EDGE MEAN LABELING OF A REGULAR GRAPHS

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Abstract

In this paper, we introduce a concept of edge odd and even mean labeling of a regular graph and also investigate the behaviour of some standard graphs.

Key words: Mean labeling, edge odd mean labeling, edge even mean labeling.

AMS Subject Classification :05c

1 Introduction

As a standard notation, assume that $G = G(V, E)$ is a finite, simple and undirected graph with p vertices and q edges. Terms and terminology as in Harary [3]. Mean labeling was introduced by S.Somasundaram and R.Ponraj in [6]. In this paper, we study the edge odd and even mean labeling of a regular graph obtained by joining some standard graphs.
Path on r vertices denoted by P_r and a cycle on r vertices is denoted by C_r. $K_{1,m}$ is called a star and it is denoted by S_m. Regular graph on n degree denoted by Q_n.
Definition 1.1 *A graph labeling is an assignment of integers, to the vertices or edges, or both of a graph. An edge labeling is a function of E to a set of labels. A graph with such a function defined is called a edge labeled graph.*

Definition 1.2 *A graph G with p vertices and q edges is a mean graph if there is an injective function f from the edges of G to $\{0, 1, 2, ...q\}$. Such that when each vertex $u \in V(G)$ is labeled with $\frac{\sum_{n=1}^{k} f(e_n)}{2}$ if $\sum f(e_n)$ is even and $\frac{\sum_{n=1}^{k} f(e_n)+1}{2}$ if $\sum f(e_n)$ is odd. Where $f(e_n)$ is the number of edge labels incident with each vertex $u \in V(G)$. Then the resulting vertices are all distinct.*

Definition 1.3 *A graph G with p vertices to be an edge odd mean graph if there is an injective function f from the edges of G to $\{1, 3, 5, ...2q - 1\}$. Such that when each vertex $u \in V(G)$ is labeled with $\frac{\sum_{n=1}^{k} f(e_n)}{2}$ if $\sum f(e_n)$ is even and $\frac{\sum_{n=1}^{k} f(e_n)+1}{2}$ if $\sum f(e_n)$ is odd. Where $f(e_n)$ is the number of edge labels incident with each vertex $u \in V(G)$. Then the resulting vertices are all distinct. Such a function is called an edge odd mean labeling.*

Definition 1.4 *A graph G with p vertices to be an edge even mean graph if there is an injective function f from the edges of G to $\{2, 4, 6, ...2q\}$. Such that the vertex labels are given by $\frac{\sum_{n=1}^{k} f(e_n)}{2}$ are distinct. Such a function is called an edge even mean labeling.*

Definition 1.5 *The join of graphs k_1 and Q_n, k_1+Q_n is obtained by joining a vertex of k_1 with every vertex of Q_n with an edge.*

Definition 1.6 *The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G (which has p vertices and p copies of G_2) and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.*

Definition 1.7 *The graph $[Q_n; S_2]$ is obtained from n copies of S_2 and the regular $Q_n : u_1, u_2, u_3, ...u_r$ by joining u_j with the vertex v_0 of the j^{th} copy of S_2 by means of an edge $1 \leq j \leq r$.*

2 Edge Odd And Even Mean Labeling

In this section, we prove that the graph Q_n, $k_1 + Q_n$, $P_r + Q_n$, $k_2 + Q_n$, $Q_n + C_r$, $Q_n \odot K_1$, $[Q_n; S_2]$, and $Q_n \odot K_{1,m}$ are edge odd and even mean labeling.

Theorem 2.1 *For $n \geq 3$, the graph Q_n has edge odd mean labeling.*
proof *The graph Q_n has 2^n vertices and $2^{n-1} \cdot n$ edges. where n is the number of edges touching with each vertex. Let $e_1, e_2, e_3, e_4, ...e_r$ be the edges of the graph Q_n. Define an edge labeling $f : E(Q_n) \to \{1, 3, 5, 7, ...2q - 1\}$ by*

$$f(e_i) = \begin{cases} 4i - 1 & \text{if } i = 1, 2, 3, 4, ...r \\ 4i - 3 & \text{if } i = 1, 2, 3, 4, ...r \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph Q_n.

