Supra – I– Compactness and Supra – I – Connectedness

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Abstract — *In 2012 Sekar and Jayakumar introduced and investigated a new class of sets and functions between supra topological spaces called supra $I-$open sets and supra $I-$continuous functions respectively. In this paper we newly originate the notions of supra $I-$ compact spaces, supra $I-$ Lindelof spaces, countably supra $I-$compact spaces and supra $I-$ connected spaces. We also interpret their several effects and characterizations.*

Keywords — Supra Topological Space, Supra $\mathbf{I}-$ Open Set, Supra I – Compact Space, Supra $\mathbf{I}-$ Lindelof Space, Countably Supra $\mathbf{I}-$ Compact Space, Supra $\mathbf{I}-$ Connected Space.

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I. Introduction

In 1983, A. S. Mashhour et al. [29] introduced the supra topological spaces and studied $S-$continuous maps and S^*-continuous maps. In 2008, R. Devi et al [9] introduced and studied a class of sets and maps between topological spaces called supra $\alpha-$open sets and $S\alpha-$continuous maps, respectively. In 2012, S. Sekar et al [33] introduced and investigated a new class of sets and functions between topological spaces called supra $\mathbf{I}-$open sets and supra $\mathbf{I}-$continuous functions respectively. Recently Krishnaveni and Vigneshwaran [16] came out with supra $\mathbf{bT}-$ closed sets and gave their properties. In 2013, Jamal M. Mustafa [13] came out with the concept of supra $\mathbf{b}-$compact and supra $\mathbf{b}-$Lindelof spaces. Now we bring up with the new concepts of supra $\mathbf{I}-$ compact, supra $\mathbf{I}-$ Lindelof, countably supra $\mathbf{I}-$ compact and supra $\mathbf{I}-$ connected spaces and present several properties and characteristics of these concepts.

II. PRELIMINARIES

DEFINITION 2.1. Let X be a nonempty set and let $\tau^* \subseteq P(X)=\{A : A \subseteq X\}$. Then τ^* is called a supra topology on X if $\phi \in \tau^*$, $X \in \tau^*$ and for all $\gamma \subseteq \tau^*$, it implies that $\bigcup\gamma \in \tau^*$. The pair (X,τ^*) is called a supra topological space. Each element $A \in \tau^*$ is called a supra open set in (X,τ^*) and the complement of A denoted by $A^C = X - C$ is called a supra closed set in (X,τ^*).

DEFINITION 2.2. Let (X,τ^*) be a supra topological space. The supra closure of a set A is denoted by $Supra-Cl(A)$ and is defined by $Supra-Cl(A)=\bigcap\{B \subseteq X : B \text{ is a supra closed set in } X \text{ such that } A \subseteq B\}$. The supra interior of a set A is denoted by $Supra-Int(A)$ and is defined by $Supra-Int(A)=\bigcup\{U \subseteq X : U \text{ is a supra open set in } X \text{ such that } U \subseteq A\}$.

DEFINITION 2.3. Let (X,τ) be a topological space and τ^* be a supra topology on $X.$ We call τ^* a supra topology associated with τ if $\tau\subseteq\tau^*.$

DEFINITION 2.4. Let (X,τ^*) be a supra topological space. A subset A of X is called a supra $\mathbf{I}-$ open set in X if $A\subseteq Supra-Int\left[Supra-Cl(A)\right].$ The complement of a supra $I-open$ set is called a supra $I-closed$ set.

DEFINITION 2.5. Let (X,τ^*) be a supra topological space. The supra $\mathbf{I}-$ closure of a set A is denoted by $Supra-I-Cl(A),$ and is defined as given in the following: $Supra-I-Cl(A)=\bigcap\{B\subseteq X: B \text{ is supra } I-closed \text{ set in } X \text{ such that } A\subseteq B\}.$

The supra $\mathbf{I}-$ interior of a set A is denoted by $Supra-I-Int(A)$ and is defined by $Supra-I-Int(A)=\bigcup\{U\subseteq X: U \text{ is supra } I-open \text{ set in } X \text{ such that } U\subseteq A\}.$ Clearly $Supra-I-Cl(A)$ is a supra $\mathbf{I}-$ closed set and $Supra-I-Int(A)$ is a supra $\mathbf{I}-$ open set.

Throughout this paper, (X,τ) and (Y,σ) will denote topological spaces and we will denote by τ^* and σ^* to be their associated supra topologies with τ and σ respectively such that $\tau\subseteq\tau^*$ and $\sigma\subseteq\sigma^*.$

THEOREM 2.6. Let (X,τ^*) be a supra topological space. Then every supra open set in X is supra $I-open$ set in $X.$

PROOF. Let A be a supra open set in $X.$ Then $A\subseteq Supra-Cl(A),$ so $Supra-Int(A)\subseteq Supra-Int\left[Supra-Cl(A)\right].$ Since $A\in\tau^*,$ so $Supra-Int(A)=A.$ Therefore $A\subseteq Supra-Int\left[Supra-Cl(A)\right].$ Hence it follows that A is supra $I-open$ set in $X.$

The converse of the above theorem need not be true as shown by the following example.

EXAMPLE 2.7. Suppose $\mathrm{X}=\{1,2,3,4,5\}$ and have the supra topology $\tau^*=\{\phi,\{1,3\},\{2,3\},\{1,2,3\},\mathrm{X}\}.$ The set $\{3\}\notin\tau^*,$ so the set $\{3\}$ is not a supra open set in $(X,\tau^*).$ Now since it clearly follows that $Supra-Int\left[Supra-Cl(\{3\})\right]=$ $Supra-Int(X)=X.$ Therefore it follows that $\{3\}$ is a supra $I-open$ set in $(X,\tau^*).$

DEFINITION 2.8. Let (X,τ^*) be a supra topological space. Then a subset A of X is called a supra semi–open set if $A\subseteq Supra-Cl\left[Supra-Int(A)\right].$

By the next two examples, we show that neither a supra $\mathbf{I}-$ open set may be a supra semi–open set nor a semi–open set may be a supra $\mathbf{I}-$ open set in a supra topological space.

