# Bianchi Type-I Dark Energy Cosmological Model With Polytropic Equation Of State In Barber's Second Self-Creation Cosmology 

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#### Abstract

In this paper, we have studied the Bianchi type-I universe with polytropic equation of state in the framework of the second self-creation theory of gravitation proposed by Barber [1]. The field equations have been solved by using (i) the power law relation between the average scale factor ' $a$ ' and the scalar field ' $\phi$ ' and (ii) the special law of variation for Hubble's parameter proposed by Berman [2]. Some physical and kinematical aspects of the models are also discussed.


Keywords - Bianchi type-I universe, Dark energy, second self creation theory.

## I. Introduction

Modified theories of gravitation provide gravitational alternatives for dark energy to explain early inflation and late time acceleration of the universe. To extend the concept of theory of general relativity, Brans and Dicke [3] formulated a scalar tensor theory of gravitation which includes a long range scalar field interacting equally with all forms of matter with the exception of electromagnetism. Barber [1] has formulated two continuous self creation theories by the general relativity and Brans and Dicke (BD) theory. The Barber's first theory is a modification of BD theory and the second theory is a modification of general relativity. Several authors Mohanty [4, 5], Pradhan [6], Singh and Kumar [7], Rao [8, 9], Reddy [10], Katore [11], Pawar [12], Naidu [13] and Santhi [14] have investigated various cosmological models in Barber's second self creation theory.

In this paper, we have studied the polytropic gas model of dark energy to explain the cosmic acceleration of the universe. In stellar astrophysics, the polytropic gas model can explain the equation of state of degenerate white dwarfs, neutron stars and also the equation of state of main sequence stars (Christensen-Dalsgaard [16]). Mukhopadhyay and Ray [17] has been investigated the idea of dark energy with polytropic gas equation of state in cosmology. Recently, several authors Karami et al.[17], Karami and Ghaffari [18], Setare et al. [19], Taji and Malekjani [20], Rahman [21] and Adhav [22] have investigated polytropic gas models in different contexts.
For a physically realistic relativistic star we expect that the matter distribution should satisfy a barotropic equation of state $p_{\Lambda}=p_{\Lambda}(\rho)$. In this paper we assume the polytropic equation of state
$p_{A}=K \rho_{A}^{1+\frac{1}{n}}$,
where $K$ is a real constant and $n$ is the polytropic index (Christensen-Dalsgaard [16]).
Motivated by the above investigations, here we take up the study of anisotropic Bianchi type-I universe with polytropic equation of state in the framework Barber's second self-creation cosmology. This is relevant because of the fact that scalar field plays a vital role in the discussion of DE models.
This paper is organized as follows. In section 2, the metric and field equations are described. Section 3 is devoted to the solution of the field equations and we obtained physical properties of model using the law of variation of parameter. Section 4 we discuss the physical properties of models and section 5 contains some concluding remarks.

## II. The Metric and Field Equations

We consider the homogeneous and anisotropic Bianchi type-I metric as

$$
\begin{equation*}
d s^{2}=-d t^{2}+A^{2} d x^{2}+B^{2} d y^{2}+C^{2} d z^{2} \tag{2}
\end{equation*}
$$

where the scale factors $A, B$ and $C$ are functions of time $t$ only.

The field equations in Barber's self creation theory are
$R_{i j}-\frac{1}{2} g_{i j} R=-8 \pi \phi^{-1}\left(T_{i j}\right)$
and
$\phi_{; k}^{k}=\frac{8 \pi \eta}{3} T_{j}^{i}$,
where $\eta$ is coupling constant.
The energy-momentum tensor for DE and DM are respectively given by
${ }^{m} T_{j}^{i}=\operatorname{diag}\left[-\rho_{m}, p_{m}, p_{m}, p_{m}\right]$
and
${ }^{\Lambda} T_{j}^{i}=\operatorname{diag}\left[-\rho_{\Lambda}, p_{\Lambda}, p_{\Lambda}, p_{\Lambda}\right]$.
(6)

