Super Fibonacci Gracefulness of Shell Related Graphs

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ABSTRACT

A graph is said to be Super Fibonacci graceful if the function f is defined as $f: V(G) \longrightarrow \{0, F_1, F_2, \dots, F_q\}$ and the induced edge labeling $f^*: E(G) \longrightarrow \{F_1, F_2, F_3, \dots, F_q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. In this paper an analysis is made on Shell, Shell-flower and Bow graphs under Super Fibonacci graceful graphs.

Keywords :

Labeling, Super Fibonacci graceful labeling, Shell, Shell-flower graph, Bow graph.

Mathematical classification: 05C78

1. INTRODUCTION:

If the vertices or edges or both assigned values subject to certain condition(s) then it is known as graph labeling.

“ Graceful labeling “was introduced by Rosa (1967)[7]. J.A. Gallian [3] studied a complete survey on graph labeling . David .W and Anthony. E. Baraaukas [1] have investigate the cycle structure of Fibonacci graceful graphs. A Fibonacci graceful labeling and Super Fibonacci graceful labeling have been introduced by Kathiresan and Amutha [5] in 2006. N. Murugesan and R. Uma [6] have obtained some Cycle- related graphs under Fibonacci graceful.

Jeba Jesintha.J. Ezhilarasi Hilda Stanley [4] are proved Bow graphs and Shell-flower graphs are graceful. In the present work, our aim is to provide the Super Fibonacci graceful labeling of Shell, Shell-flower graphs and Bow graphs.

2. DEFINITIONS

Definition 2.1.

If the vertices or edges or both assigned values subject to certain condition(s) then it is known as graph labeling.

Definition 2.2.

The function f is called a graceful labeling of agraph G if $f: V \longrightarrow \{0, 1, 2, \dots.. q\}$ is injective and the induced function $f^*: E \longrightarrow \{1, 2, 3, \dots q\}$defined as $f^*(e = uv) = |f(u) - f(v)|$ is bijective.

Definition 2.3.

The function $f: V(G) \longrightarrow \{0,1,2 \ldots . F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Fibonacci graceful if the induced edge labeling $f^* : E(G) \longrightarrow \{F_1, F_2, F_3, \ldots F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective.

Definition 2.4.

The function $f: V(G) \longrightarrow \{0, F_1, F_2, \ldots . F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Super Fibonacci graceful if the induced edge labeling $f^* : E(G) \longrightarrow \{F_1, F_2, F_3, \ldots F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective.

Definition 2.5.

A Shell graph as a cycle C_n with $(n-3)$ chords sharing a common end points called the apex. Shell graphs are denoted as $[C(n, n-3)]$.

Definition 2.6.

A Shell – flower graph as k copies of the union of the shell $C(n, n-3)$ and K_2 where one end vertex of K_2 is joined to the apex of the shell. We denote this graph by $[C(n, n-3) \cup K_2]^k$. Where the superscript k denotes the k copies of $[C(n, n-3) \cup K_2]$.

Definition 2.7.

A Bow graph is defined to be a double shell is which each shell has any order.

3.RESULTS

3.1 Theorem :

The Shell graphs are Super Fibonacci graceful .

Proof :

Let $[C(n, n-3)]$ be the Shell graph. The order of $[C(n, n-3)]$ is $p = n$ and the size of $[C(n, n-3)]$ is $q = 2n - 3$. Then the vertex set $V(G) = \{v_0, v_1, v_2, \ldots, v_{(n-1)}\}$. Let v_0 be the first vertex of the cycle C_n and let $v_1, v_2, \ldots, v_{(n-1)}$ be the vertices of the cycle C_n. The edge set $E(G) = \{e_i, e^*\}$ where $e_i = (v_0, v_i)$ and $e^* = (v_i, v_j)$.

Now let us define the function $f: V(G) \rightarrow \{0, F_1, F_2, F_3, \ldots, F_q\}$

$f(v_0) = 0,$

$f(v_1) = F_{q.}$

$f(v_i) = F_{q-2i\,.} \qquad for\ i = 1,2, \ldots, n-2$

Then the above defined function f admits the Super Fibonacci graceful labeling.

Hence the Shell graphs are Super Fibonacci graceful .

The generalized graph of $[C(n, n-3)]$ is shown in figure 1.

