Goal Programming Approach to Solve Multi-Objective Intuitionistic Fuzzy Non- Linear Programming Models

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Abstract — ***I****n this Paper Multi-objective Intuitionistic Fuzzy Non-linear Programming Problem has been solved through goal programming approach. Solving non-linear programming problem straightaway is quite difficult. So, the Multi-objective Intuitionistic Fuzzy Non-linear Programming Problem is transformed into Multi-objective Intuitionistic Fuzzy Linear Programming Problem by the use of Taylor Polynomial Series. Intuitionistic fuzzy goal programming approach is discussed by forming suitable membership and non-membership function in which the aspiration level and the tolerance levels are fixed by the decision maker or by finding the best and worst solution of each objective function. Using the membership and non-membership function of each objective, the given problem was formulated and the solutions are found by various approaches and the obtained solutions are identical. Numerical example will illustrate the efficiency of the proposed approach.*

Keywords — *Non-Linear Programming, Goal Programming, Intuitionistic Fuzzy Set.*

I. Introduction

Like Linear Programming, Non- Linear Programming is a mathematical technique for determining the optimal solutions to many business problems. Linear Programming problem is characterized by the presence of linear relationship in decision variables in the objective function as well as linear constraints. But in the real life systems, neither the objective function nor the constraints are linear functions in decision variables, i.e., the relationships are non-linear.

Samir Dey et al.,(2015) proposes an intuitionistic fuzzy optimization approach to solve a non-linear programming problem in the context of structural application. Mahapatra.G.S et al.,(2010) solved a intuitionistic multi-objective nonlinear programming problem in the context of reliability application. Based on these, Multi-objective Intuitionistic Fuzzy Non-linear Programming Problem has been solved through goal programming approach in this paper.

In the first section, Multi-objective Intuitionistic Fuzzy Non-linear Programming Problem is discussed and Taylor Polynomial series is introduced in the subsequent section. The membership and non-membership functions are formed according to the acceptance and rejection level in the third section. Various formulations of the problem are discussed in the fourth section. Step-wise solution procedure is explained in the next section. Finally, a numerical example is given and it is solved by various formulations in two cases and the obtained solutions are discussed at the end.

II. Preliminaries

A. Formulation of Multi-Objective Intuitionistic Fuzzy Non-Linear Programming Problem

Consider the Multi-objective Intuitionistic FuzzyNon-linear Programming Problem,

Max or Min$Z(X) = (\tilde{f}_1(x), \tilde{f}_2(x), \dots\dots(\tilde{f}_k(x))$ (1)

Subject to the constraints

$$g^i(x_j)\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b_i, i = 1,2,\dots,m; j = 1,2,\dots,n$$

(2)

and $x_j \geq 0, j = 1,2,\ldots\ldots,n$

In the above, g^i 's are real valued functions of n variables $x_1, x_2, \ldots\ldots, x_n$ and each $(\tilde{f}_1(x), \tilde{f}_2(x), \ldots\ldots.(\tilde{f}_k(x))$ are intuitionistic fuzzy non-linear functions.

To overcome the computational difficulties of solving non-linear programming problems, the following Taylor olynomial series is used to transform the non-linear function into linear function.

B. Taylor Polynomial Series

For each objective function $f_i(x), i = 1,2,\ldots,k$ the individual optimized value $x_i^* = (x_{i1}^*, x_{i2}^*, \ldots\ldots, x_{in}^*)$ is to be determined and transform the non-linear objectives $f_i(x), i = 1,2,\ldots,k$ into linear objectives by the use of Taylor Polynomial Series,

$$f_i(x) \cong P_i(x) = f_i(x^*) + \left[(x_1 - x_{i1}^*)\left(\frac{\partial f_i(x_i^*)}{\partial x_1}\right) + (x_2 - x_{i2}^*)\left(\frac{\partial f_i(x_i^*)}{\partial x_2}\right) + \cdots + (x_n - x_{in}^*)\left(\frac{\partial f_i(x_i^*)}{\partial x_n}\right)\right] \quad (3)$$

By replacing the non-linear function $f_i(x)$ by $f_i(x) \cong P_i(x)$ of all objective functions $f_i(x), i = 1,2,\ldots\ldots,k$ become the linear function.

Using Taylor Polynomial series, the given non-linear functions are transformed into linear functions. According to these linear functions, the following membership and non-membership functions formed.

