The Kumar – Lagrenges Formula

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ABSTRACT *- In this article, I will state the relation between f, b and c where,*

f: Function.

b: Point of open interval and closed interval i.e (a,b) and [a,b] respectively.

c: Interior point of (a,b).

KEYWORDS *- Open interval & closed intervals, Interior point and Complex function i.e. x + iy (where 'x' is the real part and 'y' is the imaginary part of the complex number).*

INTRODUCTION

In this article, I am giving a formula to determine the interior point 'c', where 'c' belongs to the open interval (a,b) for any function 'f' which is continuous on closed interval [a,b] and differentiable at 'c'. Therefore, Kumar - Lagrenges formula is given by:

$$f'(c) * b = f(b)$$

where, c belongs to the open interval (a,b).

INDENTATIONS AND EQUATIONS

Let 'f' be a function where, 'f' is continuous on closed interval [a,b], differentiable on open interval and 'c' belongs to the open interval (a,b). Then,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Which is known as, Lagrenges mean value theorem.

Now, consider,

⇨ $f'(c) = \frac{f(b) - f(a)}{b - a}$

⇨ $f'(c) * (b - a) = f(b) - f(a)$.

Now, on taking $\log_e$ on both sides we get:

⇨ $\log_e \{ f'(c) * (b - a) \} = \log_e \{ f(b) - f(a) \}$.

On applying logarithmic properties:

⇨ $\log_e f'(c) + \log_e (b - a) = \log_e \{ f(b) - f(a) \}$.

⇨ $\log_e f'(c) + \log_e [b(1 - a / b)] = \log_e \{ f(b) - f(a) \}$.

⇨ $\log_e f'(c) + \log_e b + \log_e (1 - a/b) = \log_e \{ f(b) - f(a) \}$.

⇨ $\log_e f'(c) + \log_e b + \log_e (1 - a/b) = \log_e [f(b) \{ 1 - f(a) / f(b) \}]$.

⇨ $\log_e f'(c) + \log_e b + \log_e (1 - a/b) = \log_e f(b) + \log_e \{ 1 - f(a) / f(b) \}$. ………(1)

Where $a < b$

And $f(a) < f(b)$

Now, consider:

$Log_e(1 - a/b) = -a / b - 1/2*(a/b)^2 - 1/3*(a/b)^3......$ .....(2)

Also consider:

$Log\{1 - f(a)/f(b)\} = - f(a)/f(b) - 1/2*\{f(a)/f(b)\}^2 - 1/3*\{f(a)/f(b)\}^3.....$ .....(3)

Now, substituting values of equations (2) and (3) in equation (1),

⇨ $Log_e * f'(c) + Log_e * b + \{- a / b - 1/2*(a/b)^2 - 1/3*(a/b)^3.....\} = Log_e * f(b) + \{- f(a)/f(b) - 1/2*\{ f(a) / f(b) \}^2 - 1/3*\{ f(a) /f(b) \}^3.....\}.$

⇨ $Log_e * f'(c) + Log_e * b - Log_e * f(b) = - \{ f(a)/f(b) + 1/2*\{ f(a) / f(b) \}^2 + 1/3*\{ f(a) /f(b) \}^3 + ...\} + \{ a / b + 1/2*(a/b)^2 + 1/3*(a/b)^3 + ...\}$

Using logarithmic properties, the above equation becomes:

⇨ $Log \dfrac{\{f'(c) * b\}}{f(b)} = - \{ f(a)/f(b) + 1/2*\{ f(a) / f(b) \}^2 + 1/3*\{ f(a) /f(b) \}^3 + ...\} + \{a / b + 1/2*(a/b)^2 + 1/3*(a/b)^3 + ...\}.$ .....(4)

Now put $[f(a) / f(b)] = K_1$ and $[a / b] = K_2$.

Therefore equation (4) becomes:

⇨ $Log \dfrac{\{f'(c) * b\}}{f(b)} = \{ K_2 + 1/2*K_2^2 + 1/3 *K_2^3 + ...\} - \{ K_1 + 1/2*K_1^2 + 1/3*K_1^3 + ...\}$

⇨ $Log \dfrac{\{f'(c) * b\}}{f(b)} = (k_2 - k_1) + 1/2* (k_2^2 - k_1^2) + 1/3*(k_2^3 - k_1^3) + ...$

Now, let:

$$(k_2 - k_1) + 1/2* (k_2^2 - k_1^2) + 1/3*(k_2^3 - k_1^3) + ... = P$$

Therefore, the above equation becomes:

$$Log \frac{\{f'(c) * b\}}{f(b)} = P$$

Now taking exponential on both sides of the above equation, we get:

$$\frac{f'(c) * b}{f(b)} = e^P \quad .....(5)$$

By the Euler's function, we know that

$$e^{ip} = \cos(p) + i \sin(p)$$

OR

$$e^{-P} = \cos(ip) + i \sin(ip) \quad .....(6)$$

$$e^{-ip} = cos(p) - i \sin(p) \quad .... (7)$$

$$e^{p} = \cos(ip) - i \sin(ip) \quad .... (8)$$

Therefore, substituting the value of e^p from equation (8) in (5), we get:

$$\frac{f'(c) * b}{f(b)} = \cos(ip) - i \sin(ip)$$

On comparing the real and imaginary parts, we get:

$$\frac{f'(c) * b}{f(b)} = \cos(ip) = \cos(hp) \quad .....(9)$$

After comparing the real and imaginary part in relation (9) we get:

$$- \sin(ip) = 0$$
$$\Rightarrow \sin(ip) = 0$$
$$\Rightarrow ip = 0$$

This implies that:

$$p = 0$$

Now, replacing $p = 0$ in equation (7), we get

$$\frac{f'(c) * b}{f(b)} = \cos(0)$$

$$\Rightarrow \frac{f'(c) * b}{f(b)} = 1.$$

$$\boxed{\Rightarrow f'(c) * b = f(b)}$$

Which is the required equation known as "The Kumar – Lagrenges formula".

CONCLUSION

"The Kumar - Lagrenges formula" is used to determine the interior point of any open interval (a ,b) for any differentiable function ' f ' at point c. The interior value (c) is not depend "a" and "b" at a time.

REFERENE

In this article, I have used the Lagrenges mean value theorem which I have studied in Fy – B.sc Calculus paper. This books is as per the new revised syllabus of University of Pune, June – 2008.