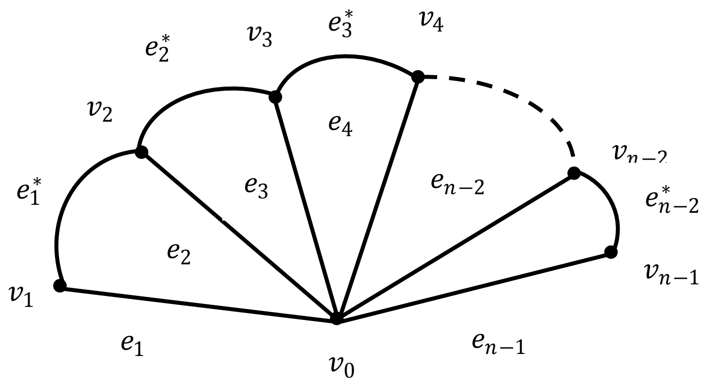

Figure 1. The Shell graph $[C(n, n-3)]$

3.2 Example:

The Shell graph $[C(4,4-3)]$ is shown in figure 2. Consider the graph $[C(4,4-3)]$, with order $p = 4$ and the size is $q = 5$.

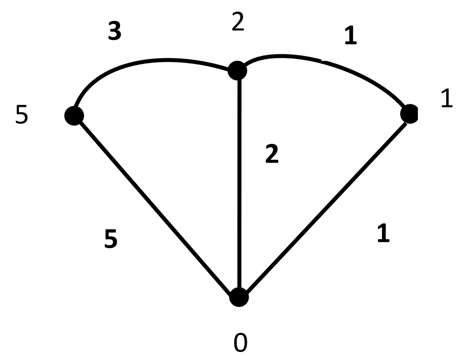

Figure 2 .The Shell graph $[C(4,4-3)]$

3.3 Theorem :

The Shell – flower graphs are Super Fibonacci graceful .

Proof:

Let $[C(n, n-3) \cup K_2]^k$ be the Shell – flower graph. The order of $[C(n, n-3) \cup K_2]^k$ is $p = kn + 1$ and the size of $[C(n, n-3) \cup K_2]^k$ is $q = k(2n-2)$. Then the vertex set $V(G) = \{v_0, v_{11}, v_{12}, \dots, v_{k(n-1)}, w_1, w_2, \dots, w_k\}$. Let v_0 be the apex vertex and $v_{11}, v_{12}, \dots, v_{k(n-1)}$ be the vertices of the k copies of shells.

Let $w_1, w_2, \dots, w_k$ be the one end vertices of K_2. The edge set $E(G) = \{e_{ij}, e_{ij}^*, e_{in}\}$ where $e_{ij} = (v_0, v_{ij})$, $e_{ij}^* = (v_i, v_j)$ and $e_{in} = (v_0, v_{in})$.

Now let us define the function $f: V(G) \rightarrow \{0, F_1, F_2, F_3 \dots F_q\}$

$f(v_0) = 0,$

$f(v_{ij}) = F_{2[(i-1)(n-1)+j]-1.}$

$$\begin{cases} for\ j = 1,2,\dots,n-1 \\ for\ i = 1,2,3,4,\dots,k \end{cases}$$

$f(w_i) = F_{2in-2i\,.}\quad for\ i = 1,2,\dots,k$

Then the above defined function f admits the Super Fibonacci graceful labeling.

Hence the Shell – flower graphs are Super Fibonacci graceful .

The generalized graph of $[C(n, n-3) \cup K_2]^k$ is shown in figure 3.

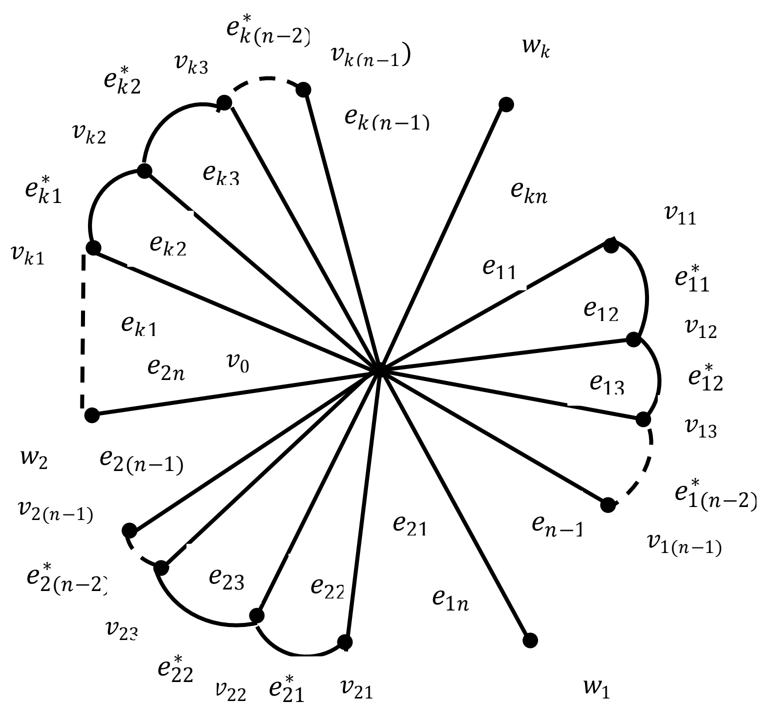

Figure 3.