C. Formulation of Membership and non-membership functions

Case (i) : The aspiration level and tolerance levels are fixed by the decision maker
Form the membership and non-membership functions for $f_i(x) \geq g_i$

$$\mu_i(x) = \begin{cases} 1 & if \quad f_1(x) \geq g_i \\ \dfrac{f_i(x) - T_i}{g_i - T_i} & if \quad T_i \leq f_i(x) \leq g_i \\ 0 & if \quad f_i(x) \leq T_i \end{cases}$$

$$\nu_i(x) = \begin{cases} 0 & if \quad f_i(x) \geq g_i \\ \dfrac{g_i - f_i(x)}{g_i - T_i^1} & if \quad T_i^1 \leq f_i(x) \leq g_i \\ 1 & if \quad f_i(x) \leq T_i^1 \end{cases}$$

and for $f_i(x) \leq g_i$,

$$\mu_i(x) = \begin{cases} 1 & if \quad f_1(x) \leq g_i \\ \dfrac{g_i - f_i(x)}{g_i - T_i} & if \quad T_i \leq f_i(x) \leq g_i \\ 0 & if \quad f_i(x) \geq T_i \end{cases}$$

$$\nu_i(x) = \begin{cases} 0 & if \quad f_i(x) \leq g_i \\ \dfrac{f_i(x) - T_i^1}{g_i - T_i^1} & if \quad T_i^1 \leq f_i(x) \leq g_i \\ 1 & if \quad f_i(x) \geq T_i^1 \end{cases}$$

Here, T_i and T_i^1 are acceptance and rejection tolerance for the ith goal.

Case (ii): The acceptance and rejection levels are fixed by finding the best and worst solutions of each objective function.
The membership and non-membership function of maximization problem is defined as follows:

$$\mu_i(f_i(x)) = \begin{cases} 0 & if \quad f_i(x) \le L_i^{acc} \\ \dfrac{f_i(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} & if \quad L_i^{acc} \le f_i(x) \le U_i^{acc} \\ 1 & if \quad f_i(x) \ge U_i^{acc} \end{cases}$$

$$\nu_i(f_i(x)) = \begin{cases} 1 & if \quad f_i(x) \le L_i^{reg} \\ \dfrac{U_i^{reg} - f_i(x)}{U_i^{reg} - L_i^{reg}} & if \quad L_i^{reg} \le f_i(x) \le U_i^{reg} \\ 0 & if \quad f_i(x) \ge U_i^{reg} \end{cases}$$

The membership and non-membership function of minimization problem is defined as:

$$\mu_i(f_i(x)) = \begin{cases} 1 & if \quad f_i(x) \le L_i^{acc} \\ \dfrac{U_i^{acc} - f_i(x)}{U_i^{acc} - L_i^{acc}} & if \quad L_i^{acc} \le f_i(x) \le U_i^{acc} \\ 0 & if \quad f_i(x) \ge U_i^{acc} \end{cases}$$

$$\nu_i(f_i(x)) = \begin{cases} 0 & if \quad f_i(x) \le L_i^{reg} \\ \dfrac{f_i(x) - L_i^{reg}}{U_i^{reg} - L_i^{reg}} & if \quad L_i^{reg} \le f_i(x) \le U_i^{reg} \\ 1 & if \quad f_i(x) \ge U_i^{reg} \end{cases}$$

Here, $U_i^{acc} = \max\{Z_i(x)\}$ and $L_i^{acc} = \min\{Z_i(x)\}$ for membership function. Moreover, $U_i^{reg} = U_i^{acc} - \varepsilon_i$, $L_i^{reg} = L_i^{acc}$ and $U_i^{reg} = U_i^{acc}$, $L_i^{reg} = L_i^{acc} + \varepsilon_i$ are non-membership function of maximization type and minimization type respectively, $\varepsilon_i, i = 1, \ldots, k$ based on the decision maker's choice.

