Heat and Mass Transfer On The Mhd Flow of an Unsteady Micropolar Fluid Along a Vertical Stretching Sheet

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Abstract — *An investigation is made to study the heat and mass transfer on an unsteady MHD micropolar fluid along a vertical stretching sheet in the presence of induced magnetic field considering both viscosity and thermal conductivity to be the inverse linear function of temperature. The governing partial differential equations are transformed into dimensionless forms using similarity transformations. The effects of temperature dependent viscosity and thermal conductivity and the other parameters involved in the problem are investigated on the velocity, micro-rotation, temperature, concentration and induced magnetic field distribution profiles by solving the governing transformed ordinary differential equations with the help of Runge-Kutta fourth order method with shooting technique and shown graphically and in tabulated form and discussed in details.*

Keywords — *Viscosity, thermal conductivity, micropolar fluid, induced magnetic field, heat transfer, mass transfer, shooting technique.*

I. INTRODUCTION

The study of the dynamics of micropolar fluids has been the field of very active research due to their numerous applications in technological field. Micropolar fluids are defined as those fluids that contain micro-constituents and can undergo rotation which affect the hydrodynamics of the flow. These fluids are distinctly non-Newtonian in nature. These types of fluids has a microstructure and exhibit microrotational effects and can support surface and body couples which are not present in the theory of Newtonian fluids. A number of flow situations such as the flow of low concentration suspensions, liquid crystals, real fluid with suspensions and animal blood etc can be studied by employing micropolar fluid theory. Initially, Eringen [1] developed the theory of microfluids which include the effect of local rotary inertia, the couple stress and inertia spin. The theory of micropolar fluids for the case where only microrotational effects and microrotational inertia exists was also developed by Eringen [2] and later on he [3] extended the theory of thermomicropolar fluids and derived the constitutive law for fluids with microstructure. Ariman et al. [4] has given an excellent review of micropolar fluids and their applications. Gorla [5] investigated the forced convective heat transfer of a micropolar fluid flow over a flat plate. Thakur et al. [6] studied the effects of variable viscosity and thermal conductivity on unsteady free convective heat and mass transfer MHD flow of micropolar fluid with constant heat flux through porous medium. The effects of variable viscosity and thermal conductivity of MHD micropolar fluid in a continuous moving flat plate was discussed by Phukan et al. [7]. Adhikari [8] studied the MHD micropolar fluid flow towards a stagnation point on a vertical surface under induced magnetic field with radiation heat flux. Unsteady MHD forced convection flow and mass transfer along a vertical stretching sheet with heat source/sink and variable fluid properties was discussed by Sharma et al. [9]. The objective of this paper is to investigate the heat and mass transfer on an unsteady MHD micropolar fluid along a vertical stretching sheet under the influence of induced magnetic field. The fluid viscosity and thermal conductivity are assumed to be the inverse linear functions of temperature following Lai and Kulacki [10]. The governing partial differential equations of motion are reduced to ordinary differential equations using similarity transformations, which are solved numerically for prescribed boundary conditions by shooting method.

II. MATHEMATICAL FORMULATION

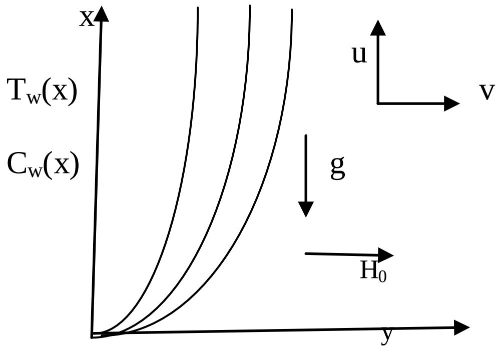


Fig.1: Physical model and coordinate system

We consider an unsteady two dimensional incompressible MHD micropolar fluid past a stretching sheet in the region y>0. The physical model and the coordinate system is depicted in Figure 1. The sheet moves in its own plane with a velocity $U_w=\frac{ax}{1-\alpha t}$ where a (>0) is the stretching parameter and α (>0) is the unsteadiness parameter and both have dimensions time^{-1}. The temperature $T_w(x,t)$ of the sheet and concentration $C_w(x,t)$ near the sheet are assumed to be vary with time t and distance x along the sheet. The magnetic Reynolds number of the flow is taken to be large enough so that induced magnetic field is not negligible. A uniform induced magnetic field of strength H_0 is assumed to be applied in the positive y-direction, normal to the vertical plate. The normal component of the induced magnetic field H_y vanishes when it reaches the wall and the parallel component H_x approaches the value of H_0. The volumetric rate of heat generation / absorption is given as

$$Q(t)=Q_0(1-\alpha t)^{-1}$$

Under the above assumptions, the governing equations are given by as follows:

Basic Equations:

Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Gauss Law of Magnetism:

