

Mathematical Model of Two Phase Newtonian Renal Blood Flow to the Kidney with Reference to Diabetes

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Abstract

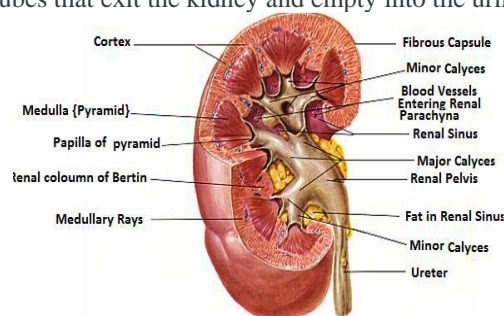
In the present paper we have formulated a mathematical model in the renal blood flow along the capillaries in case of renal disease Diabetes. Keeping in the view the nature of renal circulatory system in human body. we have considered the blood flow has two phase one of which is that of red blood cells and other is plasma. According to Fahreaus-Lindqvist effect the blood flow in two separated layers while passing through capillaries. The plasma layer which flows along the surface of the capillaries contains almost no blood cells. The second layer the core layer containing blood cells which flows in plasma along the axis of capillary. We have collected a clinical data in case of Diabetes for hematocrit v/s blood pressure. The graphical presentation for particular parametric value is much closed to the clinical observation. The overall presentation is in tensorial form and solution technique adapted is analytical as well as numerical. The role of hematocrit is explicit in the determination of blood pressure drop in case of renal disease Diabetes.

Keywords - Diabetes , Glomerular capillary ,Hematocrit , Nephron ,Pressure drop , Renal circulation

I. INTRODUCTION

A. Biophysical Problem (kidney)

The kidneys are a pair of bean-shaped structures that are located just below and posterior to the liver in the peritoneal cavity. Kidneys filter blood and purify it. All the blood in the human body is filtered many times a day by the kidneys, these organs use up almost 25 percent of the oxygen absorbed through the lungs to perform this function. Oxygen allows the kidney cells to efficiently manufacture chemical energy in the form of ATP through aerobic respiration. The filtrate coming out of the kidneys is called urine. Externally, the kidneys are surrounded by three layers, illustrated in Figure 1. The outermost layer is a tough connective tissue layer called the renal fascia. The second layer is called the perirenal fat capsule, which helps anchor the kidneys in place. The third and innermost layer is the renal capsule. Internally, the kidney has three regions—an outer cortex, a medulla in the middle, and the renal pelvis in the region called the hilum of the kidney. The hilum is the concave part of the bean-shape where blood vessels and nerves enter and exit the kidney; it is also the point of exit for the ureters. The renal cortex is granular due to the presence of nephrons—the functional unit of the kidney. The medulla consists of multiple pyramidal tissue masses, called the renal pyramids. In between the pyramids are spaces called renal columns through which the blood vessels pass. The tips of the pyramids, called renal papillae, point toward the renal pelvis. There are, on average, eight renal pyramids in each kidney. The renal pyramids along with the adjoining cortical region are called the lobes of the kidney. The renal pelvis leads to the ureter on the outside of the kidney. On the inside of the kidney, the renal pelvis branches out into two or three extensions called the major calyces, which further branch into the minor calyces. The ureters are urine-bearing tubes that exit the kidney and empty into the urinary bladder.



[7]
Fig 1

1. Function

Maintaining fluid and acid–base balance, removal of nitrogenous waste products and synthesis of hormones, such as renin, erythropoietin, and active vitamin D₃ (calcitriol) are the three major functions of kidney. The functional unit of the kidney is the nephron, which consists of a renal corpuscle, renal tubule, the proximal tubule (the part of tubule nearest to glomerulus), the loop of Henle, the distal tubule, and the collecting duct. The renal corpuscle consists of the glomerulus (tuft of capillaries) surrounded by Bowman's capsule. Each human kidney contains about one million nephrons. Proximal tubule is subdivided into Proximal convoluted tubule and Proximal straight tubule. Straight portion heads toward medulla, away from surface of kidney. The loop of Henle which participate in counter current multiplication of urine concentration includes the proximal straight tubule, thin limb and thick ascending limb. Connecting tubules connect the next segment, the short distal convoluted tubule, to the collecting duct system. Plasma is filtered in the glomerulus to form protein-free ultrafiltrate. About 60 % of this ultrafiltrate is reabsorbed in the proximal tubule. Several nephrons drain into a cortical collecting duct, which passes into an outer medullary collecting duct. In the inner medulla, inner medullary collecting ducts unite to form large papillary ducts.