Here $\rho_{m}$ and $p_{m}$ are energy density and pressure of matter respectively while $\rho_{A}$ and $p_{A}$ are respectively the energy density and pressure of the dark energy.
In co-moving coordinate system, the field equations (3) and (4) for the metric (2), reduce to following set of equations:
$\frac{\dot{A} \dot{B}}{A B}+\frac{\dot{B} \dot{C}}{B C}+\frac{\dot{A} \dot{C}}{A C}=8 \pi \phi^{-1}\left(\rho_{m}+\rho_{A}\right),(7) \frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B} \dot{C}}{B C}=-8 \pi \phi^{-1} p_{A}$,
$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A} \dot{C}}{A C}=-8 \pi \phi^{-1} p_{A}$,
$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A} \dot{B}}{A B}=-8 \pi \phi^{-1} p_{A}$,
$\ddot{\phi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \dot{\phi}=\frac{8 \pi \eta}{3}\left(\rho_{m}+\rho_{A}-3 p_{A}\right)$,
where an overhead dot denotes differentiation with respect to time $t$.
The spatial volume for Bianchi type-I metric is given by
$V=A B C$.

The average scale factor ' $a$ ' of Bianchi type-I metric is given by
$a=(A B C)^{\frac{1}{3}}$.

The mean Hubble's parameter $H$ is given by
$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)$.

The physical quantities i.e. the scalar expansion $(\boldsymbol{\theta})$, anisotropy parameter $(\boldsymbol{\Delta})$, shear scalar deceleration parameter $(q)$ are defined as
$\theta=3 H$,
$\Delta=\frac{1}{3} \sum_{i=1}^{3}\left(\frac{H_{i}-H}{H}\right)^{2}$,
where $H_{1}=\frac{\dot{A}}{A}, H_{2}=\frac{\dot{B}}{B}, H_{3}=\frac{\dot{C}}{C}$,
$\sigma^{2}=\frac{1}{2}\left(\sum_{i=1}^{3} H_{i}{ }^{2}-3 H^{2}\right)=\frac{3}{2} \Delta H^{2}$,
$q=-\frac{a \ddot{a}}{\dot{a}^{2}}$.

Subtracting Eq. (8) from Eq. (9), Eq. (9) from Eq. (10) and Eq. (8) from Eq. (10), we obtain
$\frac{d}{d t}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right) \frac{\dot{V}}{V}=0$,
$\frac{d}{d t}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right) \frac{\dot{V}}{V}=0$,
$\frac{d}{d t}\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right) \frac{\dot{V}}{V}=0$.

Integrating Eqs. (19), (20) and (21), we obtain
$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_{1}}{V}$,
$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_{2}}{V}$,
$\frac{\dot{A}}{A}-\frac{\dot{C}}{C}=\frac{k_{3}}{V}$,
where $k_{1}, k_{2}$ and $k_{3}$ are constants of integration. After solving Eqs.(22) to (24), with some simplification, we get
$\frac{A}{B}=d_{1} \exp \left(k_{1} \int \frac{d t}{V}\right)$,
$\frac{B}{C}=d_{2} \exp \left(k_{2} \int \frac{d t}{V}\right)$,
$\frac{A}{C}=d_{3} \exp \left(k_{3} \int \frac{d t}{V}\right)$,
where $d_{1}, d_{2}$ and $d_{3}$ are constants of integration. Using Eqs. (25), (26) and (27), we can write the metric functions $A, B$ and $C$ explicitly as

$$
A=a_{1} a \exp \left(b_{1} \int \frac{d t}{V}\right)
$$

$B=a_{2} a \exp \left(b_{2} \int \frac{d t}{V}\right)$,
$C=a_{3} a \exp \left(b_{3} \int \frac{d t}{V}\right)$,
where

$$
a_{1}=\left(d_{1} d_{2}\right)^{\frac{1}{3}}, a_{2}=\left(d_{1}^{-1} d_{3}\right)^{\frac{1}{3}}, a_{3}=\left(d_{2} d_{3}\right)^{-\frac{1}{3}}, b_{1}=\frac{k_{1}+k_{2}}{3}, b_{2}=\frac{k_{3}-k_{1}}{3}, b_{3}=-\frac{k_{2}+k_{3}}{3}
$$

which satisfies the relations $a_{1} a_{2} a_{3}=1$ and $b_{1}+b_{2}+b_{3}=0$.

