

# Hydromagnetic Oscillatory Flow and Heat Transfer in Dusty Viscoelastic Fluid Through Porous Medium in an Inclined Channel in The Presence of Thermal Radiation and Heat Sink

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**Abstract:** *In this study, the unsteady magnetohydrodynamic (MHD) flow and heat transfer of a dusty, viscoelastic, incompressible and electrically conducting fluid in an inclined channel in the presence of thermal radiation and heat source has been investigated. The non-dimensional equations governing the flow are solved by parametric perturbation technique under the prescribed boundary conditions. The solutions of the equations governing the flow are obtained for fluid velocity, dust particle velocity, temperature and dust particle temperature profile. The results have been expressed graphically and in tabular form to observe the effects of different flow parameters. The effect of the various parameters entering in the governing equations on the flow are evaluated numerically and discussed with the help of graphs and tables.*

**Keywords:** *Magnetohydrodynamics,, inclined channel, dusty and viscoelastic fluids.*

**Mathematical Subject Classification:** 76A10, 76D99, 76S99, 76W99.

## Introduction

Magnetohydrodynamics (MHD) convection flows of electrically conducting viscous incompressible fluids have gained significance in recent times owing to its applications in physics and engineering. In view of increasing technical applications using Magnetohydrodynamic effect, it is desirable to extend many of available viscous hydrodynamic solution to include the effect of magnetic field for those cases when the viscous fluid is electrically conducting. The study of heat transfer effects on magnetohydrodynamics (MHD) free convection flow in porous medium is also highly important due to the significant contribution of magnetic field on the performance of many systems using electrically conducting fluids and on the boundary layer flow control. The magneto hydrodynamic heat transfer has gained considerable attention because of its applications in recent advancement of space technology. Greenspan and Carrier [1] have studied the Magnetohydrodynamics (MHD) flow past a flat plate. Unsteady MHD convective heat transfer past a semi-infinite vertical porous plate with variable suction have been investigated by Kim[2]. Attia and Kotab [3] have investigated the Magnetohydrodynamics (MHD) flow between two parallel plates with heat transfer. The effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation have been studied by Seddeek [4]. The effect of heat transfer to MHD oscillatory flow in a channel filled with a porous medium have been investigated by Makinde and Mhone [5]. Seth et.al. [6] have discussed the unsteady MHD convective flow within a parallel plate rotating channel with thermal source/sink in a porous medium under slip boundary conditions. MHD flow through rotating porous medium with radiating heat transfer in the presence of fluctuating thermal diffusion has studied by Sharma and Saini [7]. Singh [8] investigated the effect of injection/ suction on convective oscillatory flow through porous medium bounded by two vertical porous plates. The numerical study of convective heat transfer through porous medium in a vertical channel with radiation effect have been dealt by Kumar and Nath [9]. Abbas et.al.[10] analysed the heat transfer due to an unsteady stretching/shrinking cylinder with partial slip condition and suction. Heat transfer with radiation and

temperature dependent heat source in MHD free convection flow in a porous medium between two vertical wavy walls has been discussed by Dada and Disu [11].

All the above investigations are free from viscoelastic fluids. The heat transfer behaviour of viscoelastic fluid between parallel plates is of great interest in many fields, like; petroleum production, food preservation and power engineering. The effect of injection on the flow of second grade fluid in the inlet region of a channel has discussed by Mishra and Panda [13]. The flow viscoelastic fluid past a porous plate has been analyzed by Ariel [14]. Maneschy et. al [15] also studied the heat transfer analysis of a non-Newtonian fluid past a porous plate. Kurtcebe and Erim [16] have characterized the heat transfer of a visco-elastic fluid in a porous channel. Unsteady Couette flow with heat transfer in a visco elastic fluid considering the Hall Effect has been investigated by Hazem [17]. Sharma and Pareek [18] studied the effect of unsteady flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free stream and suction. Visco elastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel has been studied by Singh [19]. Choudhary and Jyoti [20] discussed the heat transfer to MHD oscillatory visco elastic flow in a channel filled with porous medium. The effects of variable viscosity on the peristaltic flow of non-newtonian fluid through a porous medium in an inclined channel with slip boundary conditions have been investigated by Khan and Usman [21]. Garg et.al. [22] give the analysis of an oscillatory MHD convective flow of viscoelastic fluid through porous medium filled in a rotating vertical porous channel with heat radiation.

In the present paper we consider the dusty viscoelastic fluid. A dusty viscoelastic fluid is the mixture of viscoelastic fluid and dust particles. The dusty fluid model is important due to its application in paper industry, industrial filtration, ceramic engineering, power metallurgy, smoke emission from vehicles, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplets sprays. The volume of dust particles is generally assumed to be negligible in dynamics of dusty fluid. The flow and heat transfer of dusty fluids in a channel are extremely useful in improving the design and operation of many industrial and engineering devices. The phenomenon of flow and heat transfer in dusty fluids in two parallel plates has been analyzed by many researchers. Saffman [23] discussed on the stability of laminar flow of a dusty gas. Flow of a dusty gas through a channel with arbitrary time varying pressure gradient has been analyzed by Gupta and Gupta [24]. Unsteady flow of a conducting dusty fluid through a rectangular channel with time dependent pressure gradient has been studied by Singh [25]. Further Prasad and Ramacharyulu [26] investigated the unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient. Attia [27] studied the influence of temperature dependent viscosity on MHD channel flow of dusty fluid with heat transfer. Attia [28] discussed the unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. Influence of temperature-dependent viscosity on the MHD Couette flow of dusty fluid with heat transfer have been also studied by Attia [29]. Palani and Ganesan [30] explored the heat transfer effect on dusty gas flow past a semi infinite inclined plate. Makinde and Chinyoka [31] worked on MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition. Heat transfer in MHD oscillatory flow of dusty fluid in a rotating porous vertical channel has been investigated by Chand et.al. [33]. Attia et.al. [34] discussed the MHD flow of a dusty fluid between two infinite parallel plates with temperature dependent physical properties under exponentially decaying pressure gradient. Jalil et.al. [35] discussed an exact solution of MHD boundary layer flow of dusty fluid over a stretching surface.

