Effects of time Dependent acceleration on the flow of Blood in Artery with periodic body acceleration

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Abstract-
The aim of this paper is to develop a mathematical model describing the effect of time dependent acceleration with periodic body acceleration on the flow of blood in an artery. The flowing blood is treated to be Newtonian in character and the analytical solutions are obtained for this blood flow problem. The solution valid for the fast oscillations and a small external acceleration, are obtained for the velocity, flux and stress field. A computational analysis for the fluid mechanics of blood flow is also performed for the assumed situation. The effect of periodic body acceleration on the instantaneous flow rate, acceleration and shear stress are obtained and observed that it increases if we increase the magnitude of periodic body acceleration.

Keywords:- *Blood flow, arteries, accelerated motion, body acceleration and periodic external acceleration.*

INTRODUCTION:

The flow of blood through an artery in human being is at present difficult to measure without major surgery. It is therefore necessary to model blood problems through the arterial, either theoretically or experimentally. When developing a theoretical model, one must simplify the equations of motion sufficiently to permit the calculation of the required flow variables while at the same time maintaining the realism of the model. Various analytical and numerical approaches have been made using different simplifying assumptions.

The effect of accelerated blood flood flow in human being can be very serious, which may cause an increase in pulse rate loss of vision and venous pooling of blood in extremities.Arntzeniusetet. al. [1] and Verdouw et al. [2] obtained a very good result in this direction that indicates that blood pressure and cardiac output are raised when body acceleration synchronous with the heart beat is applied in a footward direction.

Sud[3] made an analysis of blood flow under time dependent acceleration and obtained a result which shows that high blood velocities and high shear rate capable of harming the circulation are produced under the influence of such time dependent acceleration.

Sud et al. [4] again worked on the flow through stenosed artery subject to periodic body acceleration and shows that body acceleration increases the flow rate. The pulsatile flow of blood through rigid tube under the influence of body acceleration was studied by Chaturani[6].Mandal [7] observed the effect of body acceleration on steady pulsatile flow of non Newtonian fluid through a stenosedartety. Sharma M. K. et. al.[11] studied aboutPulsatile blood flow through stenosed artery with axial translation.

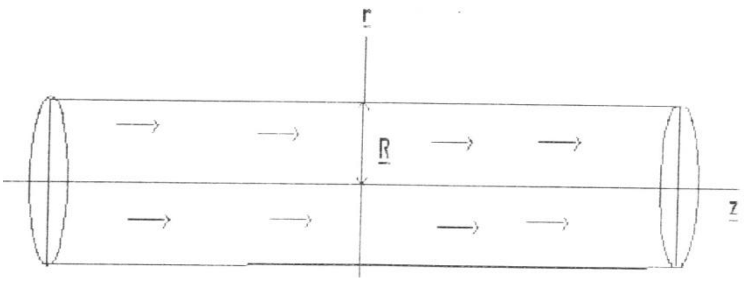
There is lot of investigation, which was made for blood flow with time dependent acceleration, and it is well known that the vibration amplitudes of mechanical equipment e.g. an aeroplane, the effect of such vibrations on the human system can be quite closely approximated by imposing a sinusoidal velocity whose amplitude grows with time on the linear acceleration of the body.

Thus a theoretical analysis for predicting the time dependent acceleration of blood flow is very important subject to investigation for the design of anti-gsuite and cordite assist devices. Therefore in this chapter a study which deals with the problem of blood under time dependent acceleration under periodic body acceleration has been made to find a mathematical model for computational result for the effect of these factor on the blood flow velocity, flow rate and shearing stress with respect to radial distance.

FORMATION OF THE PROBLEM:

To simplify the analysis. We additionally make the following suppositions:

1. The flow is laminar and there is rotational symmetry of flow.
2. The frequence of body acceleration is so small that wave effect can we neglected.
3. The variation of velocity along the tube length is small compared with the rate of change of velocity with respect to Time.
4. The artery is sufficiently along that the flows of blood along that the end effects can be ignored.
5. For simplicity consider $f = f_b$ $i.e.\omega = \omega_b$ where f & f_b the frequencies in Hz be.