## III. Solutions of Field Equations

The field equations (7) to (11) are system of five field equations in seven unknowns $A, B, C, \rho_{m}, \rho_{A}, p_{\Lambda}$ and $\phi$. Hence in order to obtain the deterministic solution we use the following two constraints:
(i) The power law relation between the average scale factor ' $a$ ' and the scalar field ' $\phi$ ' is given by $\phi=\alpha a^{\beta}$,
where $\alpha$ and $\beta$ are constants.
(ii) The special law of Hubble's parameter proposed by Berman [2] which yields the constant deceleration parameter which is defined by

$$
\begin{equation*}
H=D(A B C)^{-m / 3}=D V^{-m / 3}, \tag{32}
\end{equation*}
$$

where $D>0, m \geq 0$ are constants.
We obtain two cosmological models: A. model for $m=0$ and B. model for $m \neq 0$.

## A. for $m=0$ (Exponential Volumetric Expansion Model)

For $m=0$, From Eq. (32) the volume scale factor is given by
$V=c_{3} e^{3 D t}$,
where $c_{3}>0$ is constant of integration.

Using Eq. (33) in Eq. (31), we obtain the expression for scalar field as

$$
\begin{equation*}
\phi=\alpha c_{3}^{\beta / 3} e^{\beta D t} \tag{34}
\end{equation*}
$$

Using Eq. (33) in Eqs. (28)-(30), we obtain the scale factors as

$$
\begin{align*}
& A=a_{1}\left(c_{3}\right)^{1 / 3} \exp \left(D t-\frac{b_{1}}{3 D c_{3}} e^{-3 D t}\right), \\
& B=a_{2}\left(c_{3}\right)^{1 / 3} \exp \left(D t-\frac{b_{2}}{3 D c_{3}} e^{-3 D t}\right), \tag{35}
\end{align*}
$$

$$
C=a_{3}\left(c_{3}\right)^{1 / 3} \exp \left(D t-\frac{b_{3}}{3 D c_{3}} e^{-3 D t}\right) .
$$

Using Eqs. (35)-(37) in Eq. (7), we obtain the energy density of matter as
$\rho_{m}=\frac{\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{c_{3}^{2}} e^{-6 D t}\right)-\rho_{\Lambda}$.

Using Eqs. (35)-(37) in Eq. (8), the pressure of DE is given by
$p_{\Lambda}=\frac{-\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{c_{3}^{2}} e^{-6 D t}\right)$.

Using Eq. (39) in Eq. (1), the energy density of DE is given by
$\rho_{\Lambda}=K^{-n / n+1}\left\{\frac{-\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{c_{3}^{2}} e^{-6 D t}\right)\right\}^{n / n+1}$.

Using Eqs. (30)-(31), we obtain
The EoS parameter $\omega_{\Lambda}=\frac{p_{A}}{\rho_{A}}$ of DE is given by
$\omega_{\Lambda}=K^{n / n+1}\left\{\frac{-\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{c_{3}^{2}} e^{-6 D t}\right)\right\}^{1 / n+1}$.

Using Eqs.(35)-(37), we obtain the mean Hubble's parameter(H), scalar expansion $(\boldsymbol{\theta})$, anisotropy parameter ( $\Delta$ ), shear scalar ( $\sigma$ ) and deceleration parameter $(q)$ as
$H=D$,
$\theta=3 D$,
$\Delta=\frac{1}{3 D^{2}}\left(\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{c_{3}^{2}} e^{-6 D t}\right)$,
$\sigma^{2}=\frac{1}{2}\left(\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{c_{3}^{2}} e^{-6 D t}\right)$,
$q=-1$.

The coincidence parameter $\bar{r}=\frac{\rho_{m}}{\rho_{A}}$ i.e. the ratio of matter energy density and DE density is given by
$\bar{r}=\frac{\frac{\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{c_{3}^{2}} e^{-6 D t}\right)}{K^{-n / n+1}\left\{\frac{-\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{c_{3}^{2}} e^{-6 D t}\right)\right\}}-1$.