EXAMPLE 2.9. Suppose $\mathrm{X}=\{1,2,3,4\}$ and have the supra topology as given by $\tau^*=\{\phi,\{1,3\},\{2,3\},\{1,2,3\},\mathrm{X}\}.$ Let $\mathrm{A}=\{1,2,4\}.$ Then $\mathbf{Supra-Cl(A)=X}.$ Hence $\mathbf{A\subseteq Supra-Int\left[Supra-Cl(A)\right]=X}.$ It shows that $\mathbf{A}$ is a supra $\mathbf{I}-$ open set in X. Since $\mathbf{Supra-Cl\left[Supra-Int(A)\right]}=\phi.$ It follows that $\mathbf{A}$ is not a supra semi–open set.

EXAMPLE 2.10. Let $X=\{a,b,c\}$ and $\tau^*=\{X,\phi,\{a\},\{b\},\{a,b\}\}$ be a supra topology on $X.$ Then clearly $\{b,c\}$ is a supra semi–open set, but not a supra $\mathbf{I}-$ open set.

THEOREM 2.11. $(\mathbf{i})$ Arbitrary union of Supra $\mathbf{I}-$ open sets is always a supra $\mathbf{I}-$ open set.

$(\mathbf{ii})$ Finite intersection of supra $\mathbf{I}-$ open sets may fail to be a supra $\mathbf{I}-$ open set.

PROOF. (i) Let (X,τ^*) be a supra topological space. Let $\mathfrak{I}=\{S_i : i\in I\}$ be a family of supra $I-$ open sets in X. Let $S=\cup\mathfrak{I}=\cup\{S_i : i\in I\}$. Since for each $i\in I$, S_i is supra $I-$ open set. Hence it follows that $S_i\subseteq Supra-Int[Supra-Cl(S_i)]\subseteq Supra-Int[Supra-Cl(S)]$, $for\ all\ i\in I$. So $S_i\subseteq Supra-Int[Supra-Cl(S)]$, for all $i\in I$. Therefore clearly it follows that $S=\bigcup_{i\in I}S_i\subseteq Supra\text{-}Int[Supra\text{-}Cl(S)]$. Thus we conclude that S is supra $I-$ open set.

(ii) Let $X=\{a,b,c\}$ and $\tau^*=\{X,\phi,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\},X\}$ be a supra topology on X. Then $\{a,b\}$ and $\{b,c\}$ are supra $I-$ open sets but their intersection $\{b\}$ is a not a supra $I-$ open set.

THEOREM 2.12. (i) The arbitrary intersection of supra $I-$ closed sets is always supra $I-$ closed.

(ii) A finite union of supra $I-$ closed sets may fail to be a supra $I-$ closed set.

PROOF. (i) Follows from Theorem 2.11 (i).

(ii) Let $X=\{1,2,3,4,5\}$ and $\tau^*=\{X,\phi,\{1,2\},\{1,2,3\},\{4\},\{1,2,4\},\{3,4\},\{1,2,3,4\}\}$ be a supra topology on X. Then $\{4,5\}$ and $\{1,2,5\}$ are supra $I-$ closed sets but their union $\{1,2,4,5\}$ is a not a supra $I-$ closed set as its complement $\{3\}$ is not supra $I-$ open set.

THEOREM 2.13. Let (X,τ^*) be a supra topological space. Let A be a subset of X. Then the following statements are true.

(a) $\mathrm{Supra-I-Int}(X-A)=X-[\mathrm{Supra-I-Cl}(A)]$.

(b) $\mathrm{Supra-I-Cl}(X-A)=X-[\mathrm{Supra-I-Int}(A)]$.

(c) $Supra-I-Int(A)$ is supra $I-$ open.

(d) $Supra-I-Cl(A)$ is supra $I-$ closed.

(e) $Supra-I-Int(A)=A$ if and only if A is a supra $I-$ open set.

(f) $Supra-I-Cl(A)=A$ if and only if A is a supra $I-$ closed set.

(g) $\mathrm{Supra-I-Int}(A)=\{x\in X: \text{There } exists\ a\ supra\ I-open\ set\ U\ such\ that\ x\in U\subseteq A\}$.

(h) $\mathrm{Supra-I-Cl}(A)=\{x\in X: \text{for every supra}-I-\text{open subset } U \text{ containing } x,\ U\cap A\neq\phi\}$.

DEFINITION 2.14. A function $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is called a supra $I-$ continuous functions if the inverse image of each supra open set in Y is a supra $I-$ open set in X.

DEFINITION 2.15. A function $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is called a supra $I-$ irresolute function if $f^{-1}(V)$ is supra $I-$ closed set in (X,τ^*), for every supra $I-$ closed set V in (Y,σ^*).

DEFINITION 2.16. A function $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is called strongly supra $I-$ continuous if the inverse image $f^{-1}(V)$ of every supra $I-$ closed set V in Y is supra closed in X.

DEFINITION 2.17. A function $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is called perfectly supra $I-$ continuous if the inverse image $f^{-1}(V)$ of every supra $I-$ closed set V in Y is both supra closed and supra open in X.

THEOREM 2.18. Every continuous function is supra $I-$ continuous functions.

PROOF. Let (X,τ) and (Y,σ) be two topological spaces and let τ^* and σ^* be associated supra topologies with τ and σ respectively. Let $f: X \longrightarrow Y$ be a continuous function. Therefore $f^{-1}(A)$ is an open set in X for each open set A in Y. But, τ^* is associated with τ. That is $\tau \subseteq \tau^*$. This implies that $f^{-1}(A)$ is a supra open set in X. Since every supra open set is supra $I-$open set, this implies $f^{-1}(A)$ is supra $I-$open in X. Hence f is a supra $I-$continuous function.

The converse of the above theorem is not true as shown in the following example.

EXAMPLE 2.19. Let $X=\{a,b,c\}$ and $\tau=\{X,\phi,\{a,b\}\}$ be a topology on X. The supra topology τ^* is defined as follows, $\tau^*=\{X,\phi,\{c\},\{a,b\}\}$. Suppose that $f: X \longrightarrow X$ is a function defined as follows: $f(a)=b,\ f(b)=c,\ f(c)=a$. The inverse image of the open set $\{a,b\}$ is $\{a,c\}$ which is not an open set but it is supra $I-$open. Also $f^{-1}(\{c\})=\{b\}$ is a supra $I-$open set in X. Then f is supra $I-$continuous but it is not continuous.