GEOMETRY OF BLOOD FLOW IN ARTERIES

Consider the flow of blood in the tube of radius R. The tube is initially at time $t \le 0$. At time $t = 0$ it suddenly starts oscillating along its longitudinal direction with velocity $V_T = a_0 t \cos \omega t$ [3]Let a_0 is the acceleration in m/s^2 , $\omega = 2\Pi f$ is the angular frequency in R/sec and f is the frequency in Hz.

The imposed acceleration therefore is $a_0(\cos \omega t - \omega t \sin \omega t)$. The V_T has amplitude $a_0 t$, which increases linearly with respect to time. Now let us consider the system .subjected to periodic body accelerationF(t)[4], is given by

$$F(T) = A_0 \cos(\omega_b t + \phi)$$

Where $\omega_b = 2\Pi f_b$ is the circular frequency in Hz. ϕ is the lead angle of F(t) with respect to heart action.

The basic equation governing the flow of blood along the longitudinal direction in the tube can be written as (Batchlor, 1967)

$$.\rho \frac{\partial w}{\partial t} = \mu\left(\frac{\partial^2 w}{\partial^2 r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \quad \dots. (1)$$

While equation(1)subject to periodic body acceleration may be written as:

$$\rho \frac{\partial w}{\partial t} = A_0 \cos(\omega t + \phi) + \mu\left(\frac{\partial^2 w}{\partial^2 r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

i.e. $$\frac{\partial^2 w}{\partial^2 r^2} + \frac{1}{r}\frac{\partial w}{\partial r} = \frac{\rho}{\mu}\frac{\partial w}{\partial t} - \frac{A_0}{\mu}\cos(\omega t + \phi)$$

.... (2)

Where w is the axial velocity, ρ is the density, μ is the viscosity of the blood r is the radial distance.

The presence of thepressuregradient in the Navier stokes equation (2) was also used by womersley (1955) for analyzing the oscillatory blood flow.

The initial and boundary conditions of the problem are[3]:

$w(r,t) = 0 \ at\, t \le 0$ For all r

... (3)

$w(r,t) \to$ Finite value as $r \to 0$ for all t

... (4)

$V_T = w(R,t) = a_0 t \cos wt$ At r=R, for t>0 ...

(5)

The imposed velocity[3] V_T is such that:

$$o < t < \frac{\pi}{2\omega} \ and \ \frac{3\pi}{2\omega} < t < \frac{5\pi}{2\omega}, when V_T > 0$$ While,

$$\frac{\pi}{2\omega} < t < \frac{3\pi}{2\omega}, when V_T < 0$$

METHOD OF SOLUTION:

By applying Laplace transform and following Carslaw(1963) theory, and omitting the calculations, the solution for the flow velocity can be finally written as:

$$w(y,t)=\frac{A_0}{\rho}\sum_{n=1}^{\infty}\frac{2e^{-k\lambda_n^2 t}j_0(\lambda_n y)\sin\left(\frac{\omega t}{2}\right)}{\lambda_n j_1(\lambda_n)(k^2\lambda_n^4+\omega^2)}\left[w\cos\left(\frac{\omega t}{2}+\phi\right)-k\lambda_n^2\sin\left(\frac{\omega t}{2}+\phi\right)\right]+\frac{a_0}{2}e^{i\omega t}\frac{j_0(\alpha i^{3/2}y)}{j_0(\alpha i^{3/2})}\left[t-\frac{1}{2}\left(\frac{i}{\upsilon\omega}\right)^{1/2}R\left\{y\frac{j_1\left(\alpha i^{3/2}y\right)}{j_0(\alpha i^{3/2}y)}-\frac{j_1\left(\alpha i^{3/2}\right)}{j_0(\alpha i^{3/2})}\right\}\right]$$
$$+\frac{a_0}{2}e^{iwt}\frac{j_0(\alpha i^{3/2}y)}{j_0(\alpha i^{3/2})}\left[t-\frac{1}{2}\left(\frac{-i}{\upsilon\omega}\right)^{1/2}R\left\{y\frac{j_1\left(\alpha i^{1/2}\right)}{j_0(\alpha i^{1/2}y)}-\frac{j_1\left(\alpha i^{1/2}\right)}{j_0(\alpha i^{1/2})}\right\}\right]$$
$$-2ka_0\sum_{n=1}^{\infty}\frac{\lambda_n e^{-k\lambda_n^2 t}\left\{k^2\lambda_n^4-\omega^2\right\}j_0(\lambda_n,y)}{j_1(\lambda_n)\left(k^2\lambda_n^4+\omega^2\right)^2}$$

