A New Closure and Interior Operators via V-Closed Setsand V-Open Sets

[1]S.Sarany A, [2]Dr.K.Bageerathi
[1,2]*Assistant professors of mathematics*
[1,2]*Aditanar college of Arts and Science, Tiruchendur,*
Tamil Nadu-628215, INDIA.

Abstract

The purpose of this paper is to introduce the some operators via v –open sets and v -closed sets in topological spaces and obtain some of interesting properties of these operators.

AMS Subject Classification (2010): 54A05

Keywords: *v-open, v-closed, v - interior, v –closure.*

1. INTRODUCTION

Levine[2] initiated the study of generalized closed sets in topological spaces. In 1963 levine[3] introduced semi-open sets in topological spaces. Robert. A[4] et al., and selvi. T[6] introduced semi*-open sets and pre*-open sets respectively, using the generalized closure operator cl^* due to Dunham[1]. Saranya. S and Bageerathi. K[5]., introduced v-open sets in topological spaces. In this paper, we introduce a new operators using v-open sets and v-closed sets and study the basic properties and characterization of these operators.

2. PRELIMINARIES

Throughout this paper, spaces (X,τ) (or simply X) always mean non empty topological spaces on which no separation axioms are assumed unless explicity stated. For a subset A of a space $(X,\tau), cl(A), int(A), cl^*(A), int^*(A)$ and X/A denote the closure of A, the interior of A, g-closure of A, g-interior of A and the complement of A respectively. The following definitions and results are very useful in the subsequent sections

Definition 2.1. A subset A of a topological space (X,τ) is said to be a v-open set if $A \subseteq int^*(cl(A)) \cup cl^*(int(A))$.

3. v-INTERIOR OPERATOR

Definition 3.1. Let A be a subset of a topological space (X,τ). Then the union of all v-open sets contained in A is called the v-interior of A and it is denoted by $vint(A)$. That is, $vint(A) = \{V : V \subseteq A \text{ and } V \in v - O(X)\}$.

Remark 3.2. Since the union of v-open subsets of X is v-open in X, then $vint(A)$ is v-open in X.

Definition 3.3. Let A be a subset of a topological space X. A point $x \in X$ is called a v-interior point of A if there exists a v-open set G such that $x \in G \subseteq A$.

Theorem 3.4. Let A be a subset of a topological space (X,τ). Then

(i) $vint(A)$ is the largest v-open set contained in A.
(ii) A is v-open if and only if $vint(A)=A$.
(iii) $vint(A)$ is the set of all v-interior points of A.
(iv) A is v-open if and only if every point of A is a v-interior point of A.

Proof:

(i) Being the union of all v-open sets, $vint(A)$ is v-open and contains every v-open subset of A. Hence $vint(A)$ is the largest v-open set contained in A.
(ii) Necessity: Suppose A is v-open. Then by Definition 3.1, $A \subseteq vint(A)$. But $vint(A) \subseteq A$ and therefore, $vint(A) = A$. Sufficiency: suppose $vint(A) = A$. Then by Remark 3.2, $vint(A)$ is v-open set. Hence A is v-open.
(iii) Let $x \in vint(A) \Leftrightarrow x \in \cup \{V : V \subseteq A \text{ and } V \in v-O(X)\}$

$\Leftrightarrow$there exists a v-open set G such that x $\in G \subseteq A$.

$\Leftrightarrow$ A is a v-nbhd of x.

$\Leftrightarrow$ x is a v-interior point of A.

Hence $vint(A)$ is the set of all v-interior points of A.

(iv) Suppose A is v-open. Then by part (ii) and (iii), we have every point of A is the v-interior point of A.

Theorem 3.5. Let A and B be subsets of (X, τ). Then the following results hold.

(i) $vint(\phi) = \phi$ and $vint(X) = X$.
(ii) If B is any v-open set contained in A, then $B \subseteq vint(A)$.
(iii) If $A \subseteq B$, then $vint(A) \subseteq vint(B)$.
(iv) $int(A) \subseteq s^*int(A) \subseteq vint(A) \subseteq A$.
(v) $vint(vint(A)) = vint(A)$.