Theorem 2.2 *For $n \geq 3$, the graph Q_n has edge even mean labeling.*
proof *Define an edge labeling $f : E(Q_n) \to \{2, 4, 6, ...2q\}$ by*

$$f(e_i) = \begin{cases} 4i & \text{if } i \text{ is odd and even} \\ 4i - 2 & \text{if } i \text{ is odd and even} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph Q_n.

Theorem 2.3 *For $n \geq 3$, the graph $K_1 + Q_n$ has edge odd mean labeling.*
proof *The graph $K_1 + Q_n$ has $2^n + 1$ vertices and $2^{n-1} \cdot n + 2^n$ edges. where n is the number of edges touching with each vertex. Let $e_1, e_2, e_3, ...e_r$ be the edges of the graph Q_n.*
Define an edge labeling $f : E(K_1 + Q_n) \to \{1, 3, 5, ...2q - 1\}$ by

$$f(e_i) = \begin{cases} 4i - 3 & \text{if } i = 1, 2, 3, ...r \\ 4i - 1 & \text{if } i = 1, 2, 3, ...r \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $k_1 + Q_n$.

Example 2.4 *The edge odd mean labeling of $K_1 + Q_3$ is given in figure1.*

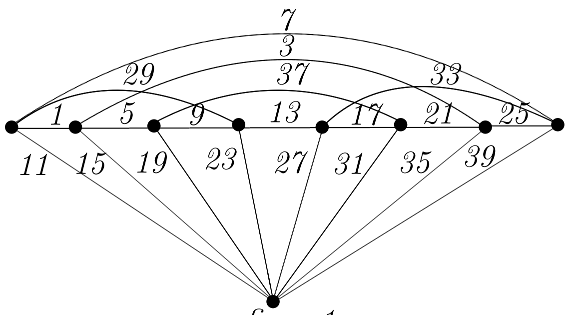

figure1

Theorem 2.5 *For $n \geq 3$, the graph $K_1 + Q_n$ has edge even mean labeling.*
proof *Define an edge labeling $f : E(K_1 + Q_n) \to \{2, 4, 6, ...2q\}$ by*

$$f(e_i) = \begin{cases} 6i - 4 & if\ i = 1, 2, 3, ...r \\ 6i - 2 & if\ i = 1, 2, 3, ...r \\ 3i & if\ i\ is\ even \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $k_1 + Q_n$.

Theorem 2.6 *For $n \geq 3$, the graph $P_r + Q_n$ has edge odd mean labeling.*
proof *The graph $P_r + Q_n$ has $2^n + r$ vertices and $2^{n-1} \cdot n + r \cdot 2^n + (r - 1)$ edges. where n is the number of edges touching with each vertex. Let $e_1, e_2, e_3, ...e_r$ be the edges of the graph $P_r + Q_n$.*
Define an edge labeling $f : E(P_r + Q_n) \to \{1, 3, 5, ...2q - 1\}$ by

$$f(e_i) = \begin{cases} 2i - 1 & if\ i\ is\ odd \\ 2i - 1 & if\ i\ is\ even \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $P_r + Q_n$.

Example 2.7 *The edge odd mean labeling of $P_r + Q_n$ is given in figure2.*

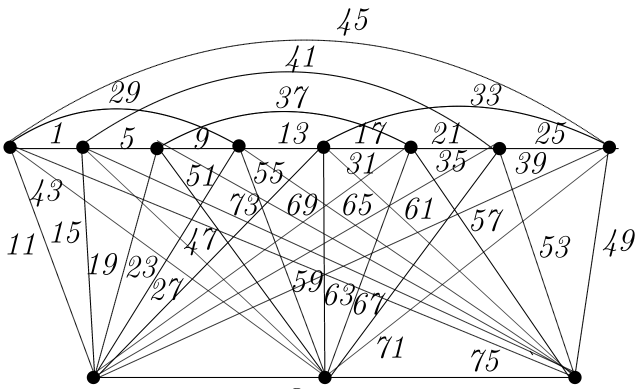

figure 2

Theorem 2.8 *For $n \geq 3$, the graph $P_r + Q_n$ has edge even mean labeling.*
proof *Define an edge labeling $f : E(P_r + Q_n) \to \{2, 4, 6, ...2q\}$ by*

$$f(e_i) = \begin{cases} 4i - 2 & if\ i = 1, 2, 3, 4, ...r \\ 4i & if\ i = 1, 2, 3, 4, ...r \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $P_r + Q_n$.