III. SUPRA $I-$ COMPACTNESS

DEFINITION 3.1. A collection $\{A_i : i \in I\}$ of supra $I-$open sets in a supra topological space (X,τ^*) is called a supra $I-$open cover of a subset B of X if $B \subseteq \cup\{A_i : i \in I\}$ holds.

DEFINITION 3.2. A supra topological space (X,τ^*) is called supra $I-$compact if every supra $I-$open cover of X has a finite subcover.

DEFINITION 3.3. A subset B of a supra topological space (X,τ^*) is said to be supra $I-$compact relative to (X,τ^*) if, for every collection $\{A_i : i \in I\}$ of supra $I-$ open subsets of X such that $B \subseteq \cup\{A_i : i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq \cup\{A_i : i \in I_0\}$.

DEFINITION 3.4. A subset B of a supra topological space (X,τ^*) is said to be supra $I-$compact if B is supra $I-$compact as a subspace of X.

THEOREM 3.5. Every supra $I-$compact space (X,τ^*) is supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a supra open cover of X. Since every supra open set in X is a supra $I-$open set in X. So $\{A_i : i \in I\}$ is a supra $I-$open cover of (X,τ^*). Since (X,τ^*) is supra $I-$compact. Therefore the supra $I-$ open cover $\{A_i : i \in I\}$ of (X,τ^*) has a finite subcover say $\{A_i : i = 1, 2, \ldots, n\}$ for X. Hence (X,τ^*) is a supra compact space.

THEOREM 3.6. Every supra $I-$closed subset of a supra $I-$compact space is supra $I-$compact relative to X.

PROOF. Let A be a supra $I-$closed subset of a supra topological space (X,τ^*). Then $A^C = X - A$ is supra $I-$open in (X,τ^*). Let $\gamma=\{A_i : i \in I\}$ be a supra $I-$ open cover of A by supra $I-$open subsets in (X,τ^*). Let $\gamma^* = \{A_i : i \in I\} \cup \{A^C\}$ be a supra $I-$ open cover of (X,τ^*). That is $X = \cup\gamma^* = (\cup\{A_i : i \in I\}) \cup A^C$. By hypothesis (X,τ^*) is supra $I-$ compact and hence γ^* is reducible to a finite subcover of (X,τ^*) say $X = A_1 \cup A_2 \cup \ldots \cup A_n \cup A^C$; $A_k \in \gamma$ for $k = 1, 2, \ldots, n$. But A and A^C are disjoint. Hence $A \subseteq A_1 \cup A_2 \cup \ldots \cup A_n$;

$A_k \in \gamma$ **for** $k = 1, 2, \ldots, n$. Thus a supra $I -$ open cover γ of A contains a finite subcover. Hence A is supra $I -$ compact relative to (X, τ^*).

THEOREM 3.7 A supra $I -$ continuous image of a supra $I -$ compact space is supra compact.

PROOF. Let $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ be a supra $I -$ continuous map from a supra $I -$ compact space X onto a supra topological space Y. Let $\{A_i : i \in I\}$ be a supra open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a supra $I -$ open cover of X, as f is supra $I -$ continuous. Since X is supra $I -$ compact, the supra $I -$ open cover of X, $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \bigcup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \bigcup\{A_i : i = 1, 2, \ldots, n\}$, then $Y = \bigcup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_i : i = 1, 2, \ldots, n\}$ is a finite subcover of $\{A_i : i \in I\}$ for Y. Hence Y is supra compact.

THEOREM 3.8. Suppose that a map $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ is supra $I -$ irresolute and a subset S of X is supra $I -$ compact relative to (X, τ^*), then the image $f(S)$ is supra $I -$ compact relative to (Y, σ^*).

PROOF. Let $\{A_i : i \in I\}$ be a collection of supra $I -$ open cover of (Y, σ^*), such that $f(S) \subseteq \cup\{A_i : i \in I\}$. Since f is supra $I -$ irresolute. Therefore $S \subseteq \cup\{f^{-1}(A_i) : i \in I\}$, where $\{f^{-1}(A_i) : i \in I\}$ is a family of supra $I -$ open sets in X. Since S is supra $I -$ compact relative to (X, τ^*), so there exists a finite subcollection $\{A_1, A_2, \ldots, A_n\}$ such that $S \subseteq \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. That is $f(S) \subseteq \cup\{A_1, A_2, \ldots, A_n\}$. Hence $f(S)$ is supra $I -$ compact relative to (Y, σ^*).

THEOREM 3.9. Suppose that a map $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ is strongly supra $I -$ continuous map from a supra compact space (X, τ^*) onto a supra topological space (Y, σ^*), then (Y, σ^*) is supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a supra open cover of (Y, σ^*). Since f is strongly supra $I -$ continuous, $\{f^{-1}(A_i) : i \in I\}$ is a supra $I -$ open cover of (X, τ^*). Again, since (X, τ^*) is supra I – compact, the supra $I -$ **open** cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ^*) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, σ^*). Hence (Y, σ^*) is supra compact.

THEOREM 3.10. Suppose that a map $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ is perfectly supra $I -$ continuous map from a supra compact space (X, τ^*) onto a supra topological space (Y, σ^*). Then (Y, σ^*) is supra compact.

PROOF. Let $\{A_i : i \in I\}$ be a supra $I -$ open cover of (Y, σ^*). Since f is perfectly supra $I -$ continuous, $\{f^{-1}(A_i) : i \in I\}$ is a supra open cover of (X, τ^*). Again, since (X, τ^*) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ^*) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$.

Therefore $X = \cup\left\{f^{-1}(A_i) : i = 1,2,\ldots,n\right\}$, which implies $f(X) = \cup\left\{A_i : i = 1,2,\ldots,n\right\}$, so that $Y = \cup\left\{A_i : i = 1, 2, \ldots, n\right\}$. That is $\left\{A_1, A_2, \ldots, A_n\right\}$ is a finite subcover of $\left\{A_i : i \in I\right\}$ for (Y, σ^*). Hence (Y, σ^*) is supra compact.