Example:

The Shell-flower $[C(4,4-3) \cup K_2]^3$is shown in figure 4. Consider the graph $[C(n, n-3) \cup K_2]^3$ with order and size is $p = 13$ and q =18 respectively.

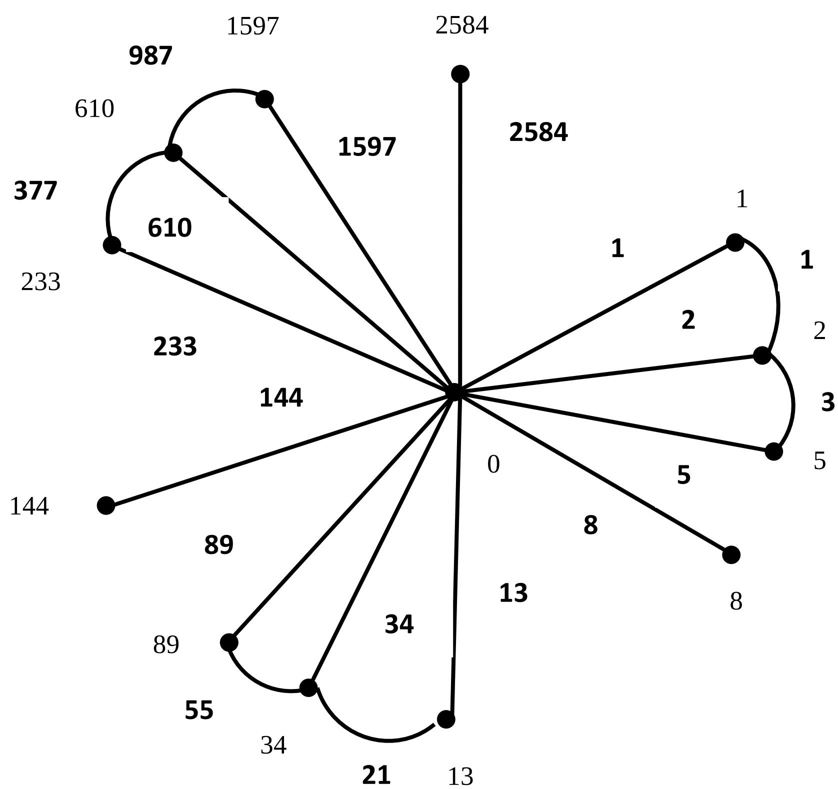

Figure 4. The Shell-flower $[C(4,4-3)\cup K_2]^3$.

3.5 Theorem :

Bow graphs are Super Fibonacci graceful .

Proof:

Let G be a Bow graph . The order of G is $p=m+n+1$ and the size of G is $q=2m+2n-2$. By the definition of Bow graph, let the vertex set $V(G)=\{v_0,v_1,v_2,\dots,v_m,u_1,u_2,\dots u_n\}$. Let v_0 be the apex vertex. Let $v_1,v_2,\dots,v_m$ be the vertices of first shell adjacent to v_0 and $u_1,u_2,\dots,u_n$ be the vertices of second shell it is adjacent to v_0. The edge set $E(G)=\{e_i,e^*,e_j',e^+\}$where $e_i=(v_0,v_i)$, $e_i^*=(v_i,v_j)$, $e_j'=(v_0,u_j)$ and $e^+=(u_i,u_j)$.

Now let us define the function $f:V(G)\to\{0,F_1,F_2,F_3,\dots,F_q\}$

$f(v_0)=0,$

$f(v_i)=F_{2i-1.}\qquad for\ i=1,2,\dots,m.$

$f(u_i)=F_{2m+2i-2.}$

$for\ i=1,2,\dots,n\ .$

$for\ m=1,2,\dots,n\ .$

Then the above define the function f admits the Super Fibonacci graceful labeling.
Hence the Bow graphs are Super Fibonacci graceful graphs.
The generalized Bow graph is shown in figure 5 .