Theorem 2.9 *For $n \geq 3$, the graph $k_2 + Q_n$ has edge odd mean labeling.*
proof *The graph $K_2 + Q_n$ has $2^n + 2$ vertices and $2^{n-1} \cdot n + 2^{n+1} + 1$ edges. where n is the number of edges touching with each vertex. Let e be an edge of the graph K_2 and let $e_1, e_2, e_3, ...e_r$ be the edges of the graph Q_n.*
Define an edge labeling $f : E(K_2 + Q_n) \rightarrow \{1, 3, 5, ...2q - 1\}$ by

$$f(e) = 1$$

$$f(e_i) = \begin{cases} 4i - 1 & 1 \leq i \leq r \quad \text{if } r \text{ is odd} \\ 2i + 3 & i \text{ is even} \\ 2i + 3 & i \text{ is odd} \qquad \text{if } r \text{ is odd} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $k_2 + Q_n$.

Example 2.10 *The edge odd mean labeling of $K_2 + Q_n$ is given in figure3.*

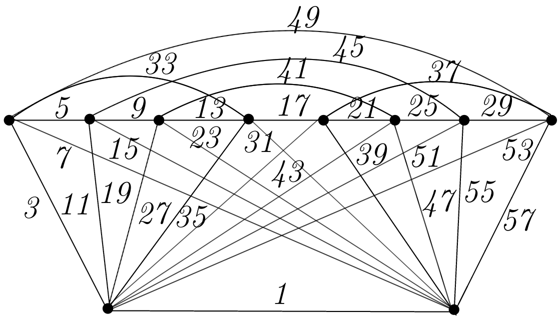

figure 3

Theorem 2.11 *For $n \geq 3$, the graph $K_2 + Q_n$ has edge even mean labeling.*
proof *Define an edge labeling $f : E(K_2 + Q_n) \rightarrow \{2, 4, 6, ...2q\}$ by*

$$f(e) = 2$$

$$f(e_i) = \begin{cases} 4i & 1 \leq i \leq r \quad \text{if } r \text{ is odd} \\ 2i + 4 & i \text{ is odd} \\ 2i + 4 & i \text{ is even} \qquad \text{if } r \text{ is even} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $k_2 + Q_n$.

Theorem 2.12 *For $n \geq 3$, the graph $Q_n + C_r$ has edge odd mean labeling.*
proof *The graph $Q_n + C_r$ has $2^n + r$ vertices and $2^{n-1} \cdot n + r \cdot 2^n + r$ edges. where n is the number of edges touching with each vertex. Let $e_1, e_2, e_3, ...e_r$ be the edges of the graph $Q_n + C_r$. Define an edge labeling $f : E(Q_n + C_r) \rightarrow \{1, 3, 5, ...2q - 1\}$ by*

$$f(e_i) = \begin{cases} 4i - 1 & if \ \ i = 1, 2, 3, ...r \\ 4i - 3 & if \ \ i = 1, 2, 3, ...r \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $Q_n + C_r$.