THEOREM 3.11. Suppose that a function $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ is supra $I-$ irresolute map from a supra $I-$ compact space (X, τ^*) onto a supra topological space (Y, σ^*). Then (Y, σ^*) is supra $I-$ compact.

PROOF. Let $\left\{A_i : i \in I\right\}$ be a supra $I-$ open cover of (Y, σ^*). Then $\left\{f^{-1}(A_i) : i \in I\right\}$ is a supra $I-$ open cover of (X, τ^*), since f is supra $I-$ irresolute. As (X, τ^*) is supra $I-$ compact, the supra $I-$ open cover $\left\{f^{-1}(A_i) : i \in I\right\}$ of (X, τ^*) has a finite subcover say $\left\{f^{-1}(A_i) : i = 1, 2, \ldots, n\right\}$. Therefore $X = \cup\left\{f^{-1}(A_i) : i = 1, 2, \ldots, n\right\}$, which implies $f(X) = \cup\left\{A_i : i = 1,2,\ldots,n\right\}$, so that $Y = \cup\left\{A_i : i = 1, 2, \ldots, n\right\}$. That is $\left\{A_1, A_2, \ldots, A_n\right\}$ is a finite subcover of $\left\{A_i : i \in I\right\}$ for (Y, σ^*). Hence (Y, σ^*) is supra $I-$ compact.

THEOREM 3.12. If (X, τ^*) is supra compact and every supra $I-$ closed set in X is also supra closed in X, then (X, τ^*) is supra $I-$ compact.

PROOF. Let $\left\{A_i : i \in I\right\}$ be a supra $I-$ open cover of X. Since every supra $I-$ closed set in X is also supra closed in X. Thus $\left\{X - A_i : i \in I\right\}$ is a supra closed cover of X and hence $\left\{A_i : i \in I\right\}$ is a supra open cover of X, Since (X, τ^*) is supra compact. So there exists a finite subcover $\left\{A_i : i = 1, 2, \ldots, n\right\}$ of $\left\{A_i : i \in I\right\}$ such that $X = \cup\left\{A_i : i = 1, 2, \ldots, n\right\}$. Hence (X, τ^*) is a supra $I-$ compact space.

THEOREM 3.13. A supra topological space (X, τ^*) is supra $I-$ compact if and only if every family of supra $I-$ closed sets of (X, τ^*) having finite intersection property has a nonempty intersection.

PROOF. Suppose (X, τ^*) is supra $I-$ compact, Let $\left\{A_i : i \in I\right\}$ be a family of supra $I-$ closed sets with finite intersection property. Suppose $\bigcap_{i \in I} A_i = \phi$, then $X - \cap\left(\left\{A_i : i \in I\right\}\right) = X$. This implies $\cup\left\{(X - A_i) : i \in I\right\} = X$. Thus $\left\{(X - A_i) : i \in I\right\}$ is a supra $I-$ open cover of (X, τ^*). Then as (X, τ^*) is supra $I-$ compact, the supra $I-$ open cover $\left\{(X - A_i) : i \in I\right\}$ of X has a finite subcover say $\left\{(X - A_i) : i = 1,2,\ldots,n\right\}$. This implies that $X = \cup\left\{(X - A_i) : i = 1,2,\ldots,n\right\}$, which implies $X = X - \cap\left\{A_i : i = 1,2,\ldots,n\right\}$, which implies $X - X = \cap\left\{A_i : i = 1,2,\ldots,n\right\}$, and which implies $\phi = \cap\left\{A_i : i = 1,2,\ldots,n\right\}$. This disproves the assumption. Hence $\cap\left\{A_i : i \in I\right\} \neq \phi$.

Conversely, suppose (X, τ^*) is not supra $I-$ compact. Then there exits a supra $I-$ open cover of (X, τ^*) say $\left\{G_i : i \in I\right\}$ having no finite subcover. This implies that for any finite subfamily $\left\{G_i : i = 1,2,\ldots,n\right\}$ of $\left\{G_i : i \in I\right\}$, we have $\cup\left\{G_i : i = 1,2,\ldots,n\right\} \neq X$, which implies $X - \left(\cup\left\{G_i : i = 1,2,\ldots,n\right\}\right) \neq X - X$, hence $\cap\left\{X - G_i : i = 1, 2,\ldots,n\right\} \neq \phi$. Therefore the family $\left\{X - G_i : i \in I\right\}$ of supra $I-$ closed sets has a finite intersection property. Then by assumption

$\cap\{X-G_i : i\in I\}\neq\phi$ which implies $X-\left(\cup\{G_i : i\in I\}\right)\neq\phi$, so that $\cup\{G_i : i\in I\}\neq X$. This implies that $\{G_i : i\in I\}$ is not a cover of (X,τ^*). This disproves the fact that $\{G_i : i\in I\}$ is a cover for (X,τ^*). Therefore any supra $I-$ open cover $\{G_i : i\in I\}$ of (X,τ^*) has a finite subcover $\{G_i : i=1, 2,\ldots, n\}$. Hence (X,τ^*) is supra $I-$compact.

THEOREM 3.14. Let A be a supra $I-$compact set relative to a supra topological space X and B be a supra $I-$closed subset of X. Then $A\cap B$ is supra $I-$ compact relative to X.

PROOF. Let A be supra $I-$ compact relative to X. Let $\{A_i : i\in I\}$ be a cover of $A\cap B$ by supra $I-$open sets in X. Then $\{A_i : i\in I\}\cup\{B^C\}$ is a cover of A by supra $I-$open sets in X, but A is supra $I-$ compact relative to X, so there exist $i_1, i_2,\ldots,i_n\in I$ such that $A\subseteq\left(\cup\{A_{i_k} : k=1,2,\ldots,n\}\right)\cup B^C$. Then $A\cap B\subseteq\cup\{A_{i_k}\cap B : k=1,2,\ldots,n\}\subseteq\cup\{A_{i_k} : k=1, 2,\ldots, n\}$. Hence $A\cap B$ is supra $I-$compact relative to X.

THEOREM 3.15. Suppose that a function $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is supra $I-$irresolute and a subset B of X is supra $I-$compact relative to X. Then $f(B)$ is supra $I-$compact relative to Y.