… (6)

Where j_0 *and* j_1 are Bessel functions of zero and first order respectively, λ_n are the zeroes of $j_0\left(i\sqrt{\frac{\omega}{\nu}}R\right)$

The kinematic viscosity $\nu=\frac{\mu}{\rho}$ *and* $k=\frac{\nu}{R^2}$

Dimensionless number $\alpha=R\left(\frac{\omega}{\nu}\right)^{1/2}$ *and* $y=\frac{r}{R}$

The expression for the rate Q can be written as[7]:

$$Q=2\pi\int_0^R rw(r,t)dr$$

… (7)

Now using equition (6)in equation (7) we get the expression for the flow rate:

$$Q=\pi Ra_0 e^{-iwt}\frac{j_1(\alpha i^{1/2}y)}{j_0(\alpha i^{1/2})}\left[t\left(\frac{-i\nu}{\omega}\right)^{1/2}+R\frac{i}{2\omega}\left\{\frac{j_2\left(\alpha i^{1/2}\right)}{j_0(\alpha i^{1/2})}-\frac{j_1\left(\alpha i^{1/2}\right)}{j_0(\alpha i^{1/2})}\right\}\right]+\pi R\frac{a_0}{2}e^{i\omega t}\frac{j_1(\alpha i^{3/2})}{j_0(\alpha i^{3/2})}\left[t\left(\frac{i\nu}{\omega}\right)^{1/2}-\frac{1}{2}\left(\frac{iR}{\omega}\right)\left\{\frac{j_2\left(\alpha i^{3/2}\right)}{j_0(\alpha i^{3/2})}-\frac{j_1\left(\alpha i^{3/2}\right)}{j_0(\alpha i^{3/2})}\right\}\right]$$
$$-2\pi R^2 ka_0\sum_{n=1}^{\infty}\left\{\frac{e^{-k\lambda_n^2 t}\left\{k^2\lambda_n^4-\omega^2\right\}}{\left(k^2\lambda_n^4+\omega^2\right)^2}+2\pi R^2\frac{A_0}{\rho}\sum_{n=1}^{\infty}\frac{2e^{-k\lambda_n^2 t}}{\lambda_n j_1(\lambda_n)(k^2\lambda_n^4+\omega^2)}\left[w\cos\left(\frac{\omega t}{2}+\phi\right)-k\lambda_n^2\sin\left(\frac{\omega t}{2}+\phi\right)\right]\sum_{r=0}^{\infty}\frac{(-1)^r\lambda_n^{2r}}{\left(2^r\angle r\right)^2(2r+2)}\right\}$$

…. (8)

The analytical solution for the velocity w(r, t) and flow rate Q (t)contains Bessel functions with complex arguments hence we shall obtain explicit solutions for small and large values of various arguments of Bessel functions.

CASE: (a) IF $.\alpha<<2$

For small values of the dimensionless number $\alpha<<2$, the zero and first order Bessel functions corresponding to the above arguments, up to two terms can be approximate as following:

$$j_0(x)\approx 1-\frac{x^2}{4},\ j_1(x)\approx\frac{x}{2}$$

Where x is the approximated argument.