Example 2.13 *The edge odd mean labeling of $Q_3 + C_3$ is given in figure4.*

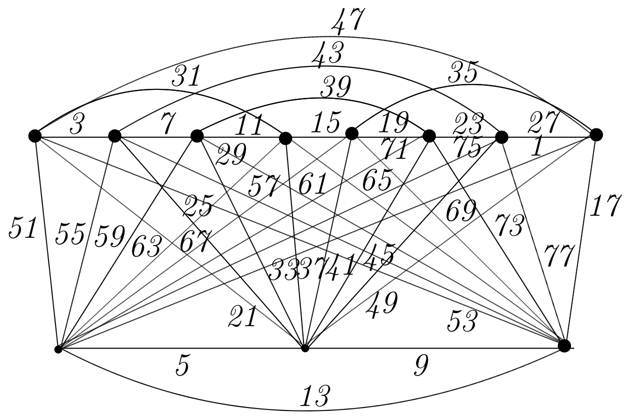

figure 4

Theorem 2.14 *For $n \geq 3$, the graph $Q_n + C_r$ has edge even mean labeling.*
proof *Define an edge labeling $f : E(Q_n + C_r) \rightarrow \{2, 4, 6, ...2q\}$ by*

$$f(e_i) = \begin{cases} 4i - 2 & if \ \ i = 1, 2, 3, ...r \\ 4i & if \ \ i = 1, 2, 3, ...r \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $Q_n + C_r$.

Theorem 2.15 *For $n \geq 3$, the graph $Q_n \odot K_1$ has edge odd mean labeling.*
proof *The graph $Q_n \odot K_1$ has 2^{n+1} vertices and $2^{n-1} \cdot n + 2^n$ edges. where n is the number of edges touching with each vertex. Let $e_1, e_2, e_3, ...e_r$ be the edges of the graph $Q_n \odot K_1$.*
Define an edge labeling $f : E(Q_n \odot K_1) \rightarrow \{1, 3, 5, ...2q - 1\}$ by

$$f(e_i) = \begin{cases} 6i-5 & \text{if } i \text{ is odd and even} \\ 6i-3 & \text{if } i \text{ is odd and even} \\ 6i-1 & \text{if } i \text{ is odd and even} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $Q_n \odot K_1$.

Example 2.16 *The edge odd mean labeling of* $Q_3 \odot K_1$ *is given in figure5.*

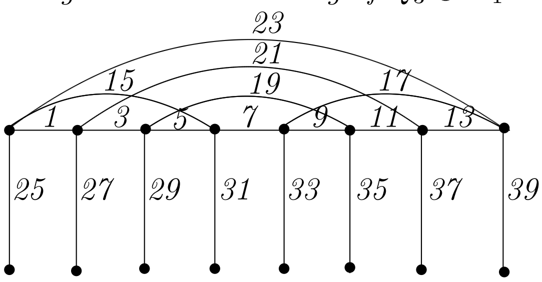

figure 5

Theorem 2.17 *For* $n \geq 3$, *the graph* $Q_n \odot K_1$ *has edge even mean labeling.*
proof *Define an edge labeling* $f : E(Q_n \odot K_1) \to \{2, 4, 6, ...2q\}$ *by*

$$f(e_i) = \begin{cases} 6i-4 & \text{if } i \text{ is odd and even} \\ 6i-2 & \text{if } i \text{ is odd and even} \\ 6i & \text{if } i \text{ is odd and even} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $Q_n \odot K_1$.

Theorem 2.18 *For* $n \geq 3$, *the graph* $[Q_n; S_2]$ *has edge odd mean labeling.*
proof *The graph* $[Q_n; S_2]$ *has* $2^n + 2^{n+1}$ *vertices and* $2^{n-1} \cdot n + 2^{n+1}$ *edges. where n is the number of edges touching with each vertex. Let* $e_1, e_2, e_3, ...e_r$ *be the edges of the graph* $[Q_n; S_2]$.
Define an edge labeling $f : E([Q_n; S_2]) \to \{1, 3, 5, ...2q-1\}$ *by*

$$f(e_i) = \begin{cases} 4i-3 & \text{if } i \text{ is odd and even} \\ 4i-1 & \text{if } i \text{ is odd and even} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $[Q_n; S_2]$.

