# Closed Neighbourhood Prime Labeling(cnp) Ofgraph : A New Labeling Technique. 

MukundBapat

Hindale,Tal: Devgad,Maharashtra,India: 416630


#### Abstract

Closed Neighbourhood of a vertex is the set of all vertices adjacento vincluding v. and is denoted by $\mathrm{N}_{[\mathrm{yl}]}$. Based on this we define a new type of labeling of graphs. We show that stars,bistars,flags,snakes, pathunions, are Closed Neighbourhood prime graphs. We also show invariance under cnpl of all non isomorphic structures of $\mathrm{Pn}\left(\mathrm{FL}\left(\mathrm{C}_{3}\right)\right)$


Key words : Closed, Neighbourhood, prime, stars, invariance,labeling etc. Subject Classification : O5C78
I.Introduction:The graphs considered in this paper are finite,simple and connected. Let $G$ be a ( $p, q$ ) graph.For definitions and terminology we depend on Gallian [3] and Harary [4]. Patel and Shrimali[6] has introduced neighbourhood prime labeling of graph.We in this labeling have considered closed neighbourhood of a vertex $v$ given by $\mathrm{N}_{[\mathrm{v}]}$. Define a bijective function f : $\mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{p}\}$ such that gcd of all labels of vertices incident with v including label of v is 1.i.e $\operatorname{gcd}\left\{\mathrm{f}(\mathrm{u}) / \mathrm{u} \in \mathrm{N}_{[v]}\right\}$ should be 1 . This is true forevery vertex v in $\mathrm{V}(\mathrm{G})$ except the isolated vertices. The graph for which such a function $f$ is defined then graph $G$ is called as Closed neighbourhoodPrime graph. (c-nhd prime )Andthe function f is called as Closed Neighbourhood Prime function.
We show that stars, bistars, flags, snakes, pathunionsare some families of Closed
Neighbourhood Prime graphs.We show that different nonisomorphic structures available on path union of flag of $\mathrm{C}_{3}$ are enpgraphs.
Following observations play important role in deciding the gcd of collection of positive numbers.

1) G.C.D. of any two consecutive integers is one.
2) If the set of numbers contains the number one the G.C.D. is equal two one.
3) If the set contain a prime and no multiple of it then G.C.D. is one.
4) If the set contains an even number say $m$ and all other numbers are odd numbers which does not have common divisor with $m$ then G.C.D. is 1 .

## II.Definitions:

3.1 Star graph $K_{1, n}$ is obtained by taking one point union of $n$ edges.It has $n+1$ points and $n$ pendent edges.
3.2 Bystar $K_{2, n}$ It is obtained by fusing $K_{2}$ with $K_{1, n}$.It has $2 n+1$ edges and $2 n+2$ vertices. 3.3 snake on $C_{3}$ i.e. $S\left(C_{3}, n\right)$ : Take a path on $n$ vertices given by $\left(v_{1}, v_{2}, \ldots v_{n}\right)$ and between every pair of consecutive vertices $v_{i}$ and $v_{i+1}$ ofPn take a new vertex wi and join it by an edge each to every $w_{i}$ and $w_{i+1}$.A $S(C 3, n)$ has $3(n-1)$ edges and $2 n+1$ vertices.
3.4 snake on $C_{4}$ i.e. $S\left(C_{4}, n\right)$ : Take a path on $n$ vertices given by $\left(v_{1}, v_{2}, \ldots v_{n}\right)$ and between every pair of consecutive vertices $v_{i}$ and $v_{i+1}$ of Pn take a new pair of vertices $w_{i}$ and $w_{i+1}$ and take new edges $\left(\mathrm{v}_{\mathrm{i}} \mathrm{w}_{\mathrm{i}}\right),\left(\mathrm{w}_{\mathrm{i}} \mathrm{w}_{\mathrm{i}+1}\right)$ and $\left(\mathrm{w}_{\mathrm{i}+1} \mathrm{v}_{\mathrm{i}+1}\right)$ A $\mathrm{S}\left(\mathrm{C}_{4}, \mathrm{n}\right)$ has $4(\mathrm{n}-1)$ edges and $3(\mathrm{n}-1)+1$ vertices. 3.5 Pathunion of $G$,i.e. $\operatorname{Pm}(\mathrm{G})$ is obtained by taking a path $\mathrm{p}_{\mathrm{m}}$ and take $m$ copies of graph $G$ Then fuse a copy each of $G$ at every vertex of path at given fixed point on $G$. It has mp vertices and $m q+m-1$ edges. Where $G$ is a $(p, q)$ graph.
3.6 Flag of a graph $G \operatorname{FL}(\mathrm{G})$ is obtained by taking a graph $\mathrm{G}=\mathrm{G}(\mathrm{p}, \mathrm{q})$. At sutable vertex of G attach a pendent edge.It has $\mathrm{p}+1$ vertices and $\mathrm{q}+1$ edges.
3.7 One point union of $k$ copies of $G$ namely $(G)^{(k)}$ is fusion of $k$ copies of graph $G$ at a fixed vertex on G.
3.8 fusion ofvertices :Let $v \in V\left(G_{1}\right), v^{\prime} \in V\left(G_{2}\right)$ where $G_{1}$ and $G_{2}$ are twographs. We fuse $v$ and v' by placing them with a single vertex say $w$ and all edges incident with $v$ in $G_{1}$ and that with $v^{\prime}$ in $\mathrm{G}_{2}$ are incident with u in the new graph $\mathrm{G}=\mathrm{G}_{1} \mathrm{FG}_{2} \cdot \operatorname{Deg}_{\mathrm{G}} \mathrm{u}=\operatorname{deg}_{\mathrm{G} 1}(\mathrm{v})_{+} \operatorname{deg}_{\mathrm{G} 2}(\mathrm{v}$ ') and $|\mathrm{V}(\mathrm{G})|=\left|\mathrm{V}\left(\mathrm{G}_{1}\right)\right|+\left|\mathrm{V}\left(\mathrm{G}_{2}\right)\right|-1,|\mathrm{E}(\mathrm{G})|=\left|\mathrm{E}\left(\mathrm{G}_{1}\right)\right|+\left|\mathrm{E}\left(\mathrm{G}_{2}\right)\right|$