Motivated by above studies we propose to study the problem of unsteady hydromagnetic oscillatory flow and heat transfer in an electrically conducting incompressible, dusty viscoelastic fluid through porous medium in an inclined channel in the presence of magnetic field, thermal radiation and heat source/sink effect.

#### ***Mathematical Formulation of the problem***

Consider an unsteady free convective flow of an incompressible, electrically conducting, radiating, dusty viscoelastic, fluid through a porous inclined channel under the effect of transversely applied magnet field of constant strength. The coordinate system is considered with x-axis along the centre of the channel and y-axis perpendicular to it. The following assumptions have been made

1. A uniform magnetic field is applied normal to the planes of the plates.

2. Boussinesq approximation is applied.
3. The dust particles are spherical in shape are uniformly distributed.
4. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field may be neglected.
5. The plates of the channel is assumed to be non conducting and kept at two different temperature.
6. The initial temperature of the fluid and the dust particle is assumed to be the same.
7. The heat due to viscous dissipation and Joule heating has been neglected.
8. The size of the dust particle has be assumed to be large as compare to the pore size of the channel.

The channel plate are considered to be infinitely long and the problem under consideration is one dimensional with velocity of fluid and dust particles as the function of the space coordinate  $y^*$  and time  $t^*$ . Under the above assumptions, the flow equations proposed by Walter [12], for the fluid known as Walter’s liquid-B for viscoelastic fluid and following Chand and Kumar[32]. The governing equations in case one dimensional takes the form

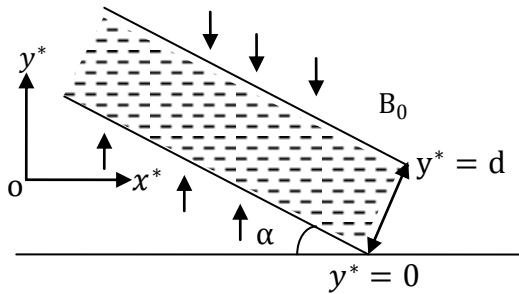


Figure.1. Geometrical interpretation of the problem.

**Equation of continuity**

$$\frac{\partial v^*}{\partial y^*} = 0, \Rightarrow v^* = \text{constant} \tag{1}$$

**Equation of motion**

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{k_0}{\rho} \left( \frac{\partial^3 u^*}{\partial y^{*2} \partial t^*} \right) + \frac{k_1 N_1}{\rho} (u_p^* - u^*) - \frac{\nu}{K^*} u^* + g\beta(T^* - T_1^*) \sin \alpha - \frac{\sigma B_0^2 u^*}{\rho}, \tag{2}$$

$$m_p \frac{\partial u_p^*}{\partial t^*} = K_1 N_1 (u^* - u_p^*), \tag{3}$$

**Equation of energy**

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q^*}{\partial y^*} - \frac{Q^*}{\rho C_p} (T^* - T_0^*) + \frac{\rho_p c_s}{\rho C_p \gamma_T} (T_p^* - T^*) \tag{4}$$

$$\frac{\partial T_p^*}{\partial t^*} = -\frac{(T_p^* - T^*)}{\gamma_T}, \tag{5}$$

**The boundary conditions relevant to the problem are**

$$\left. \begin{aligned} \text{at } y^* = 0 : u^* = u_p^* = 0, T^* = T_p^* = 0, \\ \text{at } y^* = d : u^* = u_p^* = U_0(1 + \epsilon e^{i\omega^* t^*}), \\ T^* = T_F^* + \epsilon(T_F^* - T_0^*)e^{i\omega^* t^*} \end{aligned} \right\} \tag{6}$$

Using Cogley et.al.[36], the radiative heat flux term  $q^*$  is given by

$$\frac{\partial q^*}{\partial y^*} = 4\alpha^2(T^* - T_0^*). \tag{7}$$

Where  $u^*$  is the velocity of the fluid,  $u_p^*$  is the velocity of the dust particles,  $T^*$  is the temperature of the fluid,  $T_p^*$  is the temperature of the dust particles,  $c_p$  is the specific heat capacity of the fluid at constant pressure,  $c_s$  is the specific heat capacity of the dust particles,  $k$  is the thermal conductivity of the fluid,  $q^*$  is the radiative heat flux,  $Q^*$  is the heat absorption coefficient,  $\gamma_T = \frac{3P_r\gamma_p c_s}{2c_p}$  is the temperature relaxation time,  $\gamma_p = \frac{2\rho_p d^2}{9\rho\nu}$  is velocity relaxation time

and  $P_r = \frac{\nu\rho C_p}{k}$  is the Prandtl Number.