III. Problem Formulation

Rajesh Dangwal et al.,(2012) proposed a method to solve Multi-Objective Linear Fractional Programming Problem by vague set. Here, Multi-objective Intuitionistic Fuzzy Non-linear Programming Problem has been solved in which the membership functions can be chosen as per his method. Furthermore, the non-membership should be minimized and ν_i are any one of the four possibilities defined above and each of them are greater than or equal to zero. Moreover, in intuitionistic fuzzy sets $\mu_i + \nu_i \le 1$, the membership and non-membership are formed separately but the sum of both values are less than or equal to one another. Therefore, the intuitionistic fuzzy goal programming can be formulated as:

Formulation I

For membership

$$Max \ \lambda = \sum_{i=1}^{k} \mu_i$$

Subject to

$$\mu_i = \frac{f_i(x) - T_i}{g_i - T_i} \ / \ \mu_i = \frac{g_i - f_i(x)}{g_i - T_i}$$

(or)

$$\mu_i = \frac{f_i(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} \ / \ \mu_i = \frac{U_i^{acc} - f_i(x)}{U_i^{acc} - L_i^{acc}}$$

$$\mu_i \le 1$$

$$g^i(x_j) \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b_i$$

and $x_j, \mu_i \geq 0, i = 1,2, \dots \dots, m; j = 1,2, \dots ., n$

For non-membership

$$Min\ \lambda = \sum_{i=1}^{k} \nu_i$$

Subject to

$$\nu_i = \frac{g_i - f_i(x)}{g_i - T_i^1} \ / \ \nu_i = \frac{f_i(x) - T_i^1}{g_i - T_i^1}$$

(or)

$$\nu_i = \frac{U_i^{reg} - f_i(x)}{U_i^{reg} - L_i^{reg}} \ / \ \nu_i = \frac{f_i(x) - L_i^{reg}}{U_i^{reg} - L_i^{reg}}$$

$$\nu_i \geq 0$$

$$g^i(x_j) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_i$$

and $x_j, \nu_i \geq 0, i = 1,2, \dots \dots, m; j = 1,2, \dots ., n$

To get the best achievement, the membership should be maximized and non-membership should be minimized and in intuitionistic fuzzy sets, $0 \leq \mu_i\big(f_i(x)\big) + \nu_i\big(f_i(x)\big) \leq 1$. Therefore, the given Multi-objective Intuitionistic Fuzzy Non-linear Programming Problem can be reformulated as:

Formulation II

$$Max\ \lambda = \sum_{i=1}^{k} \{\mu_i(f_i(x)) - \nu_i(f_i(x))\}$$

Subject to

$$\mu_i\big(f_i(x)\big) \geq \nu_i\big(f_i(x)\big)$$

$$\mu_i\big(f_i(x)\big) + \nu_i\big(f_i(x)\big) < 1$$

$$\mu_i\big(f_i(x)\big) + \nu_i\big(f_i(x)\big) \geq 0$$

$$g^i(x_j) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_i$$

and $x_j, \mu_i, \nu_i \geq 0, i = 1,2, \dots \dots, m; j = 1,2, \dots ., n$

Since, the highest degree of membership is 1 and the lowest degree of non-membership is 0. Therefore, in goal programming approach,

Achieve: $\{f_i(x)/(\mu_i(f_i(x)) + d_{i1}^- - d_{i1}^+ = 1\ \&\ \nu_i(Z_i(x)) + d_{i2}^- - d_{i2}^+ = 0)\}, 1 \leq i \leq j$

In which, d_{i1}^-, d_{i2}^- and d_{i1}^+, d_{i1}^+ are the under attainment and over attainment, respectively of the i^{th} goal, the deviational variables d_{i1}^+ and d_{i2}^- can be removed from the goal programming because the overachievement of membership function and underachievement of non-membership function are acceptable.

Therefore, the given problem is formulated as:

Formulation III

$$Min\ \lambda = d_{i1}^- + d_{i2}^+$$

$$\mu_i + d_{i1}^- \geq 1$$

$$\nu_i - d_{i2}^+ \leq 0$$

$$g^i(x_j) \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_i$$

and $x_j, d_{i1}^-, d_{i2}^+ \geq 0, i = 1,2, \dots \dots, k; j = 1,2, \dots ., n$

The following solution procedure is used to solve the problem without difficulty by the above formulations.

IV. Solution Procedure

Step 1 : Determine the optimal solution for each objective function $f_i(x), i = 1,2, \dots, k$ in (1) subject to the given set of constraints (2).

Step 2 : Transform the non-linear objective function $f_i(x)$ into equivalent linear objective function at the optimal points $x_i^* = (x_{i1}^*, x_{i2}^*, \dots\dots x_{in}^*)$ by Taylor Polynomial Series (3)

Step 3 :

(i) If the decision maker desires to fix the goal, then fix the aspiration level(g_i), acceptance tolerance(T_i) and rejection tolerance(T_i^1) for each of the obtained linear objective function based on the decision maker's choice.