G1 =


Figure3.1 fusion of vertex $v$ and $v^{\prime}$ at vertex $u$


## III. Results proved:

4.1 Theorem: $\mathrm{K}_{1, \mathrm{n}}$ is Closed Neighbourhood Prime graph .

Proof:Label the n -degree vertex as 1 and $\mathrm{f}(\mathrm{ui})=\mathrm{i}$ for $\mathrm{i}=2,3, . \mathrm{n}, \mathrm{n}+1$.


Fig 3.1 cnp labeling of $\mathrm{K}_{1,5}$

### 4.2 Theorem :Bistar $\mathrm{K}_{2, \mathrm{n}}$ is cnp graph.

Proof:We define it by starting with an edge (uv) and the vertices adjacent to $u$ are $u i, i=1,2, ., n$ and th e vertices adjacent to $v$ are vi, $\mathrm{i}=1,2, \ldots, \ldots, \mathrm{n}$.
Define a bijective function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{p}\}$ such that $\mathrm{f}(\mathrm{u})=1$ and $\mathrm{f}(\mathrm{v})=2$.
Further $f\left(u_{i}\right)=2 i$ and $f(v i)=2 i+1, i=1,2, \ldots, n$


Fig 3.2 cnp labeling of $\mathrm{K}_{2,7}$

### 4.3 Theorem : $\mathrm{S}\left(\mathrm{C}_{3}, \mathrm{n}\right)$ is cnp graph.

Proof:The ordinary labeling is started at extreme left with two degree vertices starting with $u_{1}$ and $u_{2}$ as label., The four degree vertices fromleft are consecutively labeled as $u_{3}, u_{5}, u_{7}, \ldots$. .Then between every 2 vertices of degree 4 vertices there is 2-degree vertex and are denoted by $\mathrm{u}_{4}, \mathrm{u} 6$, ,.all even number as suffix are 2-degree vertices.Consider a copy of $\mathrm{S}\left(\mathrm{C}_{3}, 5\right)$ with ordinary labeling is given in fig 3.3


Fig 3.3 ordinary labeling of $S\left(C_{3}, 5\right)$

Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{n}\}$ Where n is the number of vertices on graph G given by $f\left(u_{i}\right)=i$. The resultant graph is cnp .