Introducing the following non-dimensional quantities

$$x, y = \frac{x^*, y^*}{d}, u = \frac{u^*}{U_0}, u_p = \frac{u_p^*}{U_0}, \omega = \frac{d\omega^*}{U_0}, t = \frac{t^* U_0}{d}, \theta = \frac{T^* - T_0^*}{T_F^* - T_0^*}, \theta_p = \frac{T_p^* - T^*}{T_F^* - T_0^*}, R_e = \frac{U_0 d}{\nu}, H = \frac{\nu m_p}{k_1 d^2}, P_r = \frac{\nu\rho C_p}{k}, R = \frac{k_1 N_1 d^2}{\rho\nu}$$

$$K_p = \frac{d^2}{K^*}, Q = \frac{Q_0 d^2}{k}, N^2 = \frac{4\alpha^2 d^2}{k}, p = \frac{p^* d}{\nu\rho U_0}, G_r = \frac{g\beta(T_F^* - T_0^*)d^2}{\nu U_0}, M^2 = \frac{d^2 \sigma B_0^2}{\nu\rho}, K_v = \frac{k_0}{\rho d^2}, L_0 = \frac{d}{U_0 \gamma_T}.$$

Using above non-dimensional quantities, the governing equations reduces to

$$R_e \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + R_e K_v \frac{\partial^3 u}{\partial y^2 \partial t} + R(u_p - u) - (M^2 + K_p)u + G_r \sin \alpha, \tag{8}$$

$$H R_e \frac{\partial u_p}{\partial t} = u - u_p, \tag{9}$$

$$R_e P_r \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} - (N^2 + Q)T + \frac{2R}{3}(T_p - T), \tag{10}$$

$$\frac{\partial T_p}{\partial t} = -L_0(T_p - T), \tag{11}$$

And the corresponding boundary conditions in non dimensional form are

$$\left. \begin{aligned} \text{at } y = 0 : u = u_p = 0, T = T_p = 0, \\ \text{at } y = 1 : u = u_p = 1 + \epsilon e^{i\omega t}, \\ T = T_p = 1 + \epsilon e^{i\omega t} \end{aligned} \right\} \tag{12}$$

**Method of solution**

Following Sengupta and Ahmed [37], we assume the solution of the form

$$\left. \begin{aligned} u(y, t) = u_0(y) + \epsilon e^{i\omega t} u_1(y) + o(\epsilon^2) \\ u_p(y, t) = \psi_0(y) + \epsilon e^{i\omega t} \psi_1(y) + o(\epsilon^2) \\ T(y, t) = T_0(y) + \epsilon e^{i\omega t} T_1(y) + o(\epsilon^2) \\ T_p(y, t) = \xi_0(y) + \epsilon e^{i\omega t} \xi_1(y) + o(\epsilon^2) \\ -\frac{\partial P}{\partial x} = A(1 + \epsilon e^{i\omega t}) \end{aligned} \right\} \tag{13}$$

The zero order equations are given by the following expression

$$\frac{\partial^2 u_0}{\partial y^2} - (M^2 + K_p)u_0 + R(\psi_0 - u_0) = -G_r \sin \alpha - A, \tag{14} \quad u_0 - \psi_0 =$$

$$0, \tag{15} \quad \frac{\partial^2 T_0}{\partial y^2} - (N^2 + Q)T_0 - \frac{2R}{3}(\xi_0 - T_0) = 0, \tag{16}$$

$$\xi_0 - T_0 = 0, \tag{17} \quad \text{The}$$

transformed boundary conditions

$$\left. \begin{aligned} \text{at } y = 0 : u_0 = \psi_0 = 0, T_0 = \xi_0 = 0, \\ \text{at } y = 1 : u_0 = \psi_0 = 1, T_0 = \xi_0 = 1 \end{aligned} \right\} (18)$$

The first order equations are

$$\frac{\partial^2 u_1}{\partial y^2} - \frac{i\omega}{1+i\omega R_e K_v} \left[ \frac{H R R_e}{1+i\omega H R_e} + \frac{M^2 + K_p}{i\omega} + R_e \right] u_1 = \frac{-G_r \sin \alpha - A}{1+i\omega R_e K_v} (19)$$

$$i\omega R_e H \psi_1 = u_1, (20) \quad \frac{\partial^2 T_1}{\partial y^2} -$$

$$\left\{ (N^2 + Q) - i\omega \left( \frac{2R}{3(L_0 + i\omega)} + R_e P_r \right) \right\} T_1 = 0, (21) \quad \xi_1 (L_0 + i\omega) =$$

$L_0 T_1, (22)$   
 Subjected to the transformed boundary conditions

$$\left. \begin{aligned} \text{at } y = 0 : u_1 = \psi_1 = 0, T_1 = \xi_1 = 0, \\ \text{at } y = 1 : u_1 = \psi_1 = 1, T_1 = \xi_1 = 1 \end{aligned} \right\} (23)$$