PROOF. Let $\{A_i : i\in I\}$ be a cover of $f(B)$ by supra $I-$ open subsets of Y. Since f is supra $I-$irresolute. Then $\{f^{-1}(A_i) : i\in I\}$ is a cover of B by supra $I-$open subsets of X. Since B is supra $I-$ compact relative to X, so $\{f^{-1}(A_i) : i\in I\}$ has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2),\ldots, f^{-1}(A_n)\}$ for B. Then it implies that $\{A_i : i=1, 2,\ldots, n\}$ is a finite subcover of $\{A_i : i\in I\}$ for $f(B)$. So $f(B)$ is supra $I-$compact relative to Y.

IV. COUNTABLY SUPRA I – COMPACTNESS

In this section, we present the concept of countably supra $I-$compactness and its properties.

DEFINITION 4.1. A supra topological space (X,τ^*) is said to be countably supra $I-$compact if every countable supra $I-$open cover of X has a finite subcover.

THEOREM 4.2. If (X,τ^*) is a countably supra $I-$ compact space, then (X,τ^*) is countably supra compact.

PROOF. Let (X,τ^*) be a countably supra $I-$compact space. Let $\{A_i : i\in I\}$ be a countable supra open cover of (X,τ^*). Since every supra open set in X is always supra $I-$open set in X. So $\{A_i : i\in I\}$ is a countable supra $I-$ open cover of (X,τ^*). Since (X,τ^*) is countably supra $I-$ compact, so the countable supra $I-$open cover $\{A_i : i\in I\}$ of (X,τ^*) has a finite subcover say $\{A_i : i=1,2,\ldots,n\}$ for X. Hence (X,τ^*) is a countably supra compact space.

THEOREM 4.3. If (X,τ^*) is countably supra compact and every supra $I-$closed subset of X is supra closed in X, then (X,τ^*) is countably supra $I-$compact.

PROOF. Let (X,τ^*) be a countably supra compact space. Let $\{A_i : i\in I\}$ be a countable supra $I-$open cover of (X,τ^*). Since every supra $I-$ closed subset of X is supra closed in X. Thus every supra

$\mathbf{I}-$ open set in X is supra open in $\mathbf{X}$. Therefore $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ is a countable supra open cover of (X, τ^*). Since (X, τ^*) is countably supra compact, so the countable supra open cover $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ of (X, τ^*) has a finite subcover say $\{\mathbf{A_i} : \mathbf{i} = 1, 2, \ldots, \mathbf{n}\}$ for $\mathbf{X}$. Hence (X, τ^*) is a countably supra $\mathbf{I}-$ compact space.

THEOREM 4.4. Every supra $\mathbf{I}-$ compact space is countably supra $\mathbf{I}-$ compact.

PROOF. Let (X, τ^*) be a supra $\mathbf{I}-$ compact space. Let $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ be a countable supra $\mathbf{I}-$ open cover of $(\mathbf{X}, \tau^*)$. Since (X, τ^*) is supra $\mathbf{I}-$ compact, so the supra $\mathbf{I}-$ open cover $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ of (X, τ^*) has a finite subcover say $\{\mathbf{A_i} : \mathbf{i} = 1, 2, \ldots, \mathbf{n}\}$ for (X, τ^*). Hence (X, τ^*) is countably supra $\mathbf{I}-$ compact space.

THEOREM 4.5. Let $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ be a supra $\mathbf{I}-$ continuous onjective mapping. If X is countably supra $\mathbf{I}-$ compact space, then (Y, σ) is countably supra compact.

PROOF. Let $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ be a supra $\mathbf{I}-$ continuous map from a countably supra $\mathbf{I}-$ compact space (X, τ^*) onto a supra topological space (Y, σ^*). Let $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ be a countable supra open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a countable supra $\mathbf{I}-$ open cover of X, as f is supra $\mathbf{I}-$ continuous. Since X is countably supra $\mathbf{I}-$ compact. So the countable supra $\mathbf{I}-$ open cover $\{f^{-1}(A_i) : i \in I\}$ of X has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $Y = f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_i : i = 1, 2, \ldots, n\}$ is a finite subcover of $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ for $\mathbf{Y}$. Hence $\mathbf{Y}$ is countably supra compact.

THEOREM 4.6. Suppose that a map $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ is perfectly supra $\mathbf{I}-$ continuous map from a countably supra compact space (X, τ^*) onto a supra topological space (Y, σ^*). Then (Y, σ^*) is countably supra $\mathbf{I}-$ compact.

PROOF. Let $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ be a countable supra $\mathbf{I}-$ open cover of (Y, σ^*). Since f is perfectly supra $\mathbf{I}-$ continuous. So $\{f^{-1}(A_i) : i \in I\}$ is a countable supra open cover of (X, τ^*). Again, since (X, τ^*) is countably supra compact. Hence the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ^*) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \cup\{A_i : i = 1, 2, \ldots, n\}$. That is $\{\mathbf{A_1}, \mathbf{A_2}, \ldots, \mathbf{A_n}\}$ is a finite subcover of $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ for (Y, σ^*). Hence (Y, σ^*) is countably supra $\mathbf{I}-$ compact.

THEOREM 4.7. Suppose that a map $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ is strongly supra $\mathbf{I}-$ continuous map from a countably supra compact space (X, τ^*) onto a supra topological space (Y, σ^*). Then (Y, σ^*) is countably supra $\mathbf{I}-$ compact.

PROOF. Let $\{\mathbf{A_i} : \mathbf{i} \in \mathbf{I}\}$ be a countable supra $\mathbf{I}-$ open cover of (Y, σ^*). Since f is strongly supra $\mathbf{I}-$ continuous, so $\{f^{-1}(A_i) : i \in I\}$ is a countable supra open cover of (X, τ^*). Again, since (X, τ^*) is countably supra compact, so the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ^*) has a finite subcover say $\{f^{-1}(A_i): i=1,2,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i): i=1,2,...,n\}$, which implies $f(X) = \cup\{A_i : i=1,2,...,n\}$, so that $Y = \cup\{A_i : i=1,2,...,n\}$. That is $\{A_1, A_2, ..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, σ^*). Hence (Y, σ^*) is countably supra $I-$ compact.