Fig 3.4 cnp labeling of $S\left(\mathrm{C}_{3}, 5\right)$

### 4.4 Theorem The snake on $\mathrm{C}_{4}$ is enpgraph.

Proof:.In ordinary labeling we label path vertices first.This path is (v1vn) given by $\left(v_{1}, v_{2}, v_{3}, . . v_{n}\right)$.Between every consecutive two vertices of path say $v_{i}$ and $v_{i+1}(i=1,2, \ldots, n-1)$ take twonew vertices $v_{i, 1}$ and $v_{i, 2}$ and take new edges $\left(v_{i} v_{i, 1}\right)$ and $\left(v_{i, 1} v_{i, 2}\right),\left(v_{i, 2} v_{i+1}\right), i=1,2, \ldots n-1$


Fig 3.5 ordinary labeling of $S\left(C_{4}, 5\right)$


Fig 3.6 cnp labeling of $S\left(\mathrm{C}_{4}, 5\right)$

Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots . \mathrm{n}\}$ Where n is the number of vertices on graph G given by $f\left(v_{i}\right)=i$. $f\left(v_{i, j}\right)=n+(i-1) 2+j$ The resultant graph is cnp
4.5 Theorem Path union of $\mathrm{C}_{3}$ denoted by $\mathrm{Pn}\left(\mathrm{C}_{3}\right)$ is cnp graph

Proof: The ordinary labeling of $\mathrm{p}_{\mathrm{n}}\left(\mathrm{C}_{3}\right)$ is as shown in figure


Fig 3.7 ordinary labeling of $P_{6}\left(C_{3}\right)$

Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{n}\}$ Where n is the number of vertices on graph G given by $f\left(u_{i}\right)=i$. The resultant graph is cnp


Fig 3.8cnp labeling of $\mathrm{P}_{5}\left(\mathrm{C}_{3}\right)$

Theorem 4.6Flag graph of Cn i.e. $\mathrm{fl}(\mathrm{Cn})$ is cnp graph.
Proof: Let the cycle Cn be defined as ( $\mathrm{v}_{1}, \mathrm{v}_{2}, . . \mathrm{v}_{\mathrm{n}}, \mathrm{v}_{1}$ ) and flag at $\mathrm{v}_{1}$ given by vertex $\mathrm{v}_{\mathrm{n}+1}$ Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{n}\}$ Where n is the number of vertices on graph G given by $f\left(u_{i}\right)=i$. The resultant graph is cnp


Fig 3.9cnp labeling of flag $\mathrm{C}_{6}$
4.7 Path union of $\mathrm{C}_{4}$ is cnp graph.

Proof : Path union of $\mathrm{C}_{4}$ denoted by $\operatorname{Pn}\left(\mathrm{C}_{4}\right)$ is obtained by starting with a path Pn on n vertices given by $\left(v_{1}, v_{2}, \ldots v_{n}\right)$ and $n$ copies of $C_{4}$ are taken. A copy each of $C_{4}$ is fused at every vertex of Pn.Resultant graph has $5 \mathrm{n}-1$ edges and 4 n vertices. The vertices of C 4 at ith vertex of Pn are (starting with 4-degree vertex ) given by $\mathrm{v}_{\mathrm{i} 1}=\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{i} 2}, \mathrm{v}_{\mathrm{i} 3}, \mathrm{v}_{\mathrm{i} 4}, \mathrm{v}_{1}$. Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots . \mathrm{n}\}$ Where n is the number of vertices on graph $G$ given by $f\left(\mathrm{v}_{\mathrm{ij}}\right)=4(\mathrm{i}-1)+\mathrm{j}$; , $\mathrm{j}=1,2,3,4 ; \mathrm{i}=1,2, . . \mathrm{nThe}$ resultant graph is cnp.
4.8 Path union of $\mathrm{C}_{3}$ flag i.e. $\mathrm{G}=\operatorname{Pn}\left(\mathrm{FL}\left(\mathrm{C}_{3}\right)\right)$ is cnpgraph. $|\mathrm{G}(\mathrm{V})|=\mathrm{p}$ ( All three structures)

Proof. Structure 1:We start with a path Pn on $n$ vertices given by $\left(v_{1}, v_{2}, \ldots v_{n}\right)$ and $n$ copies of $G$ are taken.A copy each of $G$ is fused at every vertex of Pn. The fusion vertex being the pendent vertex of $G$. Resultant graph has $5 n-1$ edges and $5 n-1$ vertices. The vertices of $\mathrm{FL}\left(\mathrm{C}_{3}\right)$ at $\mathrm{i}^{\text {th }}$ vertex of $P n$ are (starting with path vertex ) are given by $v_{i 1}=v_{i}, v_{i 2}, v_{i 3}, v_{i 4}, v_{i 2}$. Define a function $f: V(G) \rightarrow\{1,2,3, \ldots p\}$ Where $n$ is the number of vertices on graph $G$ given by $f\left(v_{i j}\right)=4(i-1)+j, j$ $=1,2,3,4 ; i=1,2, . . n$


Fig 3.10 ordinary labeling of $\mathrm{Pn}\left(\mathrm{FL}\left(\mathrm{C}_{3}\right)\right)$