Substituting the velocity profile can be written after some simplifications as:

$$w(y,t)=a_0\left\{t+\frac{R^2}{4\nu}\left(y^2-1\right)\right\}\cos\omega t-\frac{ta_0\alpha^2}{4}\left(y^2-1\right)\sin\omega t$$
$$-2ka_0\sum_{n=1}^{\infty}\frac{\lambda_n e^{-k\lambda_n^2 t}\left(k^2\lambda_n^4-\omega^2\right)}{\left(k^2\lambda_n^4+\omega^2\right)j_1\left(\lambda_n\right)}j_0\left(\lambda_n y\right)$$
$$+\frac{A_0}{\rho}\sum_{n=1}^{\infty}\frac{2e^{-\lambda_n^2 kt}j_0\left(\lambda_n y\right)\sin\frac{\omega t}{2}}{\lambda_n j_1\left(\lambda_n\right)\left(k^2\lambda_n^4+\omega^2\right)}\left[\omega\cos\left(\frac{\omega t}{2}+\phi\right)-k\lambda_n^2\sin\left(\frac{\omega t}{2}+\phi\right)\right]$$

… (9)

The expression of the fluid acceleration f can be obtained from equation (9)
And it is as:

$$f=a_0\cos\omega t-\omega a_0 t\sin\omega t-\frac{a_0\alpha^2}{2}(y^2-1)\sin\omega t-\frac{a_0\alpha^2}{2}(y^2-1)\cos\omega t$$
$$+2k^2a_0\sum_{n=1}^{\infty}\frac{\lambda_n^3\left(k^2\lambda_n^4-\omega^2\right)e^{-k\lambda_n^2 t}j_0\left(\lambda_n y\right)}{j_1\left(\lambda_n\right)\left\{k^2\lambda_n^4+\omega^2\right\}}$$
$$+\frac{A_0}{\rho}\sum_{n=1}^{\infty}\frac{2j_0\left(\lambda_n y\right)}{\lambda_n j_1\left(\lambda_n\right)\left\{k^2\lambda_n^4+\omega^2\right\}}\left[\begin{array}{l}-\lambda_n^2 ke^{-k\lambda_n^2 t}\sin\frac{\omega t}{2}\left\{\omega\cos\left(\omega t+\phi\right)-k\lambda_n^2\sin\left(\omega t+\phi\right)\right\}\\ +e^{-k\lambda_n^2 t}\left\{\begin{array}{l}\frac{\omega^2}{2}\cos\left(\omega t+\phi\right)-\\ \frac{k\lambda_n^2\omega}{2}\left(\cos\frac{\omega t}{2}\sin\left(\frac{\omega t}{2}+\phi\right)-\sin\frac{\omega t}{2}\cos\left(\frac{\omega t}{2}+\phi\right)\right)\end{array}\right\}\end{array}\right]$$

... (10)

Using equation (9) and (10) we also calculate the values of shear stress .the shear stress can be defined as:

$$\tau=\mu\frac{dw}{df}$$

… (11)

CASE: (b) IF $\alpha>>2$ the solution valid for the large values of the dimensionless variable α , can be obtained by employing the asymptotic expression of the Bessel function. The Bessel function $j_n(x)$ of order n and argument x can be written as:

$$j_n\left(x\right)\approx\left(\frac{2}{\pi x}\right)^{1/2}coa\left(x-\left(\frac{n+1}{2}\right)\pi\right)$$

Following Mchachlan (1955) and using the asymptotic arguments and order as required in the equation (9) substituting the approximations the velocity profile can be written after some simplifications as:

$$w(y,t)=b_0\left[2t\cos\left(\omega t+\frac{b_1}{\sqrt{2}}\right)+\frac{b_1}{\omega}\left\{\cos\left(\omega t+\frac{b_1}{\sqrt{2}}\right)+\sin\left(\omega t+\frac{b_1}{\sqrt{2}}\right)\right\}\right]$$
$$-2ka_0\sum_{n=1}^{\infty}\frac{\lambda_n e^{-k\lambda_n^2 t}\left(k^2\lambda_n^4-\omega^2\right)}{\left(k^2\lambda_n^4+\omega^2\right)^2 j_1\left(\lambda_n\right)}j_0\left(\lambda_n y\right)$$
$$+\frac{A_0}{\rho}\sum_{n=1}^{\infty}\frac{2e^{-k\lambda_n^2 t}j_0\left(\lambda_n y\right)\sin\frac{\omega t}{2}}{\lambda_n j_1\left(\lambda_n\right)\left(k^2\lambda_n^4+\omega^2\right)}\left[\omega\cos\left(\frac{\omega t}{2}+\phi\right)-k\lambda_n^2\sin\left(\frac{\omega t}{2}+\phi\right)\right]$$

… (12) Where for simplicity we consider:

$$b_0 = \frac{a_0}{2\sqrt{y}} e^{\left\{\frac{\alpha}{2}(y-1)\right\}} \quad and \quad b_1 = \frac{1}{\sqrt{2}}\alpha(y-1)$$

The expression of the fluid acceleration f can be obtained from equation (11) and that is:

$$f = b_0\left[2\left\{\cos\left(\omega t+\frac{b_1}{\sqrt{2}}\right)-\omega t\sin\left(\omega t+\frac{b_1}{\sqrt{2}}\right)\right\}+\frac{b_1}{2}\left\{\sin\left(\omega t+\frac{b_1}{\sqrt{2}}\right)-\cos\left(\omega t+\frac{b_1}{\sqrt{2}}\right)\right\}\right]$$

$$+2k^2 a_0 \sum_{n=0}^{\infty} \frac{\lambda_n^3\left(k^2\lambda_n^4-\omega^2\right)e^{-k\lambda_n^2 t} j_0(\lambda_n y)}{j_1(\lambda_n)\left\{k^2\lambda_n^4+\omega^2\right\}}$$

$$+\frac{A_0}{\rho}\sum_{n=1}^{\infty}\frac{2j_0(\lambda_n y)}{\lambda_n j_1(\lambda_n)\left\{k^2\lambda_n^4+\omega^2\right\}}\left[\begin{array}{l} -\lambda_n^2 k e^{-k\lambda_n^2 t}\sin\frac{\omega t}{2}\left\{\omega\cos(\omega t+\phi)-k\lambda_n^2\sin(\omega t+\phi)\right\} \\ +e^{-k\lambda_n^2 t}\left\{\begin{array}{l}\frac{\omega^2}{2}\cos(\omega t+\phi)- \\ \frac{k\lambda_n^2\omega}{2}\left(\cos\frac{\omega t}{2}\sin\left(\frac{\omega t}{2}+\phi\right)-\sin\frac{\omega t}{2}\cos\left(\frac{\omega t}{2}+\phi\right)\right)\end{array}\right\}\end{array}\right]$$

.. .

(13)

Using equation (11) AND (12) we also calculate the value of shear stress. The shear stress can be defined as:

$$\tau = \mu\frac{dw}{df} \qquad ...\ (14)$$

RESULTS AND DISCUSSION:

To evaluate the solution we consider that the case of blood flow in small and large arteries and for the case of blood flow we consider that ρ =1050kg m^{-3} , μ =0.004 kg $m^{-1}s^{-1}$and the value of a_0 is taking as 4.905 ms^{-1} while the frequency f is 1.2 Hz. The equation (9),(10) and (11) represents solution for the small artery whereas the equation (12),(13) and (14) represents solution for the large artery. Here we consider a_0=0.2g For the different values of t we plot the variation of velocity w with respect to radial distance at four points in cycle by taken R=0.01 m and α =14.1 and for different values of t the change of shear stress τ with respect to radial distance .

It is clear that when blood flood flow in an artery under the influence of a time dependent acceleration then there are some substantial disturbances. From the above calculation we found the in this chapter that due to the periodic body acceleration the flow velocity , flow acceleration as well as shear stress increases . From figure it is also clear that there fluctuation become larger with time, as does the external acceleration.