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_x}{\partial y}=0 \tag{2}$$

Momentum Equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{\kappa}{\rho}\left(\frac{\partial N}{\partial y}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta(T-T_\infty)+g\beta'(C-C_\infty)+\frac{\mu_e}{4\pi\rho}\left(H_x\frac{\partial H_x}{\partial x}+H_y\frac{\partial H_x}{\partial y}\right) \tag{3}$$

Angular Momentum Equation:

$$\rho j\left(\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\kappa\left(2N+\frac{\partial u}{\partial y}\right)+\gamma\frac{\partial^2 N}{\partial y^2} \tag{4}$$

Energy Equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{1}{\rho c_p}(\mu+\kappa)\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho c_p}(T-T_\infty) \tag{5}$$

Magnetic Induction Equation:

$$\frac{\partial H_x}{\partial t}+u\frac{\partial H_x}{\partial x}+v\frac{\partial H_x}{\partial y}=H_x\frac{\partial u}{\partial x}+H_y\frac{\partial u}{\partial y}+\eta_e\frac{\partial^2 H_x}{\partial y^2} \tag{6}$$

Concentration Equation:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_m\frac{\partial C}{\partial y}\right) \tag{7}$$

where t is the non-dimensional time, u and v are the components of velocity along x and y- directions respectively, ρ is the fluid density, μ is the coefficient of dynamic viscosity, κ is the vortex viscosity, N is the microrotation component, g is the acceleration due to gravity, β and β′ are the coefficients of thermal and concentration expansion respectively. is the magnetic permeability, H_x and H_y are the x and y component of induced magnetic field, γ is the spin gradient viscosity, j is the micro-inertia density, T is the temperature of the fluid, λ is the thermal conductivity, c_p is the specific heat at the constant pressure, σ is the electrical conductivity, C is the concentration of the fluid within the boundary layer, T_∞ is the temperature of the fluid far away from the

sheet, C_∞ is the concentration of the fluid far away from the sheet, Q(x) is the heat generation (>0) or absorption (<0) coefficient, D_m is the molecular diffusivity of the species concentration.
The boundary conditions are given as:

$$y = 0 : u = U_w(x,t),\ \ v = 0,\ \ N = 0,\ T = T_w(x,t),\ \ \frac{\partial H_x}{\partial y} = H_y = 0,\ C=C_w(x,t)$$
$$y \to \infty : u \to 0,\quad N \to 0,\ T \to T_\infty,\ H_x = \frac{H_0 x}{1-\alpha t},\ C \to C_\infty \tag{8}$$

Where U_w is the uniform velocity of the plate, T_w and C_w are the temperature and concentration on the surface, T_∞ and C_∞ are the temperature and concentration of the fluid at infinity.
Following Lai and Kulacki, the variation of the fluid viscosity and thermal conductivity can be assumed as follows:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \delta(T-T_\infty)]\ \text{ or }\ \frac{1}{\mu} = \zeta(T-T_r)\ \text{where}\ \zeta=\frac{\delta}{\mu_\infty}\ \text{and}\ T_r = T_\infty - \frac{1}{\delta}$$
$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}[1 + \xi(T-T_\infty)]\ \text{ or }\ \frac{1}{\lambda} = \varepsilon(T-T_c)\ \text{where}\ \varepsilon=\frac{\xi}{\lambda_\infty}\ \text{and}\ T_c = T_\infty - \frac{1}{\xi} \tag{9}$$

Where μ_∞ is the viscosity at infinity, ζ and T_∞ are constants, T_r is transformed reference temperature, δ and ξ are constants based on thermal property of the fluid. Also, λ_∞ is the thermal conductivity at the infinity, ε and T_c are constants and their values depend on the reference state and thermal properties of the fluid.
To solve Eqs. (1)-(7) subject to the boundary conditions given in Eq. (8), the following similarity transformations has been introduced:

$$\eta = \sqrt{\frac{a}{\nu_\infty(1-\alpha t)}}\,y,\quad \psi(x,y,t) = \sqrt{\frac{\nu_\infty a}{(1-\alpha t)}}\,xf(\eta),\quad N = \sqrt{\frac{a^3}{\nu_\infty(1-\alpha t)^3}}\,xg(\eta),$$
$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty},\quad T_w(x,t) = T_\infty + \frac{a(1-\alpha t)^{-3/2}}{2\nu_\infty x^2}$$
$$\phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty},\quad C_w(x,t) = C_\infty + \frac{a(1-\alpha t)^{-3/2}}{2\nu_\infty x^2}$$
$$H_x = \frac{H_0 x}{1-\alpha t}h'(\eta),\quad H_y = -H_0\sqrt{\frac{\nu_\infty}{a(1-\alpha t)}}\,h(\eta)$$
$$u = \frac{\partial \psi}{\partial y},\quad v = -\frac{\partial \psi}{\partial x} \tag{10}$$

