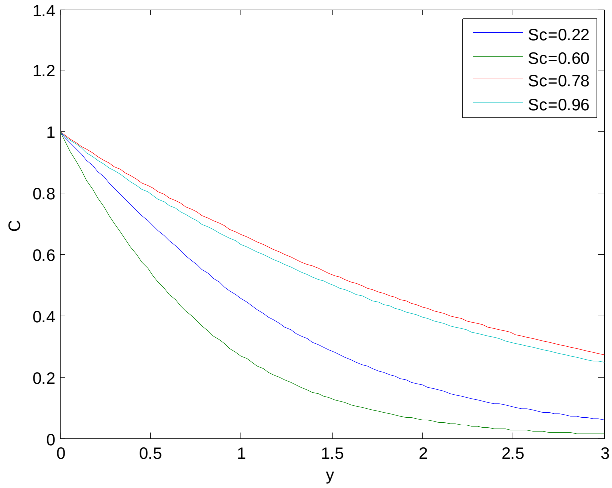

Fig 14: Effect of Sc on concentration

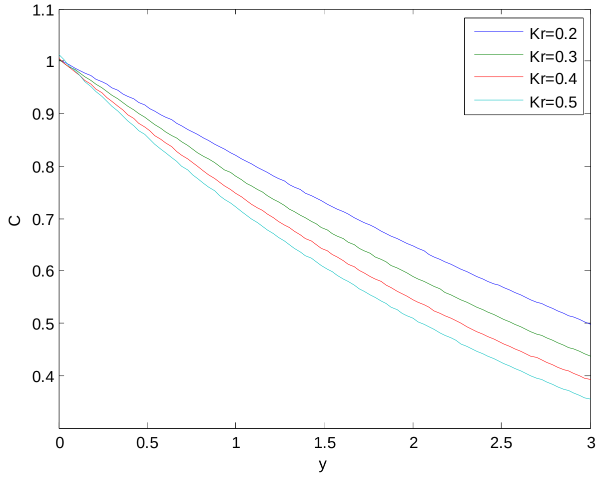

Fig 15: Effect of Kr on concentration

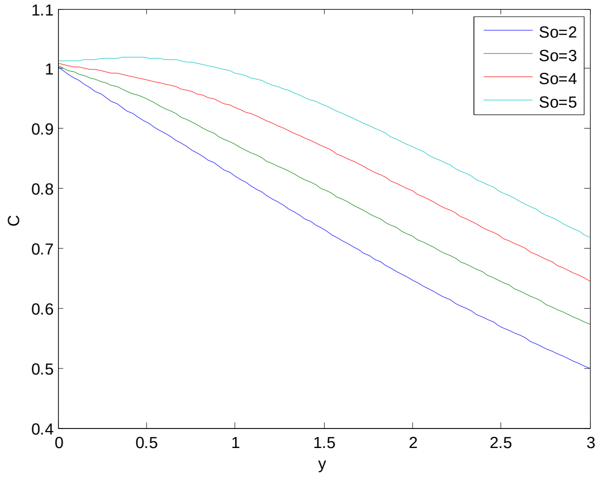

Fig 16: Effect of So on concentration

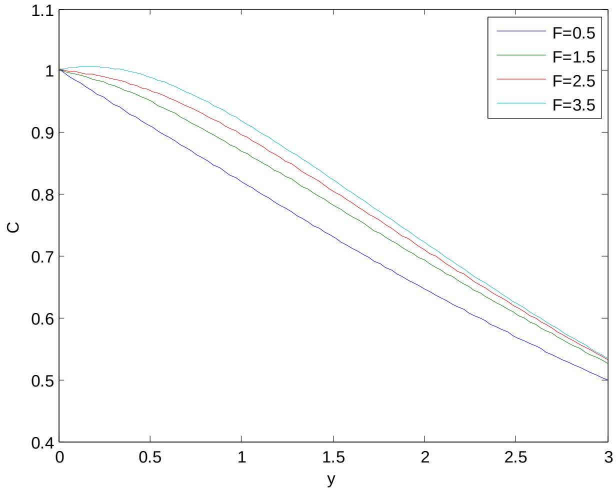

Fig 17: Effect of F on concentration

V.CONCLUSION

The problem of an unsteady free convective flow of viscous, incompressible, radiating kuvshinski fluid flow past through porous medium was formulated and solved by means of perturbation method. The following conclusion are made

- Velocity increases with an increase in Soret number, Grashof number, modified Grashof number and porosity parameter were as it decreases with an increase in Schmidt number, magnetic parameter, Prandtl number, radiation parameter, chemical reaction parameter and heat source parameter.
- Temperature decreases with an increasing in radiation parameter, Prandtl number and heat source parameter.
- Concentration increases with an increase in soret number and heat source parameter but it shows reverse phenomenon in the case of Schmidt number and chemical reaction parameter.