Proof:

(i) Since ϕ is the only v-open set contained in ϕ, then $vcl(\phi)= \phi$. Since X is v-open and $vint(X)$ is the union of all v-open sets contained in X, $vint(X) = X$.
(ii) Suppose B is v-open set contained in A. Since $vint(A)$ is the union of all v-open set contained in A, then we have $B \subseteq vint(A)$.
(iii) Suppose $A \subseteq B$. Let $x \in vint(A)$. Then x is a v-interior point of A and hence there exists a v-open set G such that $x \in G \subseteq A$. Since $A \subseteq B$, then $x \in G \subseteq B$. Therefore x is a v-interior point of A. Hence $x \in vint(B)$. This proves (iii).
(iv) Since semi*-open set is v-open, $s^*int(A) \subseteq vint(A)$. Every open set is semi*-open, $int(A) \subseteq s^*int(A)$. Therefore $int(A) \subseteq s^*int(A) \subseteq vint(A) \subseteq A$. This proves (iv).
(v) By Remark 3.2, $vint(A)$ is v-open and by Theorem 3.4, $vint(vint(A)) = vint(A)$. This proves (v).

Theorem 3.6. Let A and B are the subsets of a topological space X. Then,

(i) $vint(A) \cup vint(B) \subseteq vint(A \cup B)$.
(ii) $vint(A \cap B) \subseteq vint(A) \cap vint(B)$.

Proof:

(i) Let A and B be subsets of X. We have $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Theorem 3.5(iii), $vint(A) \subseteq vint(A \cup B)$ and $vint(B) \subseteq vint(A \cup B)$ which implies that, $vint(A) \cup vint(B) \subseteq vint(A \cup B)$. This proves (i).
(ii) We have $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Then by Theorem 3.5(iii), $vint(A \cap B) \subseteq vint(A)$ and $vint(A \cap B) \subseteq vint(B)$ which implies, $vint(A \cap B) \subseteq vint(A) \cap vint(B)$. This proves (ii).

Theorem 3.7. For any subset A of X,

(i) $int(vint(A)) = int(A)$
(ii) $vint(int(A)) = int(A)$.

Proof:(i) Since $vint(A) \subseteq A$, then $int(vint(A)) \subseteq int(A)$. By Theorem 3.5(iv), $int(A) \subseteq (vint(A))$, we have $int(A) = int(int(A)) \subseteq int(vcl(A))$. Hence $int(vint(A)) = int(A)$.

(ii) Since $int(A)$ is open and hence v-open, by Theorem 3.3, $vint(int(A)) = int(A)$.

4. v-CLOSURE OPERATOR

Definition 4.1. Let A be a subset of a topological space (X, τ). Then the intersection of all v-closed sets in X containing A is called the v-closure of A and it is denoted by $vcl(A)$. That is, $vcl(A) = \cap \{F: A \subseteq F \text{ and } F \in v - C(X)\}$.

Remark 4.2. Since the intersection of v-closed set is v-closed, then $vcl(A)$ is v-closed.

Theorem 4.3. Let A be a subset of a topological space (X, τ). Then

(i) $vcl(A)$ is the smallest v-closed set containing A.
(ii) A is v-closed if and only if $vcl(A)$=A.

Proof:

(i) Being the intersection of all v-closed sets, $vcl(A)$ is v-closed and contained in every v-closed set containing A. Hence $vcl(A)$ is the smallest v-closed set containing A.
(ii) Necessity: Suppose A is v-closed. Then by Definition 4.1, $vcl(A) \subseteq A$. But $A \subseteq vcl(A)$ and therefore $vcl(A) = A$. Sufficiency: Suppose $vcl(A) = A$. Then by Remark, $vcl(A)$ is v-closed set. Hence A is v-closed.

Theorem 4.4. Let A and B be a two subsets of a topological space (X, τ). Then the following results hold.
(i) $vcl(\phi) = \phi$ and $vcl(X) = X$.
(ii) If B is any v-closed set containing A, then $vcl(A) \subseteq B$.
(iii) If $A \subseteq B$, then $vcl(A) \subseteq vcl(B)$.
(iv) $A \subseteq vcl(A) \subseteq s^*cl(A) \subseteq cl(A)$.
(v) $vcl(vcl(A)) = vcl(A)$.