where η is the similarity parameter and ν_∞ stands for kinematic viscosity at $T=T_\infty$.
From Eqs. (8) and (9),

$$\nu = -\nu_\infty\frac{\theta_r}{\theta-\theta_r},\quad \lambda = -\lambda_\infty\frac{\theta_c}{\theta-\theta_c} \tag{11}$$

where θ_r and θ_c are the dimensionless parameters that characterise the influence of viscosity and thermal conductivity respectively and are given by,

$$\theta_r = \frac{T_r-T_\infty}{T_w-T_\infty} = -\frac{1}{\delta(T_w-T_\infty)},\quad \theta_c = \frac{T_c-T_\infty}{T_w-T_\infty} = -\frac{1}{\xi(T_w-T_\infty)} \tag{12}$$

Eqs. (1) and (2) are identically satisfied using Eq. (10) and therefore the velocity field and magnetic fields are compatible and represents the possible fluid motion.

Using Eqs. (8) - (12) in Eqs. (3) - (7) we get the following differential equations:

$$\left(\frac{\theta_r}{\theta-\theta_r} - K\right)f''' = \left[\frac{\theta_r}{(\theta-\theta_r)^2}\theta' + f - \frac{1}{2}A\eta\right]f'' - (f'+A)\,f' + Kg' + G_r\theta + G_c\phi + M[(h')^2 - hh''] \tag{13}$$
$$\Delta g'' = \frac{A}{2}(g'\eta+3g) + f'g - fg' + KG(2g+f'') \tag{14}$$
$$\theta'' = \frac{(\theta')^2}{\theta-\theta_c} + \frac{\theta-\theta_c}{\theta_c}\Pr\left[Ec\left(K-\frac{\theta_r}{\theta-\theta_r}\right)(f'')^2 + \left(f - \frac{A\eta}{2}\right)\theta' + \left(2f' - \frac{3A}{2} + S\right)\theta\right] \tag{15}$$
$$h'' = \left(\frac{A\eta}{2} - f\right)h''Pm + Ah'Pm + hf''Pm \tag{16}$$
$$\phi'' = \frac{\theta'\phi'}{\theta-\theta_r} - \frac{\theta-\theta_r}{\theta_r}Sc\left[A\left(\frac{3\phi}{2}+\frac{\phi'\eta}{2}\right) - 2f'\phi - \phi'f\right] \tag{17}$$

Where the primes denote differentiation with respect to η.
The corresponding boundary conditions are,

$$f(0)=0,\ f'(0)=1,\ g(0)=0,\ \theta(0)=1,\ h=h''=0,\ \phi(0)=1$$
$$f'(\infty)=0,\ g(\infty)=0,\ \theta(\infty)=0,\ h'(\infty)=1,\ \phi(\infty)=0 \tag{18}$$

The four important physical quantities for this problem are the skin friction co-efficient (c_f), the wall couple stress (m_w), Nusselt number (Nu) and Sherwood number (Sh), which are defined by,

$c_f = \frac{2\tau_w}{\rho U_w^2}$ where the shear stress at the surface is $\tau_w = [(\mu + k)\frac{\partial u}{\partial y} + kN]_{y=0}$,

$m_w = \Upsilon \frac{U_w^2}{2\nu_\infty x}$, Nu=$\frac{x q_w}{\lambda_\infty (T_w - T_\infty)}$ where the heat flux is $q_w = -\lambda [\frac{\partial T}{\partial y}]_{y=0}$

And Sh=-$\frac{Js}{U_w (C_w - C_\infty)}$, where Js=$[- D_m \frac{\partial C}{\partial y}]_{y=0}$

Therefore,

$c_f (R_e)^{1/2} = 2(K - \frac{\theta_r}{1-\theta_r}) f''(0)$, $m_w = \Upsilon \frac{U_w^2}{\nu_\infty (1-\alpha t)}$ $g'(0)$, $Nu(R_e)^{-1/2} = \frac{\theta_c}{1-\theta_c} \theta'(0)$ and $Sh(R_e)^{1/2} = -\frac{\theta_r}{1-\theta_r} S_c^{-1} \phi'(0)$