The expression for matter energy density $\Omega_{m}$ and dark energy density $\Omega_{\Lambda}$ are given by

$$
\begin{equation*}
\Omega_{m}=\frac{\rho_{m}}{3 H^{2}}=\frac{1}{3 D^{2}}\left\{\frac{\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{c_{3}^{2}} e^{-6 D t}\right)-K^{-n / n+1}\left(\frac{-\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{c_{3}^{2}} e^{-6 D t}\right)^{n / n+1}\right)\right\} . \tag{48}
\end{equation*}
$$

and

$$
\begin{equation*}
\Omega_{\Lambda}=\frac{\rho_{\Lambda}}{3 H^{2}}=\frac{K^{-1 / / n+1}}{3 D^{2}}\left(\frac{-\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{c_{3}^{2}} e^{-6 D t}\right)^{n / n+1}\right) . \tag{49}
\end{equation*}
$$

Using Eqs. (48) and (49), we obtain total energy density parameter $\Omega=\Omega_{m}+\Omega_{\Lambda}=\frac{1}{3 D^{2}}\left\{\frac{\phi}{8 \pi}\left(3 D^{2}+\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{c_{3}^{2}} e^{-6 D t}\right)\right\}$.

## B. Model for $m \neq 0$ (Power Law Volumetric Expansion Model)

For $m \neq 0$, from Eq. (32) the volume scale factor is given by $V=\left(m D t+c_{4}\right)^{3 / m}$,
where $c_{4}$ is a constant of integration.
Using Eq.(51) in (31), we obtain the expression for scalar field as

$$
\begin{equation*}
\phi=\alpha\left(m D t+c_{4}\right)^{\beta / m} . \tag{52}
\end{equation*}
$$

Using Eq. (51) in Eqs. (28)-(30), we obtain the scale factors as

$$
A=a_{1}\left(m D t+c_{4}\right)^{1 / m} \exp \left(\frac{b_{1}}{(m-3) D}\left(m D t+c_{4}\right)^{\frac{m-3}{m}}\right)
$$

$$
B=a_{2}\left(m D t+c_{4}\right)^{1 / m} \exp \left(\frac{b_{2}}{(m-3) D}\left(m D t+c_{4}\right)^{\frac{m-3}{m}}\right),
$$

$$
\begin{equation*}
C=a_{3}\left(m D t+c_{4}\right)^{1 / m} \exp \left(\frac{b_{3}}{(m-3) D}\left(m D t+c_{4}\right)^{\frac{m-3}{m}}\right) . \tag{54}
\end{equation*}
$$

Using Eqs. (53)-(55) in Eq. (7), we obtain the energy density of matter as
$\rho_{m}=\frac{\phi}{8 \pi}\left(\frac{3 D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1} b_{2}+b_{2} b_{3}+b_{1} b_{3}}{\left(m D t+c_{4}\right)^{6 / m}}\right)-\rho_{A}$.

Using Eqs. (53)-(55) in Eq. (8), the pressure of DE is given by

$$
p_{A}=\frac{-\phi}{8 \pi}\left(\frac{(3-2 m) D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{\left(m D t+c_{4}\right)^{5^{6} / m}}\right) .
$$

Using Eq. (57) in Eq. (1), the energy density of DE is given by

$$
\rho_{\Lambda}=K^{-n / n+1}\left\{\frac{-\phi}{8 \pi}\left(\frac{(3-2 m) D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{\left(m D t+c_{4}\right)^{6 / m}}\right)\right\}^{n / n+1}
$$

The EoS parameter $\omega_{\Lambda}=\frac{p_{\Lambda}}{\rho_{\Lambda}}$ of DE is given by

$$
\omega_{\Lambda}=K^{n / n+1}\left\{\frac{-\phi}{8 \pi}\left(\frac{(3-2 m) D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{\left(m D t+c_{4}\right)^{6 / m}}\right)\right\}^{1 / n+1} .
$$