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APPENDIX

$k_1 = \frac{Pr+\sqrt{Pr^2+4N_1F}}{2N_1}$; $N_1 = \left(1+\frac{4}{3N}\right)$; $N_2 = nPr - n^2Pr\alpha_2 - F$

$L_1 = nSc - n^2Sc\alpha_2 - KrSc$; $M_2 = M_1 - n\alpha_1 + n^2\alpha_2$; $m_3 = \frac{Sc+\sqrt{Sc^2+4KrSc}}{2}$

$m_5 = \frac{1+\sqrt{1+4M_1}}{2}$; $k_2 = \frac{ScS_0{k_1}^2}{{k_1}^2-Sck_1-KrSc}$; $k_3 = \frac{Gr}{{k_1}^2-k_1-M_1}$

$k_4 = \frac{Gm(1+k_2)}{{m_3}^2-m_3-M_1}$; $k_5 = \frac{Gmk_2}{{k_1}^2-k_1-M_1}$; $k_6 = k_5 - k_3$

$k_7 = \frac{Pre^{nt}(k_4-k_6)^2{m_5}^2}{4N_1{m_5}^2-2m_5Pr+N_2}$; $k_8 = \frac{Pre^{nt}{k_1}^2{k_6}^2}{4N_1{k_1}^2-2k_1Pr+N_2}$; $k_9 = \frac{Pre^{nt}{m_3}^2{k_4}^2}{4N_1{m_3}^2-2m_3Pr+N_2}$

$k_{10} = \frac{2Pre^{nt}(k_4-k_6)m_5k_1k_6}{N_1(k_1+m_5)^2-Pr(k_1+m_5)+N_2}$; $k_{11} = \frac{2Pre^{nt}k_4m_3k_1k_6}{N_1(k_1+m_3)^2-Pr(k_1+m_3)+N_2}$

$k_{12} = \frac{2Pre^{nt}(k_4-k_6)m_5m_3k_4}{N_1(m_3+m_5)^2-Pr(m_3+m_5)+N_2}$; $k_{13} = k_7 + k_8 + k_9 + k_{10} - k_{11} - k_{12}$

$m_7 = \frac{Pr+\sqrt{Pr^2-4N_1N_2}}{2N_1}$; $m_9 = \frac{Sc+\sqrt{Sc^2-4L_1}}{2}$; $m_{11} = \frac{1+\sqrt{1+4M_2}}{2}$

$k_{14} = \frac{ScS_0k_{13}{m_7}^2}{{m_7}^2-Scm_7+L_1}$; $k_{15} = \frac{4ScS_0k_7{m_5}^2}{4{m_5}^2-2Scm_5+L_1}$; $k_{16} = \frac{4ScS_0k_8{k_1}^2}{4{k_1}^2-2Sck_1+L_1}$

$k_{17} = \frac{4ScS_0k_9{m_3}^2}{4{m_3}^2-2Scm_3+L_1}$; $k_{18} = \frac{k_{10}ScS_0(k_1+m_5)^2}{(k_1+m_5)^2-Sc(k_1+m_5)+L_1}$; $k_{19} = \frac{k_{11}ScS_0(k_1+m_3)^2}{(k_1+m_3)^2-Sc(k_1+m_3)+L_1}$

$k_{20} = \frac{k_{12}ScS_0(m_3+m_5)^2}{(m_3+m_5)^2-Sc(m_3+m_5)+L_1}$; $k_{21} = k_{14} - k_{15} - k_{16} - k_{17} - k_{18} + k_{19} + k_{20}$

$k_{22} = \frac{Grk_{13}}{{m_7}^2-m_7-M_2}$; $k_{23} = \frac{Grk_7}{4{m_5}^2-2m_5-M_2}$; $k_{24} = \frac{Grk_8}{4{k_1}^2-2k_1-M_2}$

$k_{25} = \frac{Grk_9}{4{m_3}^2-2m_3-M_2}$; $k_{26} = \frac{Grk_{10}}{(k_1+m_5)^2-(k_1+m_5)-M_2}$; $k_{27} = \frac{Grk_{11}}{(k_1+m_3)^2-(k_1+m_3)-M_2}$

$k_{28} = \frac{Grk_{12}}{(m_3+m_5)^2-(m_3+m_5)-M_2}$; $k_{29} = \frac{Grk_{21}}{{m_9}^2-m_9-M_2}$; $k_{30} = \frac{Gmk_{14}}{{m_7}^2-m_7-M_2}$

$k_{31} = \frac{Gmk_{15}}{4{m_5}^2-2m_5-M_2}$; $k_{32} = \frac{Gmk_{16}}{4{k_1}^2-2k_1-M_2}$; $k_{33} = \frac{Gmk_{17}}{4{m_3}^2-2m_3-M_2}$

$k_{34} = \frac{Gmk_{18}}{(k_1+m_5)^2-(k_1+m_5)-M_2}$; $k_{35} = \frac{Gmk_{19}}{(k_1+m_3)^2-(k_1+m_3)-M_2}$; $k_{36} = \frac{Gmk_{20}}{(m_3+m_5)^2-(m_3+m_5)-M_2}$

$k_{37} = k_{22} - k_{23} - k_{24} - k_{25} - k_{26} + k_{27} + k_{28} + k_{29} - k_{30} + k_{31} + k_{32} + k_{33} + k_{34} - k_{35} - k_{36}$