(ii) If the decision maker may have the decision deadlock to decide the goals, then fix the acceptance and rejection levels by finding the best and worst values of each linear objective function.

Step 4 : Construct the membership and non-membership function for each linear objective function with the help of tolerance and rejection levels in step 3.

Step 5 : Construct the intuitionistic fuzzy goal programming models I , II and III as presented in section III.

Step 6 : Solve the intuitionistic fuzzy goal programming models by the use of optimization software TORA.

The following numerical example will demonstrate the efficiency of the above solution procedure. It has been solved in two proposed cases and the results are discussed at the end.

V. Numerical Example

$Max\ f_1(x) = 2x_1 + 3x_2 - 2x_1^2$
$Max\ f_2(x) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$
$Max\ f_3(x) = 3x_1 - x_1^2 + x_2$
subject to $x_1 + 4x_2 \leq 4, x_1 + x_2 \leq 2, x_1, x_2 \geq 0$
By finding the optimized solutions for each objective function subject to the given constraints,
${f_1}^{max}(0,1) = 3, {f_2}^{max}(0,1) = 4, {f_3}^{max}(2,0) = 2$
By using Taylor's series (5.1.3), the linear objective functions are,

$$f_1(x) = 3 + \left[(x_1 - 0)\left(\frac{\partial f_1(0,1)}{\partial x_1}\right) + (x_2 - 1)\left(\frac{\partial f_1(0,1)}{\partial x_2}\right)\right]$$

$$f_2(x) = 4 + \left[(x_1 - 0)\left(\frac{\partial f_2(0,1)}{\partial x_1}\right) + (x_2 - 1)\left(\frac{\partial f_2(0,1)}{\partial x_2}\right)\right]$$

$$f_3(x) = 2 + \left[(x_1 - 2)\left(\frac{\partial f_3(2,0)}{\partial x_1}\right) + (x_2 - 0)\left(\frac{\partial f_3(2,0)}{\partial x_2}\right)\right]$$

The obtained linear functions are,
$f_1(x) = 2x_1 + 3x_2, f_2(x) = 2x_1 + 2x_2 + 2, f_3(x) = -x_1 + x_2 + 4.$
The intuitionistic fuzzy goal programming is as follows:

Case (i): If the goals and tolerance values are decided by the decision maker

According to the decision maker's choice,
$f_1(x) \geq 5, f_2(x) \geq 6\ and\ f_3(x) \geq 5$
The acceptance and rejection tolerance for goal 1(G_1), goal 2(G_2) and goal 3(G_3) are as follows:
$G_1 : T_1 = 2$ and $T_1^1 = 3$
$G_2 : T_2 = 3$ and $T_2^1 = 4$
$G_3 : T_3 = 3$ and $T_3^1 = 4$
The membership and non-membership functions formed with respect to the acceptance and rejection levels defined above are:

$$\mu_1(x)=\begin{cases}1 & if \quad f_1(x)\geq 5\\ \frac{f_1(x)-3}{2} & if \quad 3\leq f_1(x)\leq 5\\ 0 & if \quad f_1(x)\leq 3\end{cases}$$

$$\nu_1(x)=\begin{cases}0 & if \quad f_1(x)\geq 5\\ \frac{5-f_1(x)}{3} & if \quad 2\leq f_1(x)\leq 5\\ 1 & if \quad f_1(x)\leq 2\end{cases}$$

$$\mu_2(x)=\begin{cases}1 & if \quad f_2(x)\geq 6\\ \frac{f_2(x)-3}{3} & if \quad 3\leq f_2(x)\leq 6\\ 0 & if \quad f_2(x)\leq 3\end{cases}$$

$$\mu_3(x)=\begin{cases}1 & if \quad f_3(x)\geq 5\\ \frac{f_3(x)-2}{3} & if \quad 2\leq f_3(x)\leq 5\\ 0 & if \quad f_3(x)\leq 2\end{cases}$$

$$\nu_3(x)=\begin{cases}0 & if \quad f_3(x)\geq 5\\ \frac{5-f_3(x)}{4} & if \quad 1\leq f_3(x)\leq 5\\ 1 & if \quad f_3(x)\leq 1\end{cases}$$