III. RESULTS AND DISCUSSION

The system of equations (Eqs.13-17) together with the boundary conditions (Eq.18) are solved for various parameters involved in the equations numerically by using the fourth order Runge-Kutta method based on shooting technique. To find the solution, the numerical values of different parameters are taken as θ_c = -5, θ_r = -5, M=.1, A=.4, G=.2, Pr=.72, Ec=.01, K=.5, Δ =.3, S_c=.2, Gr=.1, Gc=.1, Pm=.1, S=.5, V=1 unless otherwise stated. The main aim of this study is to bring out the effects of variable viscosity and thermal conductivity on the governing flow with the combination of the other flow parameters involved in the problem. The numerical computations have been carried out by developing codes for MATLAB. The results are presented graphically to get a physical insight of the problem for the dimensionless velocity profile $f'(\eta)$, dimensionless microrotation profile $g(\eta)$, temperature profile $\theta(\eta)$, induced magnetic field profile $H'(\eta)$ and concentration profile $\phi(\eta)$ with the variation of different parameters in Figures 2-26. In several practical problems, the characteristics such as skin-friction, wall couple stress, Nusselt number and Sherwood number play important roles. Therefore, the missing values of $f''(0)$, $g'(0)$, $\theta'(0)$, $H'(0)$ and $\phi'(0)$ have been derived in Tables 1-3. Figures 2-11 represent the effects of variable viscosity parameter and thermal conductivity parameter on velocity profile $f'(\eta)$, microrotation profile $g(\eta)$, temperature profile $\theta(\eta)$, induced magnetic field profile $H'(\eta)$ and concentration profile $\phi(\eta)$ respectively. From Figures 2, 4 and 6, it is observed that velocity, temperature and concentration profiles decrease with the increasing values of viscosity parameter whereas opposite trend is observed in Figures 3 and 5 for microrotation and induced magnetic field profiles. Physically, if viscosity enhances there is an increment of the total viscosity in fluid as viscosity is directly proportional to vortex viscosity that makes the fluid more viscous and the convective currents becomes weak. As a result velocity, temperature and concentration profiles reduce and microrotation and induced magnetic fieid profiles enhance. From Figures 7-11, it is observed that with the increasing values of thermal conductivity parameter, velocity, microrotation, temperature and induced magnetic field profiles reduce whereas concentration profile enhances. Due to the increase of thermal conduction within the boundary layer, the transposition of heat from region of higher temperature to the region of lower temperature increases, so temperature, velocity, microrotation and induced magnetic field profiles reduce and concentration profile increases within the boundary layer. Figures 12-15 represent effects of coupling constant parameter K on velocity profile $f'(\eta)$, microrotation profile $g(\eta)$, temperature profile $\theta(\eta)$ and induced magnetic field profile $H'(\eta)$. Since, coupling constant parameter is the ratio of vortex viscosity to the dynamic viscosity, so as K increases velocity and induced magnetic field profiles enhance and microrotation and temperature profiles are found to be reduced. Figures 16-25 exhibit effects of magnetic Prandtl number Pm and unsteadiness parameter A on velocity profile $f'(\eta)$, microrotation profile $g(\eta)$, temperature profile $\theta(\eta)$, induced magnetic field profile $H'(\eta)$ and concentration profile $\phi(\eta)$ respectively. From Figures 16-20, it is observed that velocity, microrotation, temperature, induced magnetic field and concentration profiles decrease with the increasing values of Pm. Physically, magnetic Prandtl number is defined as the ratio of momentum diffusivity (or viscosity) to the magnetic diffusivity and hence the above results are obvious. From Figures 21, 23-25, it is observed that due to the increasing values of unsteadiness parameter A, velocity, temperature, induced magnetic field and concentration profiles decrease whereas opposite trend is observed in microrotation profile from the Figure 22. Physically, since time is inversely proportional to velocity, temperature, magnetic diffusivity and mass diffusion and is directly proportional to the angular momentum and hence the above results are obvious with time parameter. Figure 26 represents the effect of Schmidt number S_c on concentration profile. Since S_c is the ratio of viscosity to the mass diffusion, so with the increasing values of S_c , the molecular mass diffusivity decreases and as a result the concentration profiles reduces. From the Tables 1-3, it is observed that, with the increasing values of viscosity parameter θ_r , thermal conductivity parameter θ_c , unsteadiness parameter A, microrotation parameter G and magnetic Prandtl number Pm, the values of $f''(0)$ and $\theta'(0)$ are decreasing. Similarly, the values of $g'(0)$ increases with increasing values of θ_r , Pm and A whereas decreases with G and θ_c. The values of $f'(0)$ decreases with θ_r, Pm and A but increases with θ_c and G. Also the values of $H'(0)$ reduces with the increasing value of θ_r , G, A and magnetic Prandtl number Pm; but opposite trend is observed with θ_c. Tables 4 and 5 display a comparison of missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ of present study with previous work of Sharma et al. [9] and a significant result has been observed by addition of the new parameters in the present work.