The solutions of

the zero order equations (14) to (17) under the boundary conditions (18) are obtained in the following form

$$u_0 = A_3 e^{a_3 y} + B_3 e^{-a_3 y} + X_1 e^{a_1 y} + Y_1 e^{-a_1 y} + Z_1, (24)$$

$$\psi_0 = u_0, (25) \quad T_0 =$$

$$A_1 e^{a_1 y} + B_1 e^{-a_1 y}, (26)$$

$$\xi_0 = T_0, (27) \quad \text{The}$$

solutions of the first order equations from (19) to (22) under the boundary conditions (23) are obtained and can be listed as below

$$u_1 = A_4 e^{a_4 y} + B_4 e^{-a_4 y} + X_2 e^{a_2 y} + Y_2 e^{-a_2 y} + Z_2, (28) \quad \psi_1 =$$

$$\frac{u_1}{(1+i\omega R_e H)}, (29) \quad T_1 = A_2 e^{a_2 y} +$$

$$B_2 e^{-a_2 y}, (30)$$

$$\xi_1 = \left( \frac{L_0}{L_0 + i\omega} \right) T_1 (31) \quad \text{The solutions of the}$$

equations (8) to (11) under the boundary conditions (12) are finally obtained the expression for the fluid velocity, dust particle velocity, fluid temperature and dust particle temperature obtained as below.

$$u = (A_3 e^{a_3 y} + B_3 e^{-a_3 y} + X_1 e^{a_1 y} + Y_1 e^{-a_1 y} + Z_1) + \varepsilon e^{i\omega t} (A_4 e^{a_4 y} + B_4 e^{-a_4 y} + X_2 e^{a_2 y} + Y_2 e^{-a_2 y} + Z_2), (32)$$

$$u_p = (A_3 e^{a_3 y} + B_3 e^{-a_3 y} + X_1 e^{a_1 y} + Y_1 e^{-a_1 y} + Z_1) + \frac{\varepsilon e^{i\omega t}}{(1+i\omega R_e H)} (A_4 e^{a_4 y} + B_4 e^{-a_4 y} + X_2 e^{a_2 y} + Y_2 e^{-a_2 y} + Z_2), (33)$$

$$T = (A_1 e^{a_1 y} + B_1 e^{-a_1 y}) + \varepsilon e^{i\omega t} (A_2 e^{a_2 y} + B_2 e^{-a_2 y}), (34)$$

$$T_p = (A_1 e^{a_1 y} + B_1 e^{-a_1 y}) + \varepsilon e^{i\omega t} \left( \frac{L_0}{L_0 + i\omega} \right) (A_2 e^{a_2 y} + B_2 e^{-a_2 y}), (35)$$

**The significant physical quantities of importance are as follows:**

Skin friction coefficient( $\tau$ ) for fluid velocity and dust particle velocity at the lower plate of the channel are:

$$\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

$$\tau = [a_3(A_3 - B_3) + a_1(X_1 - Y_1)] + \varepsilon e^{i\omega t} [a_4(A_4 - B_4) + a_2(X_2 - Y_2)], \quad (36)$$

$$\tau_p = \left( \frac{\partial u_p}{\partial y} \right)_{y=0}$$

$$\tau_p = [a_3(A_3 - B_3) + a_1(X_1 - Y_1)] + \frac{\varepsilon e^{i\omega t}}{(1+i\omega R_e H)} [a_4(A_4 - B_4) + a_2(X_2 - Y_2)], \quad (37)$$

$$\tau = \left( \frac{\partial u}{\partial y} \right)_{y=1}$$

$$\tau = [a_3(A_3 e^{a_3} - B_3 e^{-a_3}) + a_1(X_1 e^{a_1} - Y_1 e^{-a_1})] + \varepsilon e^{i\omega t} [a_4(A_4 e^{a_4} - B_4 e^{-a_4}) + a_2(X_2 e^{a_2} - Y_2 e^{-a_2})], \quad (38)$$

$$\tau_p = \left( \frac{\partial u_p}{\partial y} \right)_{y=1}$$

$$\tau_p = [a_3(A_3 e^{a_3} - B_3 e^{-a_3}) + a_1(X_1 e^{a_1} - Y_1 e^{-a_1})] + \frac{\varepsilon e^{i\omega t}}{(1+i\omega R_e H)} [a_4(A_4 e^{a_4} - B_4 e^{-a_4}) + a_2(X_2 e^{a_2} - Y_2 e^{-a_2})], \quad (39)$$

Nusselt Number (Nu) for fluid temperature and dust particle temperature at the lower plate of the channel are:

$$Nu = \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

$$Nu = [a_1(A_1 - B_1)] + \varepsilon e^{i\omega t} [a_2(A_2 - B_2)], \quad (40)$$

$$Nu_p = \left( \frac{\partial T_p}{\partial y} \right)_{y=0}$$

$$Nu(p) = [a_1(A_1 - B_1)] + \varepsilon e^{i\omega t} \left( \frac{L_0}{L_0 + i\omega} \right) [a_2(A_2 - B_2)], \quad (41)$$

$$Nu = \left( \frac{\partial T}{\partial y} \right)_{y=1}$$

$$Nu = [a_1(A_1 e^{a_1} - B_1 e^{-a_1})] + \varepsilon e^{i\omega t} [a_2(A_2 e^{a_2} - B_2 e^{-a_2})], \quad (42)$$

$$Nu_p = \left( \frac{\partial T_p}{\partial y} \right)_{y=1}$$

$$Nu(p) = [a_1(A_1 e^{a_1} - B_1 e^{-a_1})] + \varepsilon e^{i\omega t} \left( \frac{L_0}{L_0 + i\omega} \right) [a_2(A_2 e^{a_2} - B_2 e^{-a_2})], \quad (43)$$

The all constant used above have been list in the appendix.