Using these membership and non-membership function defined above, the problem will be formulated by I, II and III as follows:

Formulation I

For membership

$Max\ \lambda = 1.33x_1 + 2.5x_2 - 1.17$

Subject to

$-2\mu_1 + 2x_1 + 3x_2 = 3;$

$-3\mu_2 + 2x_1 + 2x_2 = 1;$

$3\mu_3 + x_1 - x_2 = 2\ ;$

$\mu_1 \leq 1; \mu_2 \leq 1; \mu_3 \leq 1;$

$x_1 + 4x_2 \leq 4; x_1 + x_2 \leq 2;$

And $x_1, x_2, \mu_1, \mu_2, \mu_3 \geq 0$

The optimal solution is

$x_1 = 1.33, x_2 = 0.67, \mu_1 = 0.83,\ \mu_2 = 1, \mu_3 = 0.44$

For non-membership,

$Min\ \lambda = 0.92x_1 + 1.75x_2 + 0.08$

Subject to

$3\nu_1 + 2x_1 + 3x_2 = 5;$

$4\nu_2 + 2x_1 + 2x_2 = 4;$

$4\nu_3 - x_1 + x_2 = 1;$

$\nu_1 \geq 0; \nu_2 \geq 0; \nu_3 \geq 0;$

$x_1 + 4x_2 \leq 4; x_1 + x_2 \leq 2;$

and $x_1, x_2, \nu_1, \nu_2, \nu_3 \geq 0$

The optimal solution is $x_1 = 1.33, x_2 = 0.67, \nu_1 = 0.11, \nu_2 = 0, \nu_3 = 0.42$

Formulation II

$Max\ \lambda = 2.25x_1 + 4.25x_2 - 4$

Subject to

$2x_1 + 3x_2 \leq 5; 2x_1 + 2x_2 \leq 4;$

$-2x_1 - 3x_2 \leq 1; 10x_1 + 15x_2 \geq 19;$

$7x_1 + 7x_2 \geq 8; -2x_1 - 2x_2 \leq 8;$

$7x_1 - 7x_2 \leq 5; -x_1 + x_2 \leq 1;$

$x_1 - x_2 \leq 11; x_1 + 4x_2 \leq 4;$

$x_1 + x_2 \leq 2;$

and $x_1, x_2 \geq 0$

The optimal solutions is $\boldsymbol{x_1 = 1.33, x_2 = 0.67}$
By substituting these values in the corresponding membership and non-membership functions, the following values obtained:

$$\boldsymbol{\mu_1 = 0.83, \mu_2 = 1, \mu_3 = 0.446, \nu_1 = 0.11, \nu_2 = 0, \nu_3 = 0.415}$$

Formulation III

$Min\ \lambda = d_{11}^- + d_{21}^- + d_{31}^- + d_{12}^+ + d_{22}^+ + d_{32}^+$
Subject to
$2x_1 + 3x_2 + 2d_{11}^- \geq 2; 2x_1 + 3x_2 + 3d_{12}^+ \geq 5$
$2x_1 + 2x_2 + 3d_{21}^- > 4; 2x_1 + 2x_2 + 4d_{22}^+ \geq 4;$
$-x_1 + x_2 + 3d_{31}^- \geq 1; ;-x_1 + x_2 + 4d_{32}^+ \geq 1;$
$x_1 + 4x_2 \leq 4; x_1 + x_2 \leq 2;$
and $x_1, x_2, d_{11}^-, d_{21}^-, d_{31}^-, d_{12}^+, d_{22}^+, d_{32}^+ \geq 0$

The optimal solutions are $x_1 = 1.33, x_2 = 0.67$, $d_{11}^- = 0.17, d_{21}^- = 0, d_{31}^- = 0.56, d_{12}^+ = 0.11, d_{22}^+ = 0, d_{32}^+ = 0.42$
From the membership and non-membership functions defined above, the values are,
$\mu_1 = 0.83, \mu_2 = 1, \mu_3 = 0.44, \nu_1 = 0.11, \nu_2 = 0, \nu_3 = 0.42$

Case (ii): The goals and tolerance values are fixed by finding the best and worst values of the linear functions.