Figures 2-26 for dimensionless velocity distribution f′(η), dimensionless microrotation distribution g(η) , temperature distribution $\theta(\eta)$, induced magnetic field distribution H′(η) and concentration distribution $\phi(\eta)$ with the variation of different parameters and missing value and comparison Tables 1-5 are displayed below :

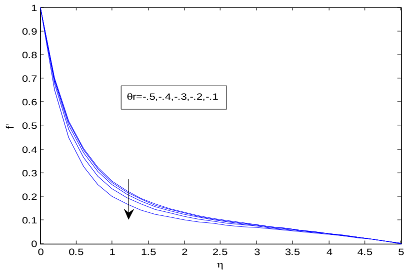


Fig. 2: Velocity for various θ_r

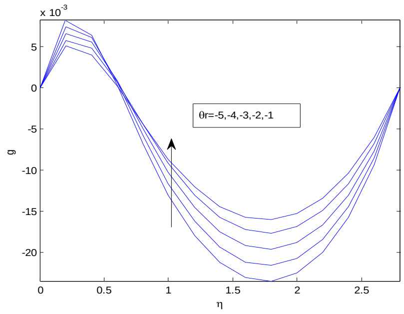


Fig.3: Microrotation for various θ_r

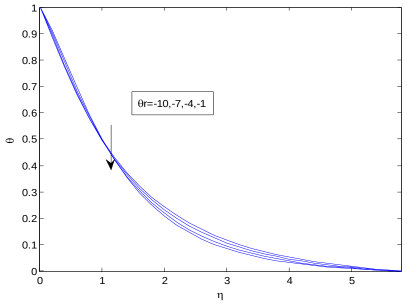


Fig.4: Temperature for various θ_r

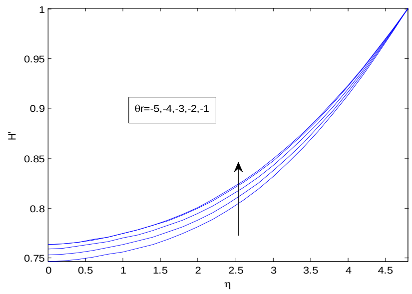


Fig. 5: Induced magnetic field for various θ_r

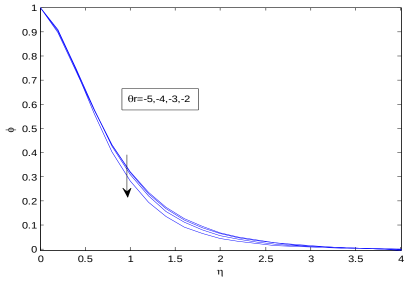


Fig. 6: Concentration for various θ_r

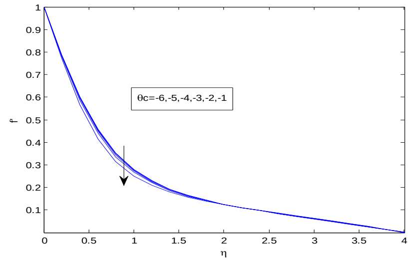


Fig. 7: Velocity for various θ_c

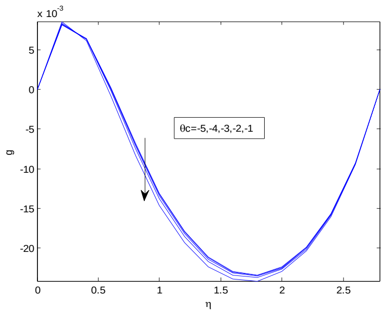


Fig.8: Microrotation for various θ_c

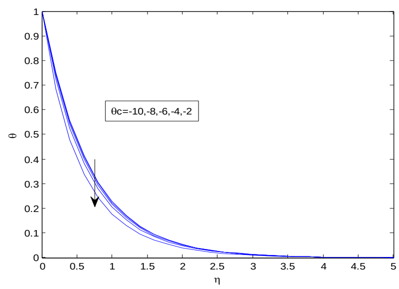
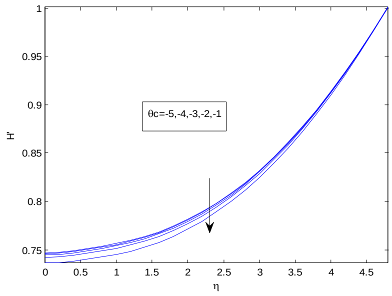