### **Results and discussions:**

The following discussion brings out the effect of the pertinent parameters such as permeability parameter ( $K_p$ ), magnetic field parameter ( $M$ ), radiation parameter ( $N$ ) and angle of inclination ( $\alpha$ ) on the fluid and dust particle velocity. Figure 2 & 6, 3 & 7 and 4 & 8 shows the effect of permeability parameter ( $K_p$ ), magnetic field parameter ( $M$ ) and radiation parameter ( $N$ ) on fluid velocity and dust particle velocity, both the fluid velocity and dust particle velocity decreases with the increasing values of permeability parameter ( $K_p$ ), magnetic field parameter ( $M$ ) and radiation parameter ( $N$ ). The effect of angle of inclination ( $\alpha$ ) on fluid velocity and dust particle velocity has been obtained in figures 5 & 9. The fluid and dust particle velocity increases with the increase in angle of inclination ( $\alpha$ ).

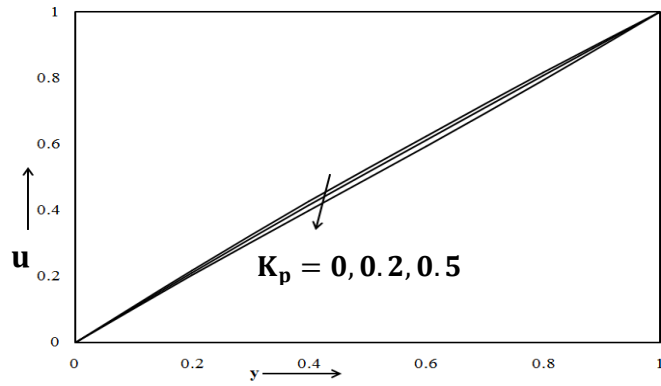


Figure.2. Fluid velocity variation for different values of permeability parameter ( $K_p$ ).

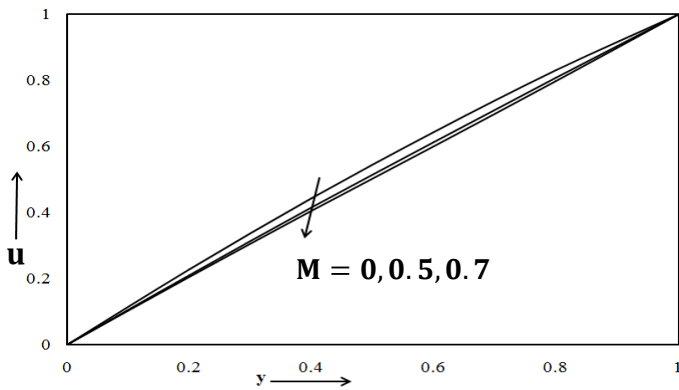


Figure.3. Fluid velocity variation for different values of magnetic field parameter ( $M$ ).

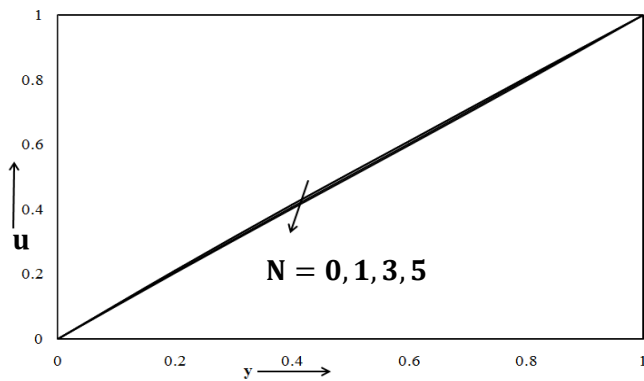


Figure.4. Fluid velocity variation for different values of Hartmann number ( $N$ ).

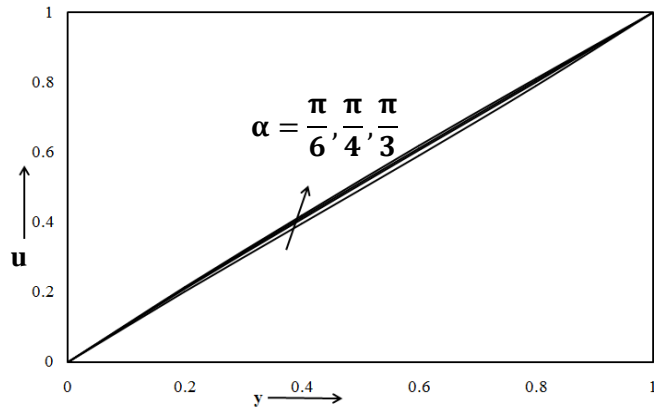


Figure.5. Fluid velocity variation for different values of angle ( $\alpha$ ).