THEOREM 4.8. The image of a countably supra $I-$ compact space under a supra $I-$ irresolute map is countably supra $I-$ compact.

PROOF. Suppose that a map $f:(X, \tau^*) \longrightarrow (Y, \sigma^*)$ is a supra $I-$ irresolute map from a countably supra $I-$ compact space (X, τ^*) onto a supra topological space (Y, σ^*). Let $\{A_i : i \in I\}$ be a countable supra $I-$ open cover of (Y, σ^*). Then $\{f^{-1}(A_i): i \in I\}$ is a countable supra $I-$ open cover of (X, τ^*), since f is supra $I-$ irresolute. As (X, τ^*) is countably supra $I-$ compact, so the countable supra $I-$ open cover $\{f^{-1}(A_i): i \in I\}$ of (X, τ^*) has a finite subcover say $\{f^{-1}(A_i): i=1, 2, ..., n\}$. Therefore $X = \cup\{f^{-1}(A_i): i=1, 2, ..., n\}$, which implies $f(X) = \cup\{A_i : i=1,2,...,n\}$, so that $Y = \cup\{A_i : i=1, 2, ..., n\}$. That is $\{A_1, A_2, ..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, σ^*). Hence (Y, σ^*) is countably supra $I-$ compact.

V. SUPRA $I-$ LINDELOF SPACE

In this section, we concentrate on the concept of supra $I-$ Lindelof space and its properties.

DEFINITION 5.1. A supra topological space (X, τ^*) is said to be supra $I-$ Lindelof space if every supra $I-$ open cover of X has a countable subcover.

THEOREM 5.2. Every supra $I-$ Lindelof space (X, τ^*) is supra Lindelof space.

PROOF. Let (X, τ^*) be a supra $I-$ Lindelof space. Let $\{A_i : i \in I\}$ be a supra open cover of (X, τ^*). Since every supra open set in X is always supra $I-$ open set in X. Therefore $\{A_i : i \in I\}$ is a supra $I-$ open cover of (X, τ^*). Since (X, τ^*) is supra $I-$ Lindelof space, so the supra $I-$ open cover $\{A_i : i \in I\}$ of (X, τ^*) has a countable subcover say $\{A_i : i=1, 2, ..., n\}$ for X. Hence (X, τ^*) is a supra Lindelof space.

THEOREM 5.3. If (X, τ^*) is supra $I-$ Lindelof space, then (X, τ) is Lindelof space.

PROOF. Let $\{A_i : i \in I\}$ be an open cover of X. Since every open set in X being a supra open set in X is also supra $I-$ open set in X. Therefore $\{A_i : i \in I\}$ is a supra $I-$ open cover of (X, τ^*). Since (X, τ^*) is supra $I-$ Lindelof, so the supra $I-$ open cover $\{A_i : i \in I\}$ of (X, τ^*) has a countable subcover say $\{A_i : i=1, 2, ..., n\}$ for X. Hence (X, τ) is a Lindelof space.

THEOREM 5.4. Every supra $I-$ compact space is supra $I-$ Lindelof space.

PROOF. Let (X, τ^*) be a supra $I-$ compact space. Let $\{A_i : i \in I\}$ be a supra $I-$ open cover of (X, τ^*). Since (X, τ^*) is supra $I-$ compact space. Then $\{A_i : i \in I\}$ has a finite subcover say $\{A_i : i=1, 2, ..., n\}$. Since every finite subcover is always countable subcover and therefore $\{A_i : i=1, 2, ..., n\}$ is a countable subcover of $\{A_i : i \in I\}$. Hence (X, τ^*) is a supra $I-$ Lindelof space.

THEOREM 5.5. A supra $\mathbf{I}-$continuous image of a supra $\mathbf{I}-$Lindelof space is supra Lindelof space.

PROOF. Let $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ be a supra $\mathbf{I}-$continuous map from a supra $\mathbf{I}-$Lindelof space X onto a supra topological space Y. Let $\{A_i : i\in I\}$ be a supra open cover of Y. Then $\{f^{-1}(A_i): i\in I\}$ is a supra $\mathbf{I}-$open cover of X, as f is supra $\mathbf{I}-$continuous. Since X is supra $\mathbf{I}-$Lindelof space, so the supra $\mathbf{I}-$open cover $\{f^{-1}(A_i): i\in I\}$ of X has a countable subcover say $\{f^{-1}(A_i): i\in I_0\}$ for some countable set $I_0\subseteq I$. Therefore $X=\mathrm{U}\{f^{-1}(A_i): i\in I_0\}$, which implies $f(X)=\mathrm{U}\{A_i : i\in I_0\}$, then $Y=\mathrm{U}\{A_i : i\in I_0\}$. That is $\{A_i : i\in I_0\}$ is a countable subcover of $\{A_i : i\in I\}$ for Y. Hence Y is a supra Lindelof space.

THEOREM 5.6. The image of a supra $\mathbf{I}-$ Lindelof space under a supra $\mathbf{I}-$ irresolute map is supra $\mathbf{I}-$ Lindelof space.

PROOF. Suppose that a map $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is supra $\mathbf{I}-$ irresolute map from a supra $\mathbf{I}-$Lindelof space (X,τ^*) onto a supra topological space (Y,σ^*). Let $\{A_i : i\in I\}$ be a supra $\mathbf{I}-$open cover of (Y,σ^*). Since f is supra $\mathbf{I}-$irresolute. Therefore $\{f^{-1}(A_i): i\in I\}$ is a supra $\mathbf{I}-$open cover of (X,τ^*). As (X,τ^*) is supra $\mathbf{I}-$ Lindelof space, so the supra $\mathbf{I}-$ open cover $\{f^{-1}(A_i): i\in I\}$ of (X,τ^*) has a countable subcover say $\{f^{-1}(A_i): i\in I_0\}$ for some countable set $I_0\subseteq I$. Therefore $X=\mathrm{U}\{f^{-1}(A_i): i\in I_0\}$, which implies $f(X)=\mathrm{U}\{A_i : i\in I_0\}$, so that $Y=\mathrm{U}\{A_i : i\in I_0\}$. That is $\{A_i : i\in I_0\}$ is a countable subcover of $\{A_i : i\in I\}$ for Y. Hence (Y,σ^*) is a supra $\mathbf{I}-$ Lindelof space.