Fig.9: Temperature for various θ_c



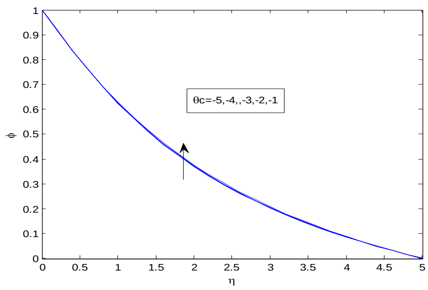


Fig.10: Induced magnetic field for various θ_c

Fig.11: Concentration for various θ_c

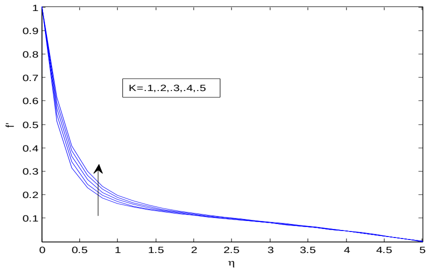


Fig.12: Velocity for various K

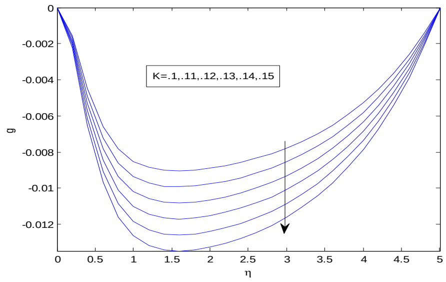


Fig.13: Microrotation for various K

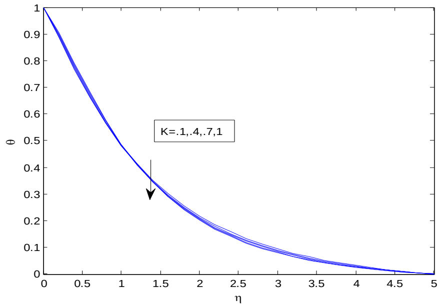


Fig.14: Temperature for various K

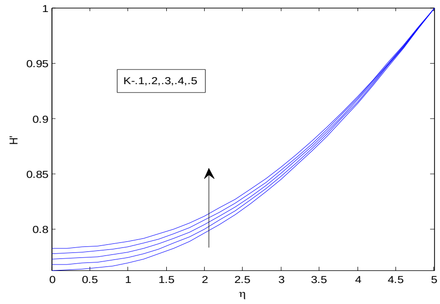


Fig.15: Induced magnetic field for various K

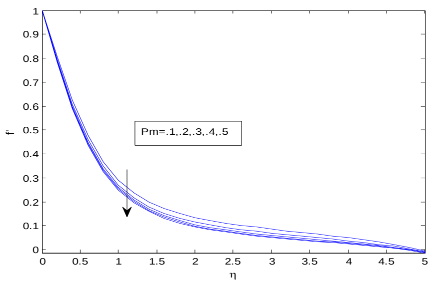


Fig.16: Velocity for various Pm

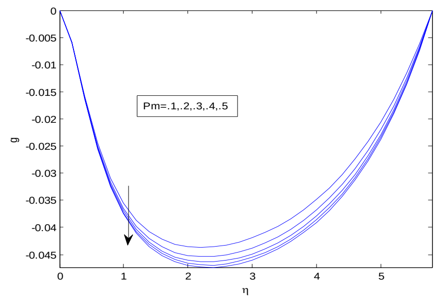


Fig.17: Microrotation for various Pm

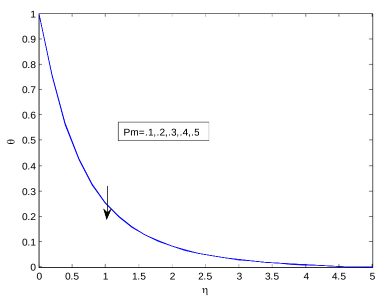


Fig.18: Temperature for various Pm

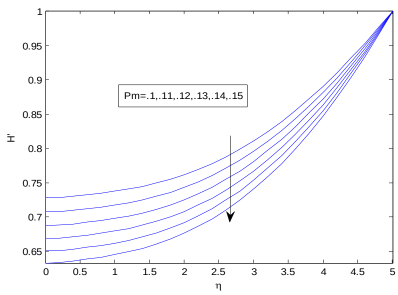


Fig.19: Induced magnetic field for various Pm

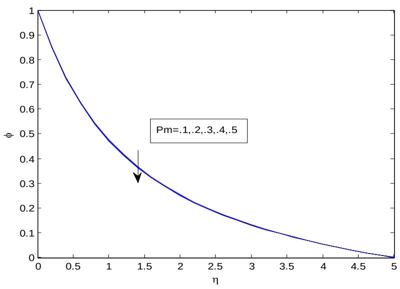


Fig.20: Concentration for various Pm

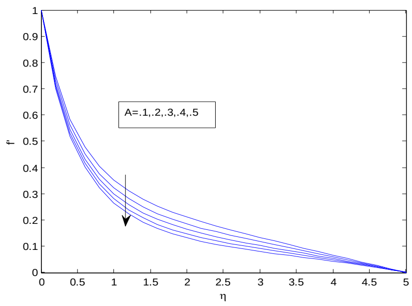


Fig.21: Velocity for various A

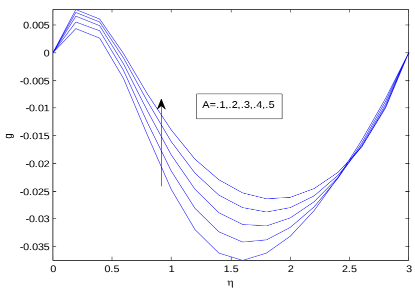


Fig.22: Microrotation for various A

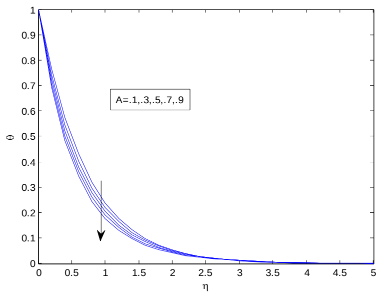


Fig.23: Temperature for various A

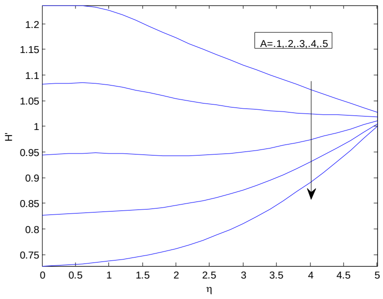


Fig.24: Induced magnetic field for various A

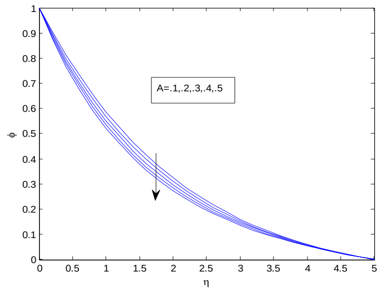


Fig.25: Concentration for various A

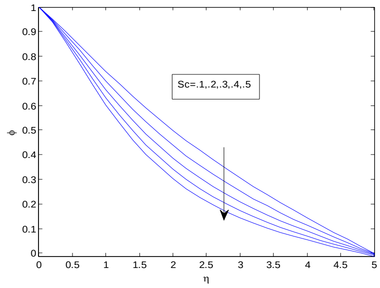


Fig.26: Concentration for various S_c

Table 1: Estimated missing values of $f''(0)$, $g'(0)$, $\theta'(0)$, $H'(0)$ and $\phi'(0)$ for various θ_r and A and θ_c =-.5, M=.1, G=.2, $\Pr$ =.72, E_c =.01, K=.5, Δ =.3, S_c=.2, Gr=.1, Gc=.1, Pm=.1, S=.5, V=1.