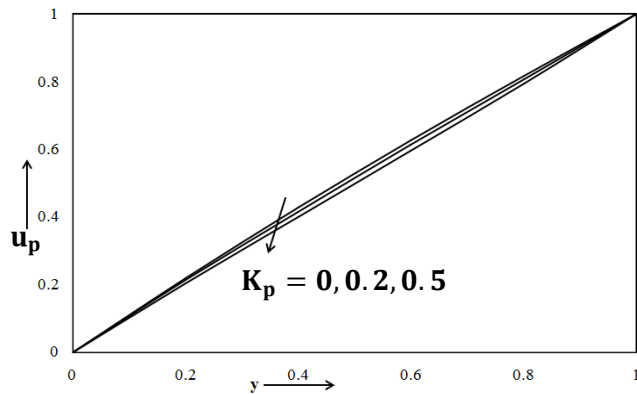


Figure.6. Dust particle velocity variation for different values of permeability parameter ( $K_p$ ).

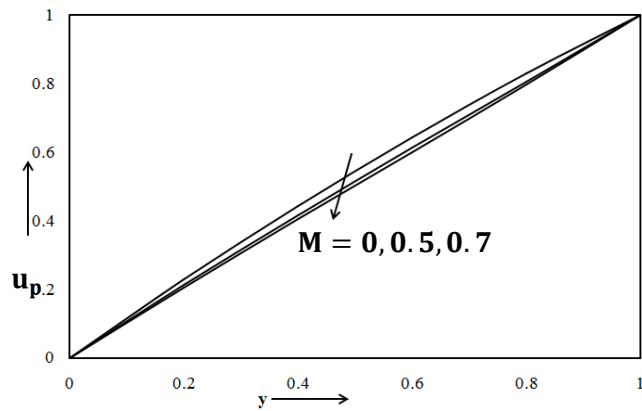


Figure.7. Dust particle velocity variation for different values of magnetic field parameter ( $M$ ).



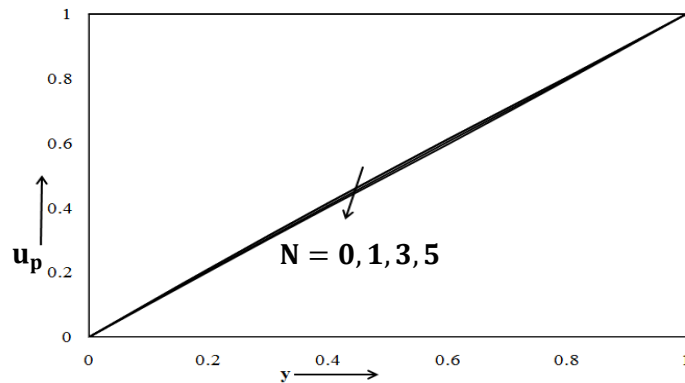


Figure.8. Dust particle velocity variation for different values of radiation parameter (N).

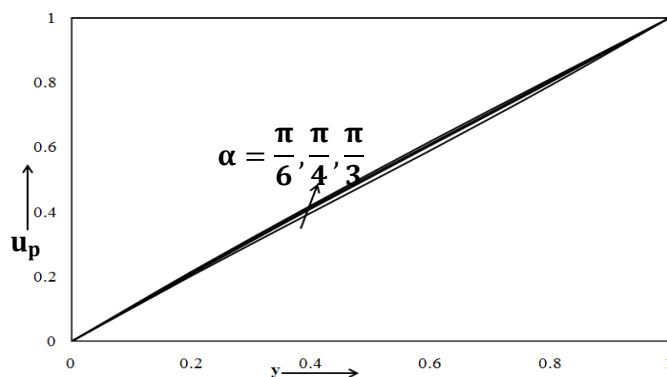


Figure.9. Dust particle velocity variation for different values of angle ( $\alpha$ ).

Tables 1 to 4 express numerically the influence of various parameters like heat sink parameter (Q), coefficient of skin friction ( $\tau$ ) and Nusselt number (Nu) for both fluid and dust particle velocity. It is evident from table 1 & 2 that both fluid and dust particle velocities decrease with the increase in heat sink parameter (Q). Table 3 & 4 express numerically the variation of fluid and dust particle velocity with frequency of oscillations ( $\omega$ ). From these tables we observed that with the increase in frequency of oscillations ( $\omega$ ), both the fluid and dust particle velocities decrease and after certain value of oscillations there is repetition of both the fluid and dust particle velocities.

Significant physical quantities of importance are shown by table 5 & 6. Table 5 & 6 shows the results for skin friction coefficient ( $\tau$ ) and Nusselt number (Nu) with magnetic field parameter (M) and radiation parameter (N) respectively at  $y = 0$  and  $y = 1$ . Table 5 depicts that with the increase in magnetic field parameter (M), skin friction coefficient ( $\tau$ ) decreases for both fluid velocity and dust particle velocity at  $y = 0$ , and the reverse pattern is obtained for both fluid and dust particle velocity at  $y = 1$ . It is seen from table 6 that with the increase in radiation parameter (N), the Nusselt number (Nu) decreases for both fluid velocity and dust particle velocity at  $y = 0$ , and increases at  $y = 1$ .

Table.1. Fluid velocity for different values of heat sink parameter (Q).