The cortex of the kidney receives the majority of renal blood flow. The renal artery branches into anterior posterior divisions, which give rise to total of five segmental arteries which enters at the hilum of the kidneys and branches into the interlobar artery. The interlobar artery then travels between the pyramids and continues as arcuate artery, which arches along the corticomedullary junction. The interlobular artery is a stem off of the arcuate artery and travels through the cortex towards the capsule. As the interlobular artery ascends towards the cortex, branches of afferent arterioles are sent to each glomerulus. The afferent arterioles give rise to the capillaries that form the glomerulus. The glomerular capillaries reunite and form the efferent arteriole which exits at the vascular pole. The efferent arterioles of cortical nephrons give rise to the peritubular capillaries. As discussed previously, in the juxtamedullary nephrons, the peritubular capillaries form the ascending and descending vasa recta. The peritubular capillaries and vasa recta then drain into the interlobular vein, which unites with others to form the arcuate vein. Then the interlobular vein exits the kidney as the renal vein. In resting adult kidney receive 1.2l to 1.3l blood per minute or 25% of cardiac output. Renal blood flow can be measured with electromagnetic or other type of flow meter or it can be determined by applying the fick principle [12]. From renal plasma flow, the renal blood flow can be calculated by dividing by one minus the hematocrit:

Hematocrit (HCT)-45%

The renal blood flow = $RPF \times 1/(1-HCT) \rightarrow 700 \times 1/(1-0.45) = 1273 \text{ml/Minute}$ [45]

2. Pressure In Renal Vessels

The behaviour of pressure in glomerular capillary has been measured directly in the rat and has been found to be considerably lower than the predicted on the basis of indirect measurement. When the mean systolic arterial pressure is 100 mmhg, then glomerular capillary pressure is about 45 mmhg. The pressure drop across the glomerulus is only 1 to 3 mmhg, but further drop occurs in the efferent arteriole so that the pressure in the peritubular capillary is about 8 mmhg. The pressure in renal vein is about 4 mmhg. The pressure gradient are similar squirrel, monkey and presumably in human with glomerular capillary pressure that is about 40% of systolic arterial pressure. [12]

3. Nephron

Each human kidney contains about one million nephrons, each capable of forming urine. Kidney cannot regenerate new nephrons, so there is decrease in nephron due to injury, disease and ageing.

4. Blood

Blood contained 7% of body weight [24][25] Blood is a constantly circulating fluid providing the body with nutrition, oxygen, and waste removal. Blood is mostly liquid, with numerous cells and proteins suspended in it, making blood "thicker" than pure water, with an average density of approx. 1060kg/m^3 [26]. The average person has about 5 liters (more than a gallon) of blood. A liquid called plasma makes up about half of the content of blood. Plasma contains proteins that help blood to clot, transport substances through the blood, and perform other functions. Blood plasma also contains glucose and other dissolved nutrients. About half of blood volume is composed of blood cells:

- Red blood cells, which carry oxygen to the tissues
- White blood cells, which fight infections
- Platelets, smaller cells that help blood to clot

Blood is conducted through blood vessels (arteries and veins). Blood is prevented from clotting in the blood vessels by their smoothness, and the finely tuned balance of clotting factors. Blood and its components are most important part of patient management treatment protocols [19][36]. The viscosity of blood depends on acting shear force and it is determined by Hematocrit value. From biological point of view blood can be considered as tissue to be

composed of various cells RBC,WBC and PLETLETS and PLASMA but from rheological point of view blood is considered as two phased liquid.

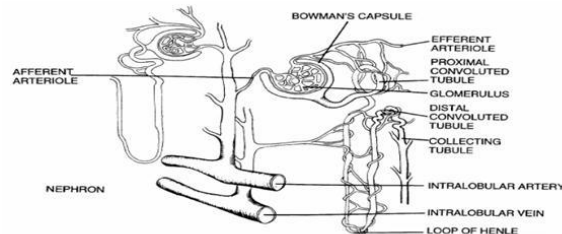


Fig 1.: Kidney anatomy and enhanced view of nephron. [13] [14]

B. Disease (diabetes)

Diabetes is the chronic disease that occurs when the pancreas does not produce enough insulin or when the body cannot effectively use the insulin it produces.