A	θ_r	$f''(0)$	$g'(0)$	$\theta'(0)$	$\Phi'(0)$	$H'(0)$
.2	-0.5	-1.49401	-0.02806	0.084896	0.190353	1.011265
	-0.4	-1.556	-0.02744	0.070182	0.078275	1.00757
	-0.3	-1.6484	-0.02669	0.046392	-0.15646	1.001574
.3	-0.5	-1.66681	-0.00794	-0.16695	-0.41431	0.901973
	-0.4	-1.72137	-0.0071	-0.17672	-0.51192	0.899079
	-0.3	-1.80103	-0.0059	-0.19115	-0.67786	0.89472
.4	-0.5	-1.77678	0.011576	-0.34949	-0.60887	0.81168

	-0.4	-1.82816	0.012709	-0.35794	-0.70953	0.809137
	-0.3	-1.90462	0.014377	-0.37043	-0.8746	0.8053

Table 2: Estimated missing values of f´´(0) , g´(0), θ´(0) ,H′(0) and ϕ´(0)for various θ_c and G and θ_r = -.5, A=.4, M=.1, Pr =.72, E_c=.01, K=.5, Δ =.3, S_c=.2, Gr=.1, Gc=.1, Pm=.1, S=.5, V=1.

G	θ_c	f″(0)	g′(0)	θ ′(0)	Φ′(0)	H′(0)
.1	-0.5	-1.84593	0.05171	-1.242	-0.61215	0.815609
	-0.4	-1.85864	0.05142	-1.51657	-0.61077	0.81661
	-0.3	-1.87585	0.05095	-1.98766	-0.6079	0.818168
.2	-0.5	-1.85685	0.01093	-1.24224	-0.61052	0.814822
	-0.4	-1.86957	0.01059	-1.51692	-0.60906	0.815826
	-0.3	-1.88678	0.01001	-1.98815	-0.60608	0.817389
.3	-0.5	-1.87712	-0.12183	-1.24178	-0.60708	0.8137
	-0.4	-1.8899	-0.12198	-1.51664	-0.60548	0.814702
	-0.3	-1.90717	-0.12233	-1.98809	-0.60231	0.816268

Table 3: Estimated missing values of f´´(0) , g´(0), θ´(0), H′(0) and ϕ´(0) for various θ_c and Pm and θ_r = -.5, A=.4, G=.2, Pr =.72, E_c=.01, K=.5, Δ=.3, S_c=.2, Gr=.1, Gc=.1, M=.1, S=.5, V=1.