Q	0	0.2	0.4	0.6	0.8	1
0	0	0.21200	0.41526	0.61272	0.80696	1.0001
1	0	0.21112	0.41381	0.61123	0.80602	1.0001
3	0	0.20962	0.41129	0.60857	0.80428	1.0001
5	0	.020848	0.40937	0.60652	0.80291	1.0001

**Table.2.**Dust particles velocity for different values of heat sink parameter (Q).

Q	0	0.2	0.4	0.6	0.8	1
0	0	0.21198	0.41523	0.61268	0.80690	1
1	0	0.21111	0.41378	0.61119	0.80569	1
3	0	0.20960	0.41126	0.60853	0.80422	1
5	0	0.20847	0.40934	0.60647	0.80285	1

**Table.3.** Fluid velocity for different values of frequency of oscillations ( $\omega$ )

$y/\omega$	0	0.2	0.4	0.6	0.8	1
1	0	0.21182	0.41497	0.61244	0.80678	1
10	0	0.21179	0.41490	0.61233	0.80663	1
20	0	0.21182	0.41497	0.61244	0.80678	1

**Table.4.** Fluid velocity for different values of frequency of oscillations ( $\omega$ )

$y/\omega$	0	0.2	0.4	0.6	0.8	1
1	0	0.21181	0.41494	0.61239	0.80672	1
10	0	0.21180	0.41492	0.61238	0.80671	1
20	0	0.21181	0.41494	0.61238	0.80671	1

**Table.5.** Skin friction coefficient( $\tau$ ):

M	0	0.2	0.5	0.7
$\tau_{y=0}$	1.1703	1.1355	1.0863	1.0552
$\tau_{py=0}$	1.1702	1.1352	1.0862	1.0551
$\tau_{y=1}$	0.80014	0.86807	0.96676	1.0305
$\tau_{py=1}$	0.80006	0.86799	0.96667	1.0304

**Table.6.** Nusselt Number (Nu):

N	0	1	3	5	7
Nu(y = 0)	0.96752	0.82492	0.29288	0.066321	0.012612
Nu(p)(y = 0)	0.96752	0.82492	0.29288	0.066321	0.012612
Nu(y = 1)	1.0659	1.3714	3.0476	5.0209	7.0150
Nu(p)(y = 1)	1.0659	1.3714	3.0476	5.0209	7.0150

### References:

- [1] H. P. Greenspan and G. F. Carrier, “The MHD flow past a flat plate”, *J. Fluid Mech.*, Vol.6, pp. 77-96, 1959.
- [2] Y. J. Kim, “Unsteady MHD convective heat transfer past a semi-infinite vertical porous plate with variable suction”, *Int. J.Engng. Sci.*, Vol. 38, pp. 833, 2000.
- [3] H. A. Attia and N. A. Kotb, “MHD flow between two parallel plates with heat transfer”, *Acta Mech.*, Vol.117, pp.215-220, 1996.
- [4] M. A. Seddeek, “The effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation”, *Int. Comm., Heat Mass Transfer*, Vol. 27(7), pp. 1037-1046, 2000.
- [5] O. D. Makinde and P.Y. Mhone, “Heat transfer to MHD oscillatory flow in a channel filled with a porous medium”, *Rom. Journ. Phys.*, Vol.50, pp.931-938, 2005.
- [6] S. Seth, R. Nandkeolyar and Md. S. Ansari, “Unsteady MHD convective flow within a parallel plate rotating channel with thermal source/sink in a porous medium under slip boundary conditions”, *Int. J. of Engng., Sci. and Tech.*, Vol. 11, pp. 1-16, 2010.
- [7] P. K. Sharma and S. K. Saini, “MHD Flow through Rotating Porous Medium with Radiating Heat Transfer in the Presence of Fluctuating Thermal Diffusion”, *Universal Journal of Applied Mathematics*, Vol. 2(2) pp. 92-98, <http://www.hrpub.org> DOI: 10.13189/ujam.2014.020203, 2014.
- [8] D. Singh, “Effect of injection/ suction on convective oscillatory flow through porous medium bounded by two vertical porous plates”, *Int. J. Physical and Mathematical Sciences*, Vol. 2, pp. 140-147, 2011.
- [9] S. T. Dinesh Kumar, P. Raveendra Nath, “A numerical study of convective heat transfer through a porous medium in a vertical channel with radiational effect”, *International Conference computational, Heat and Mass Transfer*, Vol. 127, pp.1281-1291, 2015.
- [10] Z. Abbas, S.Rasool and M.M.Rashidi, “Heat transfer analysis due to an unsteady stretching/shrinking cylinder with partial slip condition and suction”, *Ain Shams Engineering Journal*, Vol. 6, pp. 939-945, 2015.
- [11] M. S. Dada and A. B. Disu, “Heat transfer with radiation and temperature dependent heat source in MHD free convection flow in a porous medium between two vertical wavy walls”, *Journal of Nigerian Society*, Vol. 34, pp. 200-215, 2015.
- [12] K. Walters, “Non-Newtonian effects in some general elastic-viscous liquids”, *Proc. I.U.T.A.M. Symposium, Hafsa, Isreal*, pp.507-515, 1962.
- [13] S. P. Mishra and T. C. Panda, “Effect of injection on the flow of second grade fluid in the inlet region of a channel”, *Acta Mech.*, Vol.32, pp-11-17, 1979.
- [14] P. D. Ariel, “The flow of a visco elastic fluid past a porous plate”, *Acta Mech.*, Vol. 107, pp.199-204, 1984.
- [15] C. E. Maneschy, M. Massoudi and A. Ghoneimy, “Heat transfer analysis of a non-newtonian fluid past a porous plate”, *International Journal of Non-linear Mechanics*, Vol. 28(2), pp. 131-143, 1993.
- [16] C. Kurtcebe and M. Z. Erim, “Heat transfer of a visco-elastic fluid in a porous channel”, *Int.J.Heat Mass Transfer*, Vol. 48, pp. 5072-5077, 2005.
- [17] Ali Attia Hazem, ‘Unsteady coquette flow with heat transfer of a visco elastic fluid considering the Hall Effect’, *Kragujevac J. Sci.*, Vol. 27, pp.5-22, 2005.
- [18] P. R. Sharma and D. Pareek, “Unsteady flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free stream and suction”, *Bull. Cal. Math. Soc.*, Vol. 98(2), pp.97-108, 2006.
- [19] K. D. Singh, “Visco elastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel”, *Int. J. Physical and Mathematical Sciences*, Vol. 3 pp.194-205, 2012.
- [20] R. Choudhary and D. U. Jyoti, “Heat transfer to MHD oscillatory visco elastic flow in a channel filled with porous medium”, *Physics Research International* doi: 101155/2012/879537, 2012.
- [21] A. A. Khan, R. Ellahi and M. Usman (2013), The effects of variable viscosity on the peristaltic flow of non-newtonian fluid through a porous medium in an inclined channel with slip boundary conditions, *Journal of Porous Media*, Vol.2, pp.16.
- [22] B. P. Garg, K. D. Singh, A. K. Bansal, “An Oscillatory MHD Convective Flow of Viscoelastic Fluid through Porous Medium Filled in a Rotating Vertical Porous Channel with Heat Radiation”, *International Journal of Engineering and Innovative Technology (IJEIT)*, Vol. 3, Issue 12, pp.273, 2014.
- [23] P.G. Saffman, “On the stability of laminar flow of a dusty gas”, *Journal of Fluid Mechanics*, Vol.13, pp.120, 1962.
- [24] R. K. Gupta, S.C. Gupta, “Flow of a dusty gas through a channel with arbitrary time varying pressure gradient”, *Journal of Applied Mathematics and Physics*, Vol. 27, pp. 119, 1976.
- [25] K. K. Singh, “Unsteady flow of a conducting dusty fluid through a rectangular channel with time dependent pressure gradient”, *Indian Journal of Pure and Applied Mathematics*, Vol. 8 (9) , pp.1124, 1976.
- [26] V. R. Prasad, N. C. P. Ramacharyulu, “Unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient”, *Defence Science Journal*, Vol. 30 , pp. 125, 1979.
- [27] H. A. Attia, “Influence of temperature dependent viscosity on MHD channel flow of dusty fluid with heat transfer”, *Acta Mech.*, Vol. 151, pp. 89-101, 2001.
- [28] H. A. Attia, “Unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties”, *Applied Mathematical Modelling*, Vol. 26 pp. 863–875, 2002.
- [29] H. A. Attia, “Influence of temperature-dependent viscosity on the MHD Couette flow of dusty fluid with heat transfer”, *Differential Equations and Nonlinear Mechanics*, Vol. 2006, Article ID 75290, Pages 1–14, DOI 10.1155/DENM/2006/75290.