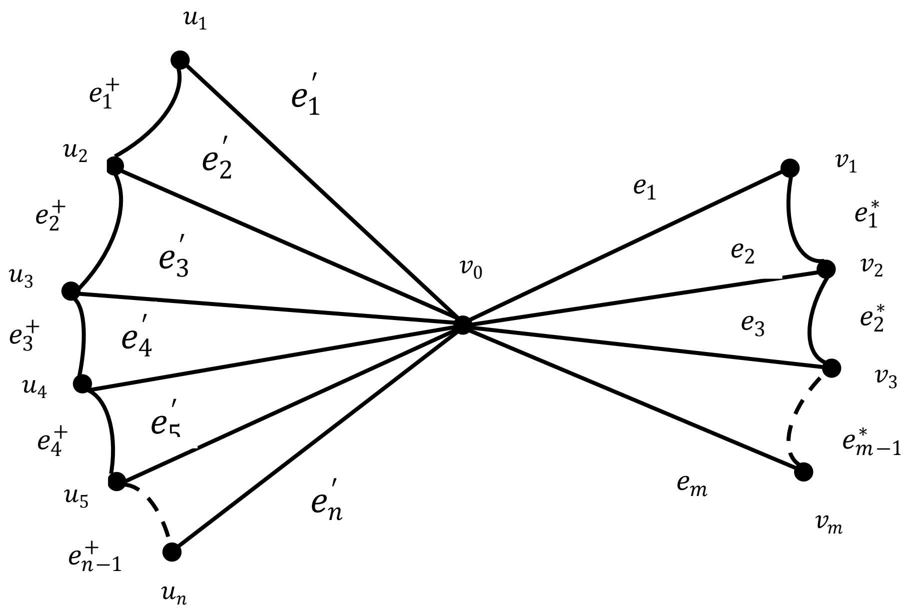

Figure 5. Bow graph

5.3.2.Example:

The Bow graph $m = 4, n = 5$ is shown in figure 6. Consider the Bow graph with order and size is $p = 10$ and $q = 16$ respectively.

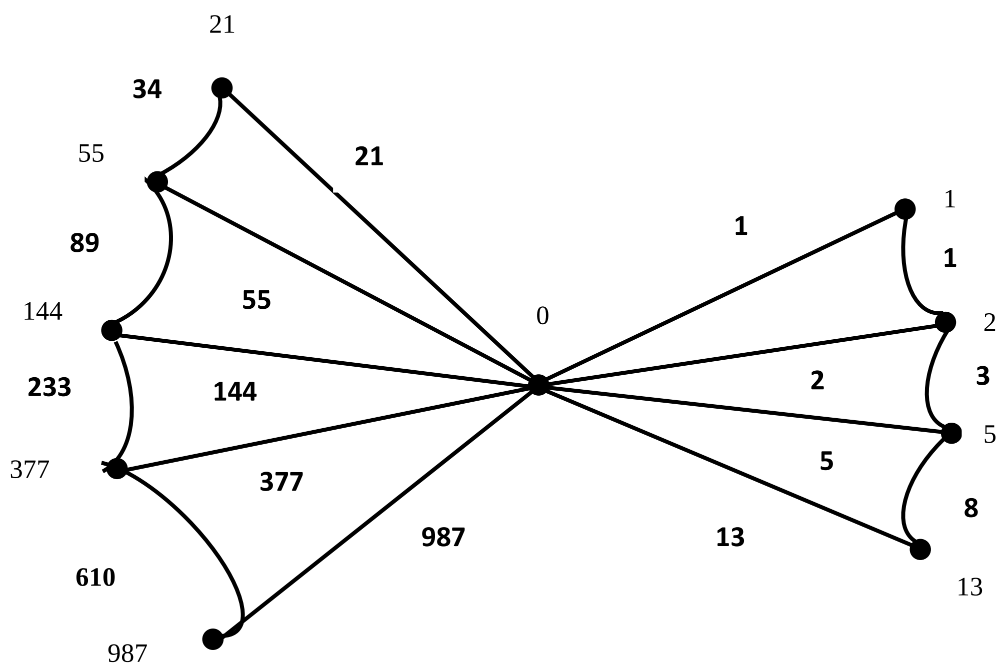

Figure 6. Bow graph $m = 4$ and $n = 5$

Conclusion:

In this paper, we have shown that Shell, Shell-flower, and Bow graphs are Super Fibonacci graceful. In future, the same process will be analyzed for other graphs.

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