THEOREM 5.7. If (X,τ^*) is supra $\mathbf{I}-$ Lindelof space and countably supra $\mathbf{I}-$ compact space, then (X,τ^*) is supra $\mathbf{I}-$compact space.

PROOF. Suppose (X,τ^*) is supra $\mathbf{I}-$ Lindelof space and countably supra $\mathbf{I}-$ compact space. Let $\{A_i : i\in I\}$ be a supra $\mathbf{I}-$ open cover of (X,τ^*). Since (X,τ^*) is supra $\mathbf{I}-$ Lindelof space, so $\{A_i : i\in I\}$ has a countable subcover say $\{A_i : i\in I_0\}$ for some countable set $I_0\subseteq I$. Therefore $\{A_i : i\in I_0\}$ is a countable supra $\mathbf{I}-$open cover of (X,τ^*). Again, since (X,τ^*) is countably supra $\mathbf{I}-$ compact space, so $\{A_i : i\in I_0\}$ has a finite subcover and say $\{A_i : i=1,2,\ldots,n\}$. Therefore $\{A_i : i=1,2,\ldots,n\}$ is a finite subcover of $\{A_i : i\in I\}$ for (X,τ^*). Hence (X,τ^*) is a supra $\mathbf{I}-$compact space.

THEOREM 5.8. If a function $f:(X,\tau^*)\longrightarrow(Y,\sigma^*)$ is supra $\mathbf{I}-$ irresolute and a subset B of X is supra $\mathbf{I}-$Lindelof relative to X, then $f(B)$ is supra $\mathbf{I}-$Lindelof relative to Y.

PROOF. Let $\{A_i : i\in I\}$ be a cover of $f(B)$ by supra $\mathbf{I}-$open subsets of Y. By hypothesis f is supra $\mathbf{I}-$irresolute and so $\{f^{-1}(A_i): i\in I\}$ is a cover of B by supra $\mathbf{I}-$ open subsets of X. Since B is supra $\mathbf{I}-$Lindelof relative to X, $\{f^{-1}(A_i): i\in I\}$ has a countable subcover say $\{f^{-1}(A_i): i\in I_0\}$ for B, where I_0 is a countable subset of I. Now $\{A_i : i\in I_0\}$ is a countable subcover of $\{A_i : i\in I\}$ for $f(B)$. So $f(B)$ is supra $\mathbf{I}-$ Lindelof relative to Y.

VI. SUPRA $\mathbf{I}-$CONNECTEDNESS

DEFINITION 6.1. A supra topological space (X, τ^*) is said to be supra connected if X cannot be written as a disjoint union of two nonempty supra open subsets of X. A subset of (X, τ^*) is supra connected if it is supra connected as a subspace.

DEFINITION 6.2. A supra topological space (X, τ^*) is said to be supra $\mathbf{I}-$connected if X cannot be written as a disjoint union of two nonempty supra $\mathbf{I}-$open sets. A subset of (X, τ^*) is supra $\mathbf{I}-$connected if it is supra $\mathbf{I}-$ connected as a subspace.

THEOREM 6.3. Every supra $\mathbf{I}-$connected space (X, τ^*) is supra connected.

PROOF. Let A and B be two nonempty disjoint proper supra open sets in $\mathbf{X}$. Since every supra open set is supra $\mathbf{I}-$ open set. Therefore A and B are nonempty disjoint proper supra $\mathbf{I}-$ open sets in $\mathbf{X}$. By hypothesis $\mathbf{X}$ is supra $\mathbf{I}-$connected space. Hence $\mathbf{X} \neq \mathbf{A} \cup \mathbf{B}$. Therefore $\mathbf{X}$ is supra $\mathbf{I}-$connected.

The converse of the above theorem need not be true in general, which follows from the following example.

EXAMPLE 6.4. Let $X = \{1, 2, 3, 4\}$ and $\tau^* = \{\phi, \{1, 2\}, \{1, 2, 3\}, X\}$. Then (X, τ^*) is a supra topological space. Since X cannot be written as a disjoint union of any two nonempty supra open sets. Therefore (X, τ^*) is a supra connected topological space. We notice that both $\{1\}$ and $\{2,3,4\}$ are supra $\mathbf{I-open}$ sets in (X, τ^*) because $\{1\} \subseteq Supra-Int[Supra-Cl(\{1\})] = Supra-Int(X) = X$ and $\{2,3,4\} \subseteq Supra-Int[Supra-Cl(\{2,3,4\})] = Supra-Int(X) = X$. Therefore $\{1\}$ and $\{2,3,4\}$ are nonempty disjoint supra $\mathbf{I}-$open sets such that $X = \{1\} \cup \{2, 3, 4\}$. Hence (X, τ^*) is not a supra $\mathbf{I}-$connected space.

THEOREM 6.5. Let (X, τ^*) be a supra topological space. Then the following statements are equivalent

(i) (X, τ^*) is supra $\mathbf{I}-$connected.

(ii) The only subsets of (X, τ^*) which are both supra $\mathbf{I}-$ open and supra $\mathbf{I}-$closed are the empty sets ϕ and X.

(iii) Each supra $\mathbf{I}-$continuous map of (X, τ^*) into a discrete space (Y, σ^*) with at least two points is a constant map.

PROOF. $(i) \Rightarrow (ii)$: Let G be a nonempty proper supra $\mathbf{I}-$open and supra $\mathbf{I}-$closed subset of (X, τ^*). Then $X-G$ is also both supra $\mathbf{I}-$ open and supra $\mathbf{I}-$ closed set. Then $X = G \cup (X-G)$ is a disjoint union of two nonempty supra $\mathbf{I}-$open sets, which contradicts the fact that (X, τ^*) is supra $\mathbf{I}-$connected. Hence $G = \phi$ or $G = X$.

$(ii) \Rightarrow (i)$: Suppose that $X = A \cup B$ where A and B are disjoint nonempty supra I – open subsets of (X, τ^*). Since $A = X - B$, then A is both supra $\mathbf{I}-$ open and supra $\mathbf{I}-$ closed set. By assumption $A = \phi$ or $A = X$, which is a contradiction. Hence (X, τ^*) is supra $\mathbf{I}-$connected.