- [30]
- [31] G. Palani and P. Ganesan, “Heat transfer effect on dusty gas flow past a semi infinite inclined plate”, *Forschung im Ingsenteurwesen*, Vol.71(3-4),pp. 223-230, 2007.
- [32] O.D. Makinde, T. Chinyoka, “MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition”, *Computers and Mathematics with Applications*, Vol. 60, pp.660-669, 2010.
- [33] K. Chand and S. Kumar, “Effect of hall current and rotation on chemically reacting and radiating MHD oscillatory dusty viscoelastic flow through porous vertical channel”, *International Journal of Applied and Computation*, Vol. 5(3), pp.17-32, 2013.
- [34] K. Chand, K. D. Singh and S. Sharma, “Heat transfer in MHD oscillatory flow of dusty fluid in a rotating porous vertical channel”, *Indian J. Pure Appl. Math.*, 819-835, 2014.
- [35] H. A. Attia, A. L. Aboul- Hassan, M. A. M. Abdeen and A. El-Din Abdin, “MHD flow of a dusty fluid between two infinite parallel plates with temperature dependent physical properties under exponentially decaying pressure gradient”, *Bulgarian Chemical Communications*, Vol. 46(2) pp. 320 – 329, 2014.
- [36] M. Jalil, S. Asghar and S. Yasmeen, “An exact solution of MHD boundary layer flow of Dusty fluid over a stretching surface”, *Mathematical Problems in Engineering*, Vol. 2017, Article ID 2307469, 5 pages, <https://doi.org/10.1155/2017/2307469>, 2017.
- [37] A. C. Cogley, W. G. Vincenti, S. E. Gilles, “Differential approximation for radiative transfer in a non-gray gas near equilibrium”, *Am. Inst. Aeronaut, Astronaut*, Vol. 6, pp. 551-553, 1968.
- [38] S. Sengupta and N. Ahmed, “MHD free convective Chemically reactive flow of a dissipative fluid with thermal diffusion, fluctuating wall temperature and concentration in velocity slip regime,” *Int. J. of Appl. Math and Mech.*, vol. 10(4), pp.27-54, 2014.