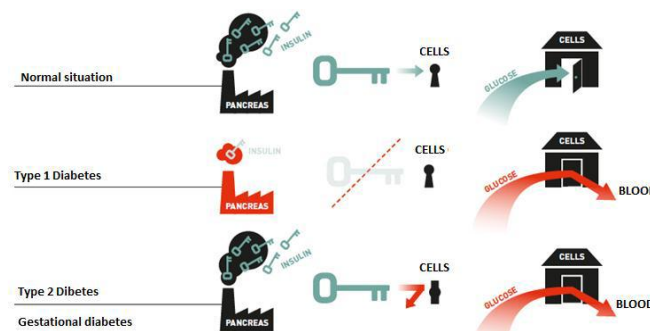


Fig3

C. Renal capillaries

Renal circulation has two capillary beds, the glomerular and pertibular capillaries,which are arranged in series and separated by the efferent arterioles that help,regulate the hydrostatic pressures both sets of capillaries.High hydrostatic pressure in the glomerular capillaries(about 60mm Hg) causes rapid fluid filtration,whereas much lower hydrostatic pressure in the pertibular capillaries(about 13mm Hg)permits rapid fluid reabsorption[32].the pertibular capillaries empty into the vessels of the venous system, which run parallel to the arteriolar vessels and progressively forms the interlobular vein arcuate vein,inter lobar vein and renal vein,which leaves kidney beside the renal artery and ureter.The glomerulus is a tuft of small blood vessels called capillaries located within Bowman’s capsule within the kidney[33].

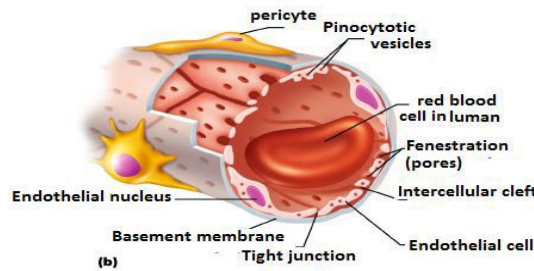


Fig 4

1. Both Layers Are Newtonian, Description Of The Problem

How the blood flow in capillaries is possible as we know that these vessels are far enough from the heart as well as thin. It’s a natural question because the blood flows very slowly in arterioles where there is high viscosity. The satisfactory answer of this problem is given by Fahreaus-Lindqvist effect. According to this effect the blood flows in two separated layers while passing through capillaries. The plasma layer containing almost no blood cells. The second layer is that of blood cells. The second layer is that of blood cells which float in plasma on the axis of the capillary. In this process the effective blood viscosity depends upon radius of the capillary. That’s why the effective viscosity decreases, as the radius and thus the blood flow becomes possible.

II. REAL MODEL

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the bloods plasma and 2nd phase of blood is RBCs.[35]. The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one is of plasma and other one is of blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels [36]. Plasma is a liquid containing semi permeable packages of RBCs

The behaviour of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behaviour [37]. We have selected generalized three-dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3 called as 3-dim Euclidean space. Here we have some quantities related to moving blood in cylindrical vessels:

Blood velocity $V^k=V^k(x^i,t), k=1,2,3$ blood pressure $P=p(x^i,t)$ and density $\rho=\rho(x^i,t)$ where x^i be the coordinates of any point in space and $i=1,2,3$. If let us consider that the both phases- plasma and blood cells are equally distributed in whole blood. Then blood treated as homogeneous mixture. When there is absence of source and sink in any region of flowing fluid, the fluid mass is conserved in that region. As we observed that there is no source or sink in the whole circuit of the human blood circulatory system, the heart behaves merely like a pumping station, so the law of conservation of mass can well be applied to hemodynamic [38]. Since, whole blood flow circuit of the kidney is called a Renal Circulatory System. Hence renal circulatory system is a sub system of human circulatory system. Blood enter in kidney by arteries and out by veins and in a kidney no source or sink.

Mass of enter the blood = mass of outer the blood Therefore law of conservation of mass can also be applied for renal circulatory system. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells.

Let X is the volume portion covered by the blood cells in unit volume. And X can be replaced by $H/100$, where H is the hematocrit the volume percentage of blood cells. Then the volume portion covered by plasma will be $1-X$. if the mass ratio of blood cells to plasma is r , then clearly

$$r = \frac{x \rho_c}{(1-x) \rho_p}$$

Where ρ_c and ρ_p and are densities of blood cells and blood plasma respectively. Usually this mass ratio is not constant; even then this may be supposed to be constant in present context [39]. The both phase of blood, i.e., blood cells and plasma move with a common velocity. Campbell and Pitcher have presented a model for this situation. According to this model we consider the two phases of blood separately [40]. Hence according to principle of conservation of mass, the equations of continuity for the two phases are as follows [41].

$$\frac{\partial(x\rho_c)}{\partial t} + (X\rho_c V^i)_{,i} = 0$$

$$\frac{\partial((1-X)\rho_p)}{\partial t} + ((1-X)\rho_p V^i)_{,i} = 0$$

Where v is the common velocity of the two phases blood cells and plasma and $(X\rho_c V^i)_{,i}$ is co-variant derivative of $(X\rho_c V^i)$ with respect to X^i

In the same way $((1-X)\rho_p V^i)_{,i}$ with respect to X^i .