Pm	θ_c	f″(0)	g′(0)	θ ′(0)	Φ′(0)	H′(0)
.1	-0.5	-1.77678	0.011576	-0.34949	-0.60887	0.81168
	-0.4	-1.82816	0.012709	-0.35794	-0.70953	0.809137
	-0.3	-1.90462	0.014377	-0.37043	-0.8746	0.8053
.2	-0.5	-1.81054	0.01267	-0.36421	-0.62377	0.702587
	-0.4	-1.86301	0.0138	-0.37308	-0.72509	0.698509
	-0.3	-1.94132	0.015461	-0.3862	-0.89079	0.692375
.3	-0.5	-1.83018	0.013265	-0.3727	-0.6322	0.631238
	-0.4	-1.88325	0.014388	-0.3818	-0.7339	0.626115
	-0.3	-1.9626	0.016038	-0.39527	-0.89995	0.618433

Table 4: Comparison of missing values of f´´(0) , θ´(0) and ϕ´(0) for various values of A .

Previous work, Sharma et al [10]				Present work		
A	f″(0)	θ′(0)	Φ′(0)	f″(0)	θ′(0)	Φ′(0)
.2	-2.08074	-0.52859	-0.3099	-1.37548	0.102311	0.246808
.3	-2.13502	-0.64495	-0.37101	-1.56212	-0.14961	-0.2358
.4	-2.18574	-0.74768	-0.42744	-1.68935	-0.33481	-0.40749

Table 5: Comparison of missing values of f´´(0) , θ´(0) and ϕ´(0) for various values of Gr.

Previous work, Sharma et al [10]				Present work		
Gr	f″(0)	θ′(0)	Φ′(0)	f″(0)	θ′(0)	Φ′(0)
.1	-2.02171	-0.39331	-0.24254	-1.96859	-0.36733	-0.29491
.2	-1.97715	-0.38864	-0.24223	-1.87887	-0.34914	-0.28426
.3	-1.93264	-0.38399	-0.24191	-1.79141	-0.3314	-0.27381

IV. CONCLUSION

From the above study, the following significant observations has been found:

1. When the effects of variable viscosity and thermal conductivity are taken into account, the flow characteristics are significantly changed as shown in the Tables 4-5.

2. Velocity reduces with the increasing values of viscosity parameter θ_r , thermal conductivity parameter θ_c , magnetic Prandtl number Pm and unsteadiness parameter A; but reverse trend happens with coupling constant parameter K.

3. Temperature decreases with the increasing values of viscosity parameter θ_r , thermal conductivity parameter θ_c , magnetic Prandtl number Pm, unsteadiness parameter A and coupling constant parameter K.

4. Microrotation enhances due to the increasing values of viscosity parameter θ_r , and unsteadiness parameter A, whereas it decreases with the increasing values of thermal conductivity parameter θ_c , coupling constant parameter K and magnetic Prandtl number Pm .

5. Concentration reduces with the increasing values of viscosity parameter θ_r , Schmidt number Sc , magnetic Prandtl number Pm and unsteadiness parameter A; but reverse trend happens with thermal conductivity parameter θ_c.

6. Induced magnetic field reduces with the increasing values of thermal conductivity parameter θ_c , magnetic Prandtl number Pm and unsteadiness parameter A; but increases with viscosity parameter θ_r and coupling constant parameter K.

7. With the increasing values of viscosity parameter θ_r and thermal conductivity parameter θ_c the values of $f''(0)$ and $\theta'(0)$ decrease.

8. It is hoped that the findings of this investigation may be useful for further studies in the field of fluid mechanics.

V. NOMENCLATURE

t	Non dimensional time,
(u,v)	Velocity components of the fluid,
ρ	Fluid density,
μ	Coefficient of dynamic viscosity,
k	Vortex viscosity,
N	Microrotation component,
g	Acceleration due to gravity,
(β,β')	Coefficients of thermal and concentration expansion,
μ_e	Magnetic permeability,
(H_x,H_y)	x and y component of induced magnetic field,
Υ	Spin gradient viscosity,
j	Micro-inertia density,
T	Temperature of the fluid,
λ	Thermal conductivity ,
c_p	Specific heat at the constant pressure ,
σ	Electrical conductivity,
C	Concentration of the fluid within the boundary layer,
T_∞	Temperature of the fluid far away from the sheet,
C_∞	Concentration of the fluid far away from the sheet,
$Q(x)$	Heat generation (>0) or absorption (<0) coefficient,
D_m	Molecular diffusivity of the species concentration,
K	Coupling constant parameter,
A	Unsteadiness parameter ,
G_r	Grashof nummber ,
G_c	Modified Grashof number,
Δ	Material constant ,
G	Local microrotation parameter,
P_r	Prandtl number ,
E_c	Eckert number
S_c	Schmidt number ,

P_m Magnetic Prandtl number,

η_e Magnetic diffusivity,

M Induced magnetic parameter,

S Heat generation/absorption parameter.

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