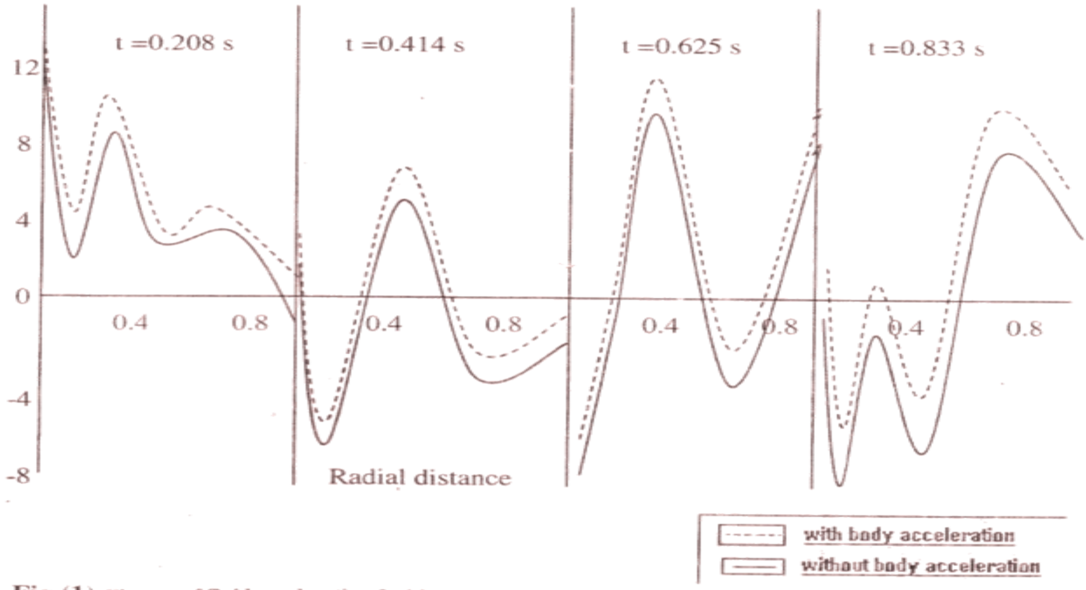
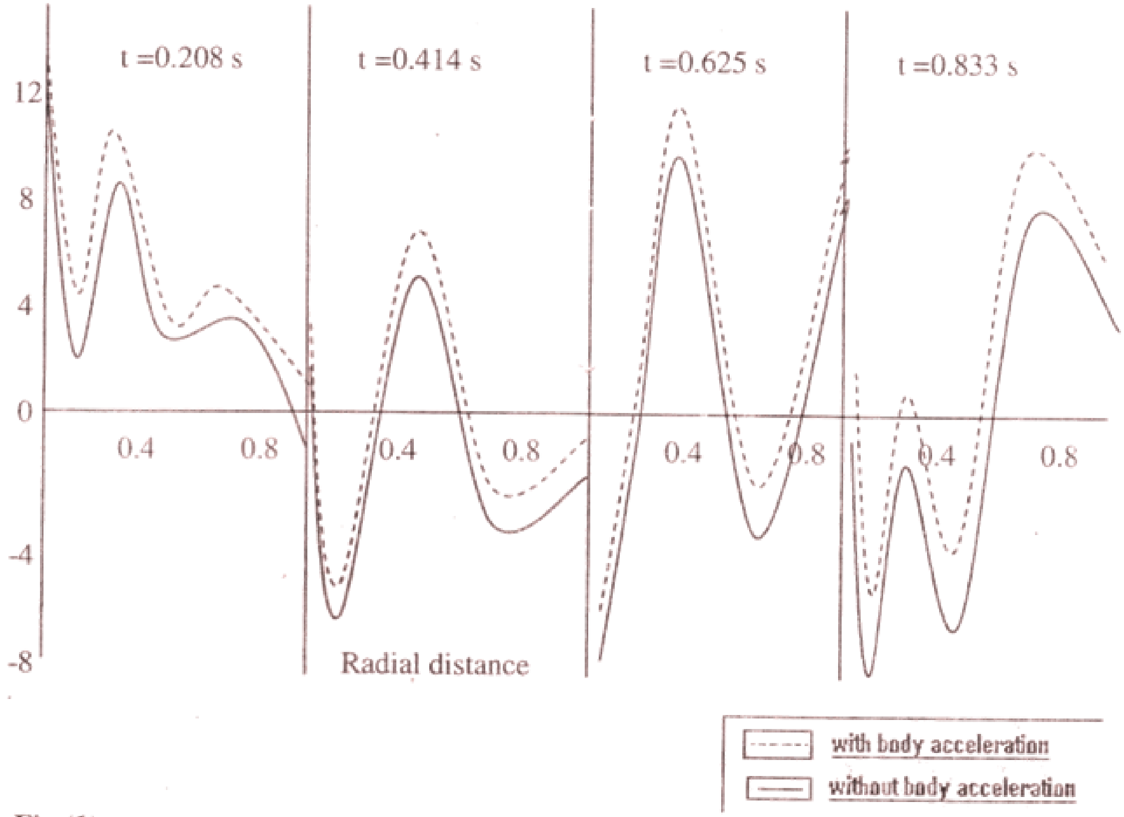