If we define uniform density ρ_m as follows:

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \dots\dots(1)$$

Then the equations can be combined together as follows,

$$\frac{\partial(\rho_m)}{\partial t} + (\rho_m c)_{,i} = 0$$

As we know that blood is incompressible fluid, hence ρ_m will be a constant quantity. Thus the equation of continuity for blood flow takes the following form:

$$V^i_{,i} = 0$$

$$\frac{\partial v^i}{\partial x^i} + \frac{v^i \partial \sqrt{g}}{\sqrt{g} \partial x^i} = \frac{1}{\sqrt{g}} (\sqrt{g} V^i)_{,i} = 0$$

According to this principle, the total momentum of any fluid system is conserved in absence of external force. So the law of conservation of momentum can well apply to renal circulatory system. In other words, the rate of

change of momentum of a fluid particle with respect to time equals to external force exerted on it. This is also called Newton's 2nd law of motion.

So, the rate of change of momentum is equal to sum of about two mentioned forces, which may be symbolically presented as follows.

$$\frac{dp}{dt} = -P + F \text{ where, } \frac{dp}{dt} = \text{rate of change of momentum}$$

P=Internal pressures

F=viscous force

The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood [42]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^i)v_{,j}^i = -Xp_{,j} g^{ij} + X\eta_c (g^{jk} v_{,k}^i)_{,j} \quad \dots(2)$$

Similarly, taking the viscosity coefficient of plasma to be η_p the equation of motion of plasma will be as follows:

$$(1 - X)\rho_p \frac{\partial v^i}{\partial t} + ((1 - X)\rho_p v^j)v_{,j}^i = -(1 - X)p_{,j} g^{ij} + (1 - X)\eta_p (g^{jk} v_{,k}^i)_{,j} \quad \dots(3)$$

Now adding equation (2) and (3) and using relation (1), the equation of blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j)v_{,j}^i = -p_{,j} g^{ij} + \eta_m (g^{jk} v_{,k}^i)_{,j}$$

Where $\eta_m = X\eta_c + (1 - X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

Generally, blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

(1) Newtonian equation

$\tau = \eta e$, where η is the viscosity coefficient

This is found to hold good in the broad blood vessels where there is low hematocrit [43].

(2) The non-Newtonian power law equation

$$\tau = \eta e^n$$

This is found to be conformable for strain rate between 5 and 200

$$0.68 \leq n \leq 0.80 \quad [44]$$

The non-Newtonian Herschel-Bulkley equation [45]

$$\tau = \eta e^n + \tau_0 (\tau \geq \tau_0)$$

$$e = 0 (\tau < \tau_0)$$

It holds good when blood shows yield stress τ_0 .

We notice that the yield stress arise because blood cells form aggregates in the form of rouleaux at low strain rate.

If $\tau < \tau_0$, no blood flow takes place. It is found that yield stress is given by the following formula

$$\tau_0^{\frac{1}{3}} = \frac{A(H - H_m)}{100}$$

Where,

$$A = (0.08 \pm 0.002 \text{ dyne/cm}^2)^{\frac{1}{3}}$$

H is normal haematocrit and H_m is the haematocrit below which there is no yield stress.

III. HEMATOCRIT

This is the ratio of the volume of red cells to the volume of whole blood. Normal range for hematocrit is different between the sexes and is approximately 45% to 52% for men and 37% to 48% for women. This is usually measured by spinning down a sample of blood in a test tube, which causes the red blood cells to pack at the bottom of the tube. The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as grams per deciliter).[24][32][33][34][35]

IV. BOUNDARY CONDITIONS

1. The velocity of blood flow on the axis of capillaries at $r=0$ will be maximum and finite, say $V_0 =$ maximum velocity.

2. The velocity of blood flow on the wall blood vessel at $r=R$, where, R is the radius of capillary, will be zero. This condition is well known as no-slip condition.