Example 2.19 *The edge odd mean labeling of* $[Q_3; S_2]$ *is given in figure6.*

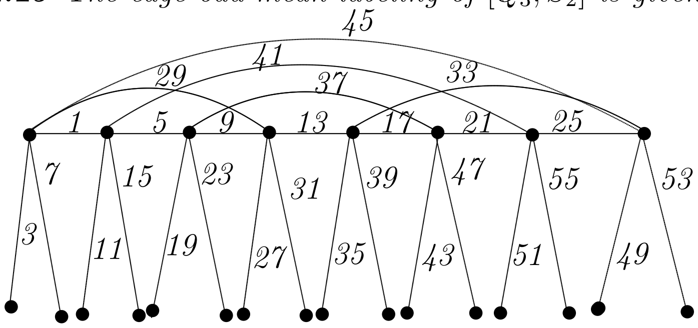

figure 6

Theorem 2.20 *For* $n \geq 3$*,the graph* $[Q_n; S_2]$ *has edge even mean labeling.*
proof *Define an edge labeling* $f : E([Q_n; S_2]) \to \{2, 4, 6, ...2q\}$ *by*

$$f(e_i) = \begin{cases} 4i - 2 & if \ \ i = 1, 2, 3, ...r \\ 4i & if \ \ i = 1, 2, 3, ...r \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $[Q_n; S_2]$.

Theorem 2.21 *For* $n, m \geq 3$*, the graph* $Q_n \odot K_{1,m}$ *has edge odd mean labeling.*
proof *The graph* $Q_n \odot K_{1,m}$ *has* $2^n + m \cdot 2^n$ *vertices and* $2^{n-1} \cdot n + m \cdot 2^n$ *edges. where n is the number of edges touching with each vertex. Let* $e_1, e_2, e_3, ...e_r$ *be the edges of the graph* $Q_n \odot K_{1,m}$.
Define an edge labeling $f : E(Q_n \odot K_{1,m}) \to \{1, 3, 5, ...2q - 1\}$ *by*

$$f(e_i) = \begin{cases} 8i - 7 & if \ \ i = 1, 2, 3, ...r \\ 8i - 5 & if \ \ i \ is \ odd \ and \ even \\ 8i - 3 & if \ \ i = 1, 2, 3, ...r \\ 8i - 1 & if \ \ i \ is \ odd \ and \ even \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge odd mean labeling for the graph $Q_3 \odot K_{1,3}$.

Example 2.22 *The edge odd mean labeling of* $Q_3 \odot K_{1,3}$ *is given in figure7.*

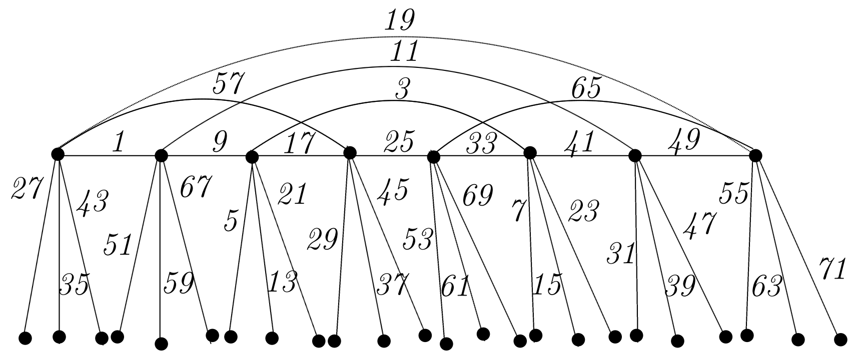

figure 7

Theorem 2.23 *For $n, m \geq 3$, the graph $Q_n \odot K_{1,m}$ has edge even mean labeling.*
proof *Define an edge labeling $f : E(Q_n \odot K_{1,m}) \rightarrow \{2, 4, 6, ...2q\}$ by*

$$f(e_i) = \begin{cases} 8i - 6 & \textit{if } i = 1, 2, 3, ...r \\ 8i - 4 & \textit{if } i \textit{ is odd and even} \\ 8i - 2 & \textit{if } i = 1, 2, 3, ...r \\ 8i & \textit{if } i \textit{ is odd and even} \end{cases}$$

Therefore mean of the vertex labels are all distinct. Hence the function f provides edge even mean labeling for the graph $Q_3 \odot K_{1,3}$.

3 Conclusion

The main focus of this paper is resolving the edge mean labeling of a regular graphs for some standard graphs.

4 Acknowledgement

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