The best and worst values of each of the objective functions are,
$Max\ f_1(x) = 4.67;\ Min\ f_1(x) = 0$;
$Maxf_2(x) = 6;\ Min\ f_2(x) = 2;$
$Max\ f_3(x) = 5;\ Min\ f_3(x) = 2;$

According to these values, the membership and non-membership functions are,

$$\mu_1(x) = \begin{cases} 1 & if \quad f_1(x) \geq 4.67 \\ 4.67 & if \quad 0 \leq f_1(x) \leq 4.67 \\ 0 & if \quad f_1(x) \leq 0 \end{cases}$$

$$\nu_1(x) = \begin{cases} 0 & if \quad f_1(x) \geq 4.67 - \varepsilon_1 \\ \dfrac{(4.67 - \varepsilon_1) - f_1(x)}{(4.67 - \varepsilon_1)} & if \quad 0 \leq f_1(x) \leq 4.67 - \varepsilon_1 \\ 1 & if \quad f_1(x) \leq 0 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & if \quad f_2(x) \geq 6 \\ \dfrac{f_2(x) - 2}{4} & if \quad 2 \leq f_2(x) \leq 6 \\ 0 & if \quad f_2(x) \leq 2 \end{cases}$$

$$\nu_2(x) = \begin{cases} 0 & if \quad f_2(x) \geq 6 - \varepsilon_2 \\ \dfrac{(6 - \varepsilon_2) - f_2(x)}{(6 - \varepsilon_2) - 2} & if \quad 2 \leq f_2(x) \leq 6 - \varepsilon_2 \\ 1 & if \quad f_2(x) \leq 2 \end{cases}$$

$$\mu_3(x) = \begin{cases} 1 & if \quad f_3(x) \geq 5 \\ \dfrac{f_3(x) - 2}{3} & if \quad 2 \leq f_3(x) \leq 5 \\ 0 & if \quad f_3(x) \leq 2 \end{cases}$$

$$\nu_3(x)=\begin{cases}0 & if \quad f_3(x)\geq 5-\varepsilon_3\\ \dfrac{(5-\varepsilon_3)-f_3(x)}{(5-\varepsilon_3)-2} & if \quad 2\leq f_3(x)\leq 5-\varepsilon_3\\ 1 & if \quad f_3(x)\leq 2\end{cases}$$

Using these membership and non-membership function defined above, the problem will be formulated by I, II and III for the values $\epsilon_1 = 0.02, \epsilon_2 = 0.03\ and\ \epsilon_3 = 1$ as follows:

Formulation I

For membership

$Max\ \lambda = 0.598x_1 + 1.472x_2 + 0.66$

Subject to

$4.67\mu_1 - 2x_1 - 3x_2 = 0;$

$4\mu_2 - 2x_1 - 2x_2 = 0;$

$3\mu_3 + x_1 - x_2 = 2\ ;$

$\mu_1 \leq 1; \mu_2 \leq 1; \mu_3 \leq 1;$

$x_1 + 4x_2 \leq 4; x_1 + x_2 \leq 2;$

And $x_1, x_2, \mu_1, \mu_2, \mu_3 \geq 0$

The optimal solution is $x_1 = 1.33, x_2 = 0.67, \mu_1 = 1, \mu_2 = 1, \mu_3 = 0.44$

For non-membership,

$Min\ \lambda = -0.434x_1 - 1.645x_2$

Subject to

$4.65\nu_1 + 2x_1 + 3x_2 = 4.65;$

$3.97\nu_2 + 2x_1 + 2x_2 = 3.97;$

$2\nu_3 - x_1 + x_2 = 0;$

$\nu_1 \geq 0; \nu_2 \geq 0; \nu_3 \geq 0;$

$x_1 + 4x_2 \leq 4; x_1 + x_2 \leq 2;$

and $x_1, x_2, \nu_1, \nu_2, \nu_3 \geq 0$

The optimal solution is $x_1 = 1.33, x_2 = 0.67, \nu_1 = 0, \nu_2 = 0, \nu_3 = 0.32$

Formulation II

$Max\ \lambda = 1.028x_1 + 3.117x_2 + 1.2$

Subject to

$18.64x_1 + 27.96x_2 \geq 21.72; 0.04x_1 + 0.06x_2 \geq 0;$

$0.04x_1 + 0.06x_2 \leq 21.72; 15.94x_1 + 15.94x_2 \geq 15.88;$

$0.06x_1 + 0.06x_2 \geq 0; 0.06x_1 + 0.06x_2 \leq 15.88;$

$5x_1 - 5x_2 \leq 4; x_1 - x_2 \leq 2;$

$-x_1 + x_2 \leq 4; x_1 + 4x_2 \leq 4;$

$x_1 + x_2 \leq 2;$

and $x_1, x_2 \geq 0$

The optimal solutions is $x_1 = 1.33, x_2 = 0.67$

By substituting these values in the corresponding membership and non-membership functions, the following values obtained:

$\mu_1 = 1, \mu_2 = 1, \mu_3 = 0.45, \nu_1 = 0, \nu_2 = 0, \nu_3 = 0.32$

Formulation III

$Min\ \lambda = d_{11}^- + d_{21}^- + d_{31}^- + d_{12}^+ + d_{22}^+ + d_{32}^+$

Subject to

$2x_1 + 3x_2 + 4.67d_{11}^- \geq 4.67;\ 2x_1 + 3x_2 + 4.65d_{12}^+ \geq 4.65;$

$2x_1 + 2x_2 + 4d_{21}^- \geq 4;\ 2x_1 + 2x_2 + 3.97d_{22}^+ \geq 3.97;$

$-x_1 + x_2 + 3d_{31}^- \geq 1;\ x_1 - x_2 - 2d_{32}^+ \leq 0;$

$x_1 + 4x_2 \leq 4;\ x_1 + x_2 \leq 2;$

and $x_1, x_2, d_{11}^-, d_{21}^-, d_{31}^-, d_{12}^+, d_{22}^+, d_{32}^+ \geq 0$

The optimal solutions are $x_1 = 1.33, x_2 = 0.67, d_{11}^- = 0, d_{21}^- = 0, d_{31}^- = 0.55, d_{12}^+ = 0, d_{22}^+ = 0, d_{32}^+ = 0.32$
From the membership and non-membership functions defined above, the values are,
$\mu_1 = 1, \mu_2 = 1, \mu_3 = 0.45, \nu_1 = 0, \nu_2 = 0, \nu_3 = 0.32$

Comparison of Optimal Solutions for Case (i)

Table : 1

Approach	**Solution Points** (x_1, x_2)	**Achievement Level** (μ_1, μ_2, μ_3)	**Non-achievement level** (ν_1, ν_2, ν_3)
Formulation I	(1.33,0.67)	(0.83,1,0.44)	(0.11,0,0.42)
Formulation II	(1.33,0.67)	(0.83,1,0.44)	(0.11,0,0.42)
Formulation III	(1.33,0.67)	(0.83,1,0.44)	(0.11,0,0.42)

Comparison of Optimal of Solutions for Case (ii)

Table : 2

Approach	**Solution Points** (x_1, x_2)	**Achievement Level** (μ_1, μ_2, μ_3)	**Non-achievement level** (ν_1, ν_2, ν_3)
Formulation I	(1.33,0.67)	(1,1,0.45)	(0,0,0.32)
Formulation II	(1.33,0.67)	(1,1,0.45)	(0,0,0.32)
Formulation III	(1.33,0.67)	(1,1,0.45)	(0,0,0.32)

In comparison of the optimal solutions in the three formulations under two various cases,

The values are identical in all the formulations and the possibility of the achievements under case (i), the goal 1 is 83%, goal 2 is 100% and goal 3 is 45%. The non-achievement level of goal 1 is 11%, goal 2 is 0% and goal 3 is 42%.

The values are identical in all the formulations and the possibility of the achievements under case (ii), the goal 1 is 100%, goal 2 is 100% and goal 3 is 45%. The possibilities of non-achievement of the goal 1 is 0% , goal 2 is 0% and goal 3 is 32%. i.e. the first two goals are fully achieved and the third goal is achieved by 45%.

Hence, the solutions are identical in all the formulations and in addition the second case gives more efficient solution than the first one. Moreover, in the second case the epsilon (ε) value can be changed to get the better solution.

VI. Conclusion

Multi-objective Intuitionistic Fuzzy Non-linear programming problem has been solved in two various cases under three formulations and the obtained solutions are compared at the end. Taylor's Polynomial Series is used to convert the non-linear into linear to solve the problem rather easily. The decision maker's achievement level was satisfied with good percentage in all the objectives. The stepwise solution procedure will helpful to the reader to solve the multi-objective non-linear programming problem with better solution. Moreover, the numerical example clearly explained for better understanding of the solution procedure.

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