V. MATHEMATICAL MODELLING

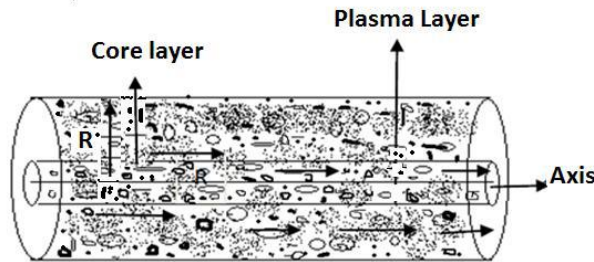
We consider the two-layer blood flow to be Newtonian. The first layer is that of plasma while second one is core layer. Let the viscosity of plasma layer be η_p and that of core layer η_m .

Where $\eta_m = X\eta_c + (1 - X)\eta_p$ where

η_c = viscosity of blood cells η_p = Viscosity of Plasma layer

η_m =Viscosity of core layer

and X is portion of blood cells in unit . $X = \frac{H}{100}$



Vessels Capillary

Fig5

VI. EQUATION OF CONTINUITY

Now we describe the basic equations for Power law blood flow as follows: In tensorial form as follows:

$$\frac{1}{\sqrt{g}} (\sqrt{g} v^i)_{,i} = 0$$

VII. EQUATION OF MOTION

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v_{,j}^i = -\rho_{,j} g^{ij} + \eta_m (g^{jk} v_{,k}^i)_{,j}$$

$$\rho_m = X\rho_c + (1 - X)\rho_p$$

Where

ρ_m =density of mixture blood

ρ_c = density of plasma

ρ_p = density of blood cells

The blood flow in capillary is symmetrical w.r.t. axis.

Hence, v_θ, v_z, v_r and p do not depend upon θ . Since only one component of velocity which is along axis is effective.

We have,

$$v_\theta = 0, v_z = V, v_r = 0$$

Since, flow is steady,

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

$$\frac{\partial v_z}{\partial z} = 0, v_z = V(r),$$

r-component

$$\rho_m (0) = -\frac{\partial p}{\partial r} + \eta_m (0), \frac{\partial p}{\partial r} = 0, P = p(z), \theta\text{-component}$$

$$\rho_m (0) = 0 + \eta_m (0)$$

z- component

$$\rho_m v_z \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r} \frac{\partial}{\partial r} \left\{ r \frac{\partial v_z}{\partial r} \right\} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

$$\rho_m v_r \frac{\partial v(r)}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r} \frac{\partial}{\partial r} \left\{ r \frac{\partial v(r)}{\partial r} \right\} + \frac{\partial^2 v(r)}{\partial z^2} \right]$$

and pressure p depends on z.

$$p = -\frac{\partial p}{\partial z}$$

By using first and second boundary condition, we get

$$V = \frac{p}{4\eta_m} (R^2 - r^2)$$

The velocity of plasma layer is obtained by replacing η_m by η_p in formula of Newtonian model, which is as follows:

$$V_p = \frac{p}{4\eta_p} (R^2 - r^2); R-\delta \leq r \leq R$$

The velocity of core layer can also be obtained in a similar way as follows:

$$V_p = \frac{p}{4\eta_p} (R^2 - r^2); R-\delta \leq r \leq R$$

$$v_m = \frac{p}{4\eta_m} (R^2 - r^2) + \frac{p}{4\eta_m} [R^2 - (R-\delta)^2] \left[\frac{\eta_m}{\eta_p} - 1 \right]; 0 \leq r \leq R-\delta$$

Where R is the radius of the capillary and δ is the thickness of the plasma layer. δ is supposed to be independent of R. [15]

VIII. BIO-PHYSICAL INTERPRETATION

The blood flow in capillary is

$$Q = \int_0^{R-\delta} v_m 2\pi r dr + \int_{R-\delta}^R v_p 2\pi r dr$$

$$Q = \int_0^{R-\delta} \left[\frac{p}{4\eta_m} (R^2 - r^2) + \frac{p}{4\eta_m} [R^2 - (R-\delta)^2] \left(\frac{\eta_m}{\eta_p} - 1 \right) \right] 2\pi r dr + \int_{R-\delta}^R \frac{p}{4\eta_p} (R^2 - r^2) 2\pi r dr$$

$$Q = \frac{\pi p R^4}{8\eta_p} \left[1 - \left(1 - \frac{\delta}{R} \right)^4 \left(1 - \frac{\eta_p}{\eta_m} \right) \right]$$

$$Q = \frac{\pi P R^4}{8\eta} , \text{ where } \eta = \eta_p \left[1 - \left(1 - \frac{\delta}{R} \right)^4 \left(1 - \frac{\eta_p}{\eta_m} \right) \right]^{-1}$$