Fig (1) Change of fluid acceleration f with respect to radial distance at four point in a cycle(R=0.01)



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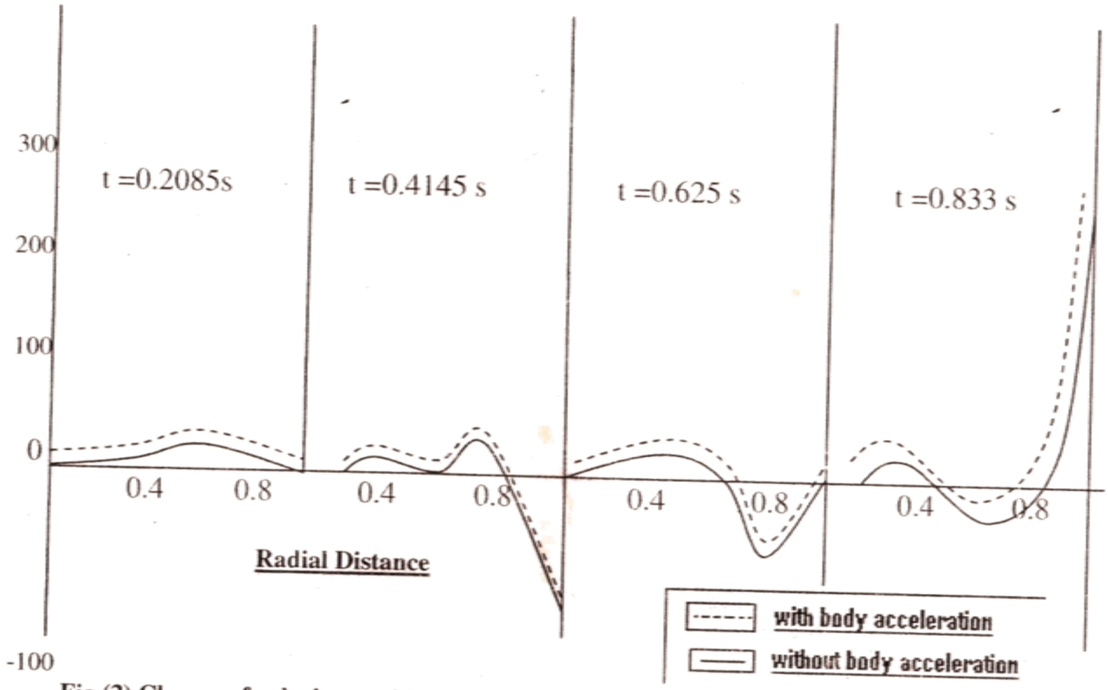

Fig (2) Change of velocity w with respect to radial distance at four points in a cycle (R=0.01)

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