$(ii) \Rightarrow (iii)$: Let $f : (X, \tau^*) \longrightarrow (Y, \sigma^*)$ be a supra $\mathbf{I}-$ continuous map, where (Y, σ^*) is discrete space with at least two points. Then $f^{-1}(y)$ is supra $\mathbf{I}-$ closed and supra $\mathbf{I}-$ open for each

$y \in Y$. Thus $\left(X, \tau^{*}\right)$ is covered by supra $\mathrm{I}-$closed and supra $\mathrm{I}-$ open covering $\left\{f^{-1}(y): y \in Y\right\}$. By assumption, $f^{-1}(y)=\phi$ or $f^{-1}(y)=X$ for each $y \in Y$. If $f^{-1}(y)=\phi$ for each $y \in Y$, then f fails to be a map. Therefore their exists at least one point say $y^{*} \in Y$ such that $f^{-1}\left(y^{*}\right)=X$. This shows that f is a constant map.

$(iii) \Rightarrow (ii)$: Let G be both supra $\mathrm{I}-$ open and supra $\mathrm{I}-$closed nonempty set in $\left(X, \tau^{*}\right)$. Suppose $G \neq X$. Then $f:\left(X, \tau^{*}\right) \longrightarrow\left(Y, \sigma^{*}\right)$ is a nonconstant supra $\mathrm{I}-$ continuous map defined by $f(G)=\{a\}$ and $f(X-G)=\{b\}$ where $a \neq b$ and $a, b \in Y$. By assumption, f is constant so we conclude that $G=X$.

THEOREM 6.6. If $f:\left(X, \tau^{*}\right) \longrightarrow\left(Y, \sigma^{*}\right)$ is a supra $\mathrm{I}-$continuous surjection and $\left(X, \tau^{*}\right)$ is supra $\mathrm{I}-$connected, then $\left(Y, \sigma^{*}\right)$ is supra connected.

PROOF. Suppose $\left(Y, \sigma^{*}\right)$ is not supra connected. Let $Y=A \cup B$, where A and B are disjoint nonempty supra open subsets of $\left(Y, \sigma^{*}\right)$. Since f is supra $\mathrm{I}-$continuous and onto, so $X=f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty supra $\mathrm{I}-$ open subsets of $\left(X, \tau^{*}\right)$. This disproves the fact that $\left(X, \tau^{*}\right)$ is supra $\mathrm{I}-$connected. Hence $\left(Y, \sigma^{*}\right)$ is supra connected.

THEOREM 6.7. If $f:\left(X, \tau^{*}\right) \longrightarrow\left(Y, \sigma^{*}\right)$ is a supra $\mathrm{I}-$ irresolute surjection and X is supra $\mathrm{I}-$connected, then Y is supra $\mathrm{I}-$connected.

PROOF. Suppose that Y is not supra $\mathrm{I}-$ connected. Let $Y=A \cup B$, where A and B are disjoint nonempty supra $\mathrm{I}-$ open sets in Y. Since f is supra $\mathrm{I}-$ irresolute and onto, so $X=f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty supra $\mathrm{I}-$ open sets in $\left(X, \tau^{*}\right)$. This contradicts the fact that $\left(X, \tau^{*}\right)$ is supra $\mathrm{I}-$ connected. Hence $\left(Y, \sigma^{*}\right)$ is supra $\mathrm{I}-$connected.

THEOREM 6.8. If every supra $\mathrm{I}-$closed set in $\mathbf{X}$ is supra closed in $\mathbf{X}$ and $\mathbf{X}$ is supra connected, then $\mathbf{X}$ is supra $\mathrm{I}-$connected.

PROOF. Suppose that $\mathbf{X}$ is supra connected. Then $\mathbf{X}$ cannot be expressed as a disjoint union of two nonempty proper supra open subset of $\mathbf{X}$. Suppose $\mathbf{X}$ is not supra $\mathrm{I}-$ connected space. Let A and B be any two nonempty supra $\mathrm{I}-$ open subsets of $\mathbf{X}$ such that $X=A \cup B$, where $A \cap B=\phi$. Since every supra $\mathrm{I}-$closed set in $\mathbf{X}$ is supra closed in $\mathbf{X}$. Therefore every supra $\mathrm{I}-$open set in $\mathbf{X}$ is supra open in $\mathbf{X}$. Hence A and B are supra open subsets of $\mathbf{X}$, which contradicts the fact that $\mathbf{X}$ is supra connected. Therefore X is supra $\mathrm{I}-$connected.

THEOREM 6.9. If two supra $\mathrm{I}-$open sets C and D form a separation of $\mathbf{X}$ and if Y is supra $\mathrm{I}-$connected subspace of $\mathbf{X}$, then Y lies entirely within C or D.

PROOF. By hypothesis C and D are both supra $\mathrm{I}-$open sets in $\mathbf{X}$. The sets $\mathbf{C} \cap \mathbf{Y}$ and $\mathbf{D} \cap \mathbf{Y}$ are supra $\mathrm{I}-$open in $\mathbf{Y}$. These two sets are disjoint and their union is $\mathbf{Y}$. If they were both nonempty, then they would constitute a separation of $\mathbf{Y}$. Therefore, one of them is empty. Hence $\mathbf{Y}$ must lie entirely in C or D.

THEOREM 6.10. Let A be a supra $\mathrm{I}-$connected subspace of $\mathbf{X}$. If $A \subseteq B \subseteq Supra-I-Cl(A)$, then B is also supra $\mathrm{I}-$connected.

PROOF. Let A be supra $\mathrm{I}-$connected. Let $A \subseteq B \subseteq Supra-I-Cl(A)$. Suppose that $B=C \cup D$ is a separation of B by supra $\mathrm{I}-$open sets. Thus by the previous theorem A must lie entirely in C or D. Suppose that $A \subseteq C$, then it implies that $Supra-I-Cl(A) \subseteq Supra-I-Cl(C)$. Since

$Supra-I-Cl(C)$ and D are disjoint, B cannot intersect D. This disproves the fact that D is nonempty subset of B. So $D=\phi$ which implies B is supra $I-$ connected.

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