Clinical data (male Diabetic, age 45yrs)

Sno.	H.B.	Hematocrit	Pressure drop(mmhg)
1	9.1	26.8	140/90mmhg=18664.8/11998.8p
2	9.2	27.8	140/90mmhg=18664.8/11998.8p
3	9.4	28.9	130/90mmhg=17331.6/11998p
4	8.8	27	130/90mmhg=17331.6/11998p
5	9.8	29	140/90mmhg=18664.8/11998.8p

In this model we can find directly the relationship between the Hematocrit and blood pressure drop.

$$p = -\frac{dp}{dz} \rightarrow p \int_{z_i}^{z_f} dz = - \int_{p_i}^{p_f} dp \rightarrow p(z_f - z_i) = (p_i - p_f)$$

$$p = \frac{(p_i - p_f)}{(z_f - z_i)} = \frac{\Delta p}{\text{length of capillary}} \dots\dots\dots(1)$$

$$\text{Now } Q = \frac{\pi P R^4}{8\eta} , \text{ where } \eta = \eta_p \left[1 - \left(1 - \frac{\delta}{R} \right)^4 \left(1 - \frac{\eta_p}{\eta_m} \right) \right]^{-1} \dots(2)$$

From (1) and (2)

$$\Delta p = \frac{8Q\eta_p \times \text{length of capillary}}{\pi R^4 \left(\left[1 - \left(1 - \frac{\delta}{R} \right)^4 \left(1 - \frac{\eta_p}{\eta_m} \right) \right] \right)} \dots\dots\dots(3)$$

We have η_p = Viscosity of plasma = 1.2×10^{-3} ps
 R = radius of capillary = 0.0965 m
 δ = Thickness of plasma 10^{-6} m
 Q = flow flux of blood in renal capillary = 0.01833 pa

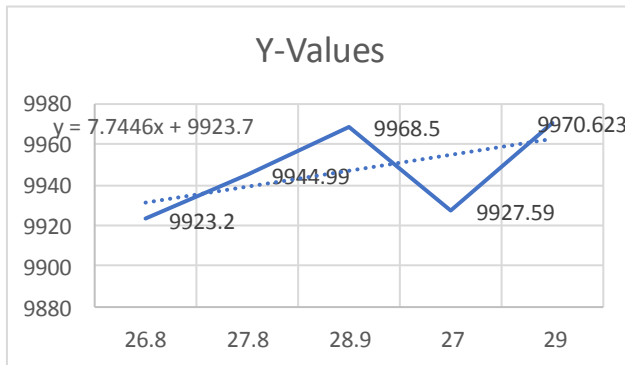
$$\eta_m = X\eta_c + (1 - X)\eta_p$$

Substituting in (3) we have

$$\Delta p = \frac{1.939 \times 10^{-4} \times H + 0.0117}{1.5766 \times 10^{-8} \times H + 1.2802 \times 10^{-6}}$$

IX. RESULT AND DISCUSSION

S. no	1	2	3	4	5
H	26.8	27.8	28.9	27	29
P	99	99	99	99	997
D	23.20	44.99	68.50	27.59	0.623



X. CONCLUSION

In Bio physical Interpretation, we have taken clinical data regarding with Blood Pressure and Hematocrit of Diabetic Patient. And we get the relation

$$\Delta p = \frac{1.939 \times 10^{-4} \times H + 0.0117}{1.5766 \times 10^{-8} \times H + 1.2802 \times 10^{-6}}$$

, by using the Two phase Newtonian Model (Newtonian Power law) and draw the graph between Blood pressure drop and Hematocrit in Renal capillary and trend of graph shows the relation between Blood Pressure drop and Hematocrit as linear as $y = 7.744x + 9970.62$. This linear relation approves the two-phase relation $\eta_m = \eta_c X + \eta_p (1 - X)$ where $X = H/100$ And slope of trend line is 7.744.

Hence the two phase Newtonian model is verified in clinical data of the Diabetic patient and pressure drop is proportional to Hematocrit, It is remarkable that velocity of plasma layer is taken as if whole capillary is filled with plasma. Again, the velocity of core layer is taken as if the core layer blood is filled in whole capillary. The relative velocity of the both layers is also added to it. **Remark**
 These information are beneficial to medical science and give idea of blood requirement during operation

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