Using Eqs. (53)-(55), we obtain the mean Hubble's parameter(H), scalar expansion $(\boldsymbol{\theta})$, anisotropy parameter ( $\Delta$ ), shear scalar $(\sigma)$ and deceleration parameter $(q)$ as
$H=\frac{D}{\left(m D t+c_{4}\right)}$,
$\theta=\frac{3 D}{\left(m D t+c_{4}\right)}$,
$\Delta=\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{3 D^{2}}\left(m D t+c_{4}\right)^{2(m-3) / m},(62) \sigma^{2}=\frac{b_{1}^{2}+b_{2}^{2}+b_{3}^{2}}{2}\left(m D t+c_{4}\right)^{-6 / m}$,
$q=m-1$.
(64)

The coincidence parameter $\bar{r}=\frac{\rho_{m}}{\rho_{A}}$ i.e. the ratio of matter energy density and DE density is given by
$\bar{r}=\frac{\frac{\phi}{8 \pi}\left(\frac{3 D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1} b_{2}+b_{2} b_{3}+b_{1} b_{3}}{\left(m D t+c_{4}\right)^{4 / m}}\right)}{K^{-n / n+1}\left\{\frac{-\phi}{8 \pi}\left(\frac{(3-2 m) D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{\left(m D t+c_{4}\right)^{6 / m}}\right)\right\}}-1$.

The expression for matter energy density $\Omega_{m}$ and dark energy density $\Omega_{\Lambda}$ are given by $\Omega_{m}=\frac{\rho_{m}}{3 H^{2}}=\frac{\left(m D t+c_{4}\right)^{2}}{3 D^{2}}\left\{\frac{\phi}{8 \pi}\left(\frac{3 D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1} b_{2}+b_{2} b_{3}+b_{1} b_{3}}{\left(m D t+c_{4}\right)^{6 / m}}\right)-K^{-n / n+1}\left(\frac{-\phi}{8 \pi}\left(\frac{(3-2 m) D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{\left(m D t+c_{4}\right)^{6 / m}}\right)\right)\right\}$.
and
$\Omega_{\Lambda}=\frac{\rho_{\Lambda}}{3 H^{2}}=\frac{K^{-n / n+1}\left(m D t+c_{4}\right)^{2}}{3 D^{2}}\left(\frac{-\phi}{8 \pi}\left(\frac{(3-2 m) D^{2}}{\left(m D t+c_{4}\right)^{2}}+\frac{b_{1}^{2}+b_{2}^{2}+b_{1} b_{2}}{\left(m D t+c_{4}\right)^{6 / m}}\right)\right)^{n / n+1}$.

Using Eqs. (66) and (67), we obtain total energy density parameter $\Omega=\Omega_{m}+\Omega_{\Lambda}=\frac{\phi}{24 \pi D^{2}}\left(3 D^{2}+\left(b_{1} b_{2}+b_{2} b_{3}+b_{1} b_{3}\right)\left(m D t+c_{4}\right)^{2(m-3) / m}\right)$.

The physical and kinematical properties of the model are as follows

## A. Barber's Scalar Function $\phi$

From Fig.1, it is observed that, for both the models, the scalar function $\phi$ is increasing function of time.


Fig. 1 The plot of scalar function $\phi$ versus cosmic time $t$ for $D=1, m=0,0.5, \quad c_{3}=0.2, c_{4}=0.1$

## B. Expansion Scalar $\theta$



Fig. 2 The plot of expansion scalar $\theta$ versus cosmic time $t$ for $D=1, m=0,0.5, c_{3}=0.2, c_{4}=0.1$
From Fig. 2, it is observed that, for model $m=0$ (i.e. exponential volumetric expansion model) the expansion scalar $\theta$ is constant throughout the evolution of the universe and $d H / d t=0$, which implies that greater the value of Hubble's parameter faster the rate of expansion of universe. Whereas for model $m \neq 0$ (i.e. power law volumetric expansion model) the expansion $\operatorname{scalar} \theta$ is infinite at $t=0$ and it tends to zero at some finite time.