Proof:

(i) Since ϕ is v-closed and $vcl(\phi)$ is the intersection of all v-closed sets containing ϕ, $vcl(\phi) = \phi$. since X is the only v-closed set containing X, then $vcl(X) = X$.
(ii) Suppose B is v-closed set containing A. Since $vcl(A)$ is the intersection of all v-closed set containing A, then we have $vcl(A) \subseteq B$.
(iii) suppose $A \subseteq B$. Let F be any v-closed set containing B. Since $A \subseteq B$, then $A \subseteq F$ and hence by part (ii), $vcl(A) \subseteq F$. Therefore $vcl(A) \subseteq \cap \{F / B \subseteq F$ and F is v-closed$\} = vcl(B)$. This proves (iii).
(iv) Since semi*-closed set is v-closed, $vcl(A) \subseteq s^*cl(A)$ and every closed set is v-closed, $vcl(A) \subseteq cl(A)$. Therefore $A \subseteq vcl(A) \subseteq s^*cl(A) \subseteq cl(A)$. This proves (iv).
(v) By Remark 4.2, $vcl(A)$ is v-closed and by Theorem 4.3, $vcl(vcl(A)) = vcl(A)$. This proves (v).

Theorem 4.5. Let A and B be subsets of a topological space (X, τ). Then,

(i) $vcl(A) \cup vcl(B) \subseteq vcl(A \cup B)$.
(ii) $vcl(A \cap B) \subseteq vcl(A) \cap vcl(B)$.

Proof:(i) Let A and B be subsets of X. We have $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Theorem 4.4 (iii), $vcl(A) \subseteq vcl(A \cup B)$ and $vcl(B) \subseteq vcl(A \cup B)$ which implies that, $vcl(A) \cup vcl(B) \subseteq vcl(A \cup B)$. This proves (i). (ii) We have $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Then by Theorem 4.4(iii), $vcl(A \cap B) \subseteq vcl(A)$ and $vcl(A \cap B) \subseteq vcl(B)$ which implies, $vcl(A \cap B) \subseteq vcl(A) \cap vcl(B)$. This proves (ii).

Theorem 4.6. For a subset A of X and $x \in X$, $x \in vcl(A)$ if and only if $V \cap A \neq \phi$ for every v-open set V containing x.

Proof: Necessity: Let $x \in vcl(A)$. Suppose there is a v-open set V containing x such that $V \cap A = \phi$. Then $A \subseteq X \backslash V$ and $X \backslash V$ is v-closed and hence $vcl(A) \subseteq X \backslash V$. Since $x \in vcl(A)$, then $x \in X \backslash V$ which contradicts to $x \in V$.

Sufficiency: Assume that $V \cap A \neq \phi$ for every v-open set V containing x. Suppose $x \notin vcl(A)$. Then there exists a v-closed set F such that $A \subseteq F$ and $x \notin F$. Therefore $x \in X \backslash F$, $A \cap (X \backslash F) = \phi$ and $X \backslash F$ is v-open. This is a contradiction to our assumption. Hence $x \in vcl(A)$.

Theorem 4.7. For any subset A of X,

(i) $cl(vcl(A)) = cl(A)$
(ii) $vcl(cl(A)) = cl(A)$.

Proof:(i) Since $A \subseteq vcl(A)$, then $cl(A) \subseteq cl(vcl(A))$. By Theorem 4.4(iv), $vcl(A) \subseteq cl(A)$, we have $cl(vcl(A)) \subseteq cl(cl(A)) = cl(A)$. Hence $cl(vcl(A)) = cl(A)$. (ii) Since $cl(A)$ is closed and hence v-closed, by Theorem 4.3, $vcl(cl(A)) = cl(A)$.

REFERENCES

[1] Dunham, W., A New Closure Operator for Non-T1 Topologies, *Kyungpook Math. J*. 22 (1982), 55-60.
[2] Levine, N., Semi-Open Sets and Semi-Continuity in Topological Space, *Amer. Math. Monthly*. 70 (1963), 36-41.
[3] Levine, N., Generalized Closed Sets in Topology, *Rend. Circ. Mat. Palermo*.19 (2) (1970), 89-96.
[4] Robert, A., and Pious Missier, S. A New Class of Nearly Open Sets, International Journal of mathametical archive, International Journal of Mathematical Archive-3(7), 2012, 2575-2582.
[5] Saranya, S., and Bageerathi. K., International Journal of Mathematical Archive-8(12), 2017, 140-144.
[6] Selvi. T and A. Punitha Dharani, Some new class of nearly closed and open sets, Asian Journal of Current Engineering and Maths 1:5 Sep – Oct (2012) 305-307.