Fig 3.11cnp labeling of $\mathrm{P} 3\left(\mathrm{FL}\left(\mathrm{C}_{3}\right)\right)$
Fig 3.8 One degree vertex is fused with path vertex to obtain structure 1

Structure 2:We start with a path Pn on $n$ vertices given by $\left(v_{1}, v_{2}, \ldots v_{n}\right)$ and $n$ copies of $G$ are taken. A copy each of $G$ is fused at every vertex of Pn. The fusion vertex being the degree threevertex of G. Resultant graph has 5n-1 edges and 5n-1 vertices. The vertices of $\mathrm{FL}\left(\mathrm{C}_{3}\right)$ at $\mathrm{i}^{\text {th }}$ vertex of Pn are (starting with path vertex) are given by $\mathrm{v}_{\mathrm{i} 1}=\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{i} 2}, \mathrm{v}_{\mathrm{i} 3}, \mathrm{v}_{\mathrm{i} 1}, \mathrm{v}_{\mathrm{i} 4}$,. Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{p}\}$ where n is the number of vertices on graph G given by $f\left(v_{i j}\right)=4(i-1)+j, j=1,2,3,4 ; i=1,2, . . n$


Fig 3.12 $\mathrm{Pn}\left(\mathrm{fl}\left(\mathrm{C}_{3}\right)\right)$ with ordinary labeling


Fig 3.14Pn(fl( $\left.\mathrm{C}_{3}\right)$ )with ordinary labeling


Fig 3.13The degree 3 vertex is used to fuse with path vertices.

$$
4(n-1)+1
$$

Note that all pendent vertices have received labels of type $4 \mathrm{x}, \mathrm{x}=1,2, \ldots$
Structure 3:We start with a path Pn on $n$ vertices given by $\left(v_{1}, v_{2}, \ldots v_{n}\right)$ and $n$ copies of $G$ are taken. A copy each of $G$ is fused at every vertex of Pn. The fusion vertex being the one of degree two vertices of G. Using x inplace y or conversely will result in isomorphic structure. Resultant graph has $5 \mathrm{n}-1$ edges and


Fig 3.15Pn( $\left(\mathrm{fl}\left(\mathrm{C}_{3}\right)\right.$ )with ordinary labeling
$5 n-1$ vertices. The vertices of the copy of $\mathrm{FL}\left(\mathrm{C}_{3}\right)$ fused at $\mathrm{i}^{\text {th }}$ vertex of Pn are (starting with end vertex of path ) are given by $\mathrm{v}_{\mathrm{i} 1}=\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{i} 2}, \mathrm{v}_{\mathrm{i} 3}, \mathrm{v}_{\mathrm{i} 1}, \mathrm{v}_{\mathrm{i} 4}$, Define a function $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{1,2,3, \ldots \mathrm{p}\}$ where $n$ is the number of vertices on graph $G$ given by $f\left(v_{i j}\right)=4(i-1)+j, j=1,2,3,4 ; i=1,2, . . n$


Fig 3.16 P2(fl( $\left.\mathrm{C}_{3}\right)$ with cnp labeling


Fig 3.17The 3- degree vertex is fused with path vertex to obtain structure 3

## 5.Conclusions:

We have defined a new type of graph labeling in which vertex label is obtained depending on labels of neighbourhood vertices including the vertex itself.
We have shown that snakes and path union of C3 and that of C4 are cnp .It is necessary to evaluate same for general case Cn . We have also shown the structural invariance under cnp labeling of $\mathrm{Pn}\left(\mathrm{FL}\left(\mathrm{C}_{3}\right)\right)$. Here also it is necessary to evaluate invariance under cnpl of different structures of pathunion flag of Cn

## References :

[1] Bapat M.V. Some vertex prime graphs and a new type of graph labeling Vol 47 part 1 yr2017 pg 23-29 IJMTT
[2] BapatMukund V. Ph.D.Thesis ,University of Mumbai 2004.
[3] Joe Gallian Dynamic survey of graph labeling 2016
[4] Harary,GraphTheory,Narosa publishing ,New Delhi
[5] Jonh Clark, D. A. Holtan Graph theory by allied publisher and world scientist
[6] S. K. Patel and N. P. Shrimali, Neighborhood-prime labeling, Internat. J. Math. and Soft Comput., 5, no. 2, (2015) 135-143.
[7] M. Sundaram, R. Ponraj and S. Somasundaram, EP-cordial labeling of graphs, Varahmihir J. Math. Sci., 7 (2007) 183-194.