## C. Shear Scalar $\sigma$

From Fig. 3, it is observed that, in both the models, the shear scalar $\sigma \rightarrow \infty$ as $t \rightarrow 0$ and it tends to zero as time is increases.


Fig. 3 The plot of shear scalar $\sigma$ versus cosmic time $t$ for $D=1, m=0,0.5, c_{3}=0.2, c_{4}=0.1$

## D. The Anisotropy Parameter $\Delta$

Fig. 4 is the plot of anisotropy parameter $\Delta$ versus time $t$. It shows that, in both the models anisotropy parameter $\Delta \rightarrow \infty$ as $t \rightarrow 0$ and it tends to zero as time is increases.


Fig. 4 The plot of anisotropy parameter $\Delta$ versus cosmic time $t$ for $D=1, m=0,0.5, c_{3}=0.2, c_{4}=0.1$
The anisotropy parameter $\Delta \neq 0$, therefore the models does not approach isotropy.

## $E$. The deceleration parameter $q$

We have the sign of q indicates whether the models inflates or not. The positive sign of $q$ indicates the model is decelerating where as negative sign of $q$ indicates the inflationary model. From Eqs. (46) and (64) it is observed that, the deceleration parameter the deceleration parameter $q$ is negative for $m=0$ and $0<m<1$, which shows that the universe undergoes accelerating expansion. The model for $m=0$ is inflationary whereas model for $m \neq 0$ is inflationary for $0<m<1$ but for $m>1$, the model is decelerating model.

## F. The EoS Parameter $\omega_{\Lambda}$

The behaviour of equation of state parameter in terms of cosmic time $t$ is shown in Fig. 5. This figure shows that the EoS parameter for model $m=0$ is varies in phantom region $\left(\omega_{\Lambda}<-1\right)$, whereas the EoS parameter for model $m \neq 0$ is starts from phantom region $\left(\omega_{\Lambda}<-1\right)$, increases rapidly and attains the value $\left(\omega_{\Lambda}=-1\right)$ after some finite time $t$. i.e. our model approaches to $\Lambda$ CDM model after some finite time. The $\Lambda$ CDM models are best candidate for describing the cosmological evolution of the universe. Hence our models are in good agreement with well established theoretical results (Spergel [23], Riess [24, 25], Astier [26] and Bamba [27].


Fig. 5 The plot of EoS parameter $\omega_{A}$ versus cosmic time $t$ for $D=1, m=0,0.5, c_{3}=0.2, c_{4}=0.1$

## G. Coincidence Parameter

The coincidence parameter is the ratio of two energy densities ( $\bar{r}=\frac{\rho_{m}}{\rho_{A}}$ ). From Eqs. (47) and (65), it is observed that the coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem(unlike $\Lambda \mathrm{CDM}$ ).

## H. Overall Density Parameter $\Omega$

From Fig. 6, it is observe that for $0<m<1$ the overall density parameter approaches to one as $T \rightarrow \infty$. Thus derived models predict a flat universe at late time.


Fig. 6 The plot of total energy density $\Omega$ versus cosmic time $t$ for $D=1, m=0,0.5, c_{3}=0.2, c_{4}=0.1$

## V. CONCLUSION

In this paper, we have we have studied the anisotropic Bianchi type-I cosmological models with polytropic DE in the framework of Barber's second self creation cosmology (1982). The exact solutions of field equations have been obtained by considering (i) the power law relation between the average scale factor ' $a$ ' and the scalar field ' $\phi$ ' and (ii) the special law of variation for Hubble's parameter proposed by Berman (1983). Model for $m=0$ is non-singular, since spatial volume $V=c_{3} e^{3 D t}$ not vanishes at any values of $t$. Therefore model
$m=0$ does not have any physical singularity. But for model $m \neq 0$ have a big bang singularity at $t=-c_{4} / m D$, since spatial volume $V=\left(m D t+c_{4}\right)^{3 / m}$ is zero at $t=-c_{4} / m D$. The physical and kinematical parameters such as Barber's scalar function, anisotropic parameter, shear scalar, expansion scalar, deceleration parameter, EoS parameter, coincidence parameter and total energy density are discussed.

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