Spherically Symmetric Space-Time with Magnatic Field and Zero Mass Scalar Field

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Abstract: *Einstein's field equation for a homogeneous spherically symmetric space-time with the field of gravitations viz: Magnetic field and zero mass scalar field has been considered. The solution for the cosmic parameters 'λ' and 'μ', which are function of 't' only are obtained in three cases viz: (i)* $m \neq 0,\ n \neq 0, v \neq 0$ *(ii)* $m = 0, n \neq 0, v \neq 0$ *(iii)* $n = 0,\ m \neq 0,\ v \neq 0$, *here m and n are the components of electromagnetic potential where as V is the scalar potential of the zero mass scalar field. The variation of* $\lambda,\ \mu, m, n$ *and* V *with the cosmic time has been discussed followed by the concluding remark.*

Keywords: Spherically Symmetric, Electromagnetic Field, Zero Rest Mass Scalar Field, Radiation.

I. Introduction:

The Einstein's theory of gravity is exceedingly compelling and stands out sharp and clear as it agrees with experiments. It describes gravity entirely in terms of geometry without depending on prior geometry. The spherical symmetry has immediate relation to our intuitive notions pertaining to the structure of the nature. Every physical theory carries its own mathematical structure in form of tensor equation and the validity of the theory is usually studied through the solutions of the mathematical structure.

The scalar fields represent matter field with spin loss quanta. The zero rest mass scalar field describes long range interactions where as the massive scalar field describes short range interaction. Photon particles carrying the energy of radiation in the electromagnetic field are associated with forces between the electric charges. They are similar to the relativistic particles with zero rest mass. In this paper we have taken zero rest mass scalar field along with electromagnetic field in a spherically symmetric metric to construct a new model of the universe.

II. Metric and Field Equations:

In this paper, we consider a spherically symmetric space-time of the form

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + e^{\mu} dt^2 , \tag{1}$$

where $\lambda(t)$ and $\mu(t)$ are the cosmic scale factors.

The field equation of electromagnetic field is,

$$E_{ij} = \frac{1}{4\pi}\left(-F_{ia} F_j^a + \frac{1}{4} g_{ij} F_{ab} F^{ab} \right), \tag{2}$$

where the electromagnetic field tensor (F_{ij}) satisfies the following equations

$$F_{ij} = \phi_{j,i} - \phi_{i,j} \tag{3}$$

and

$$F^{ij}_{;j} = 4\pi \sigma U^i , \tag{4}$$

where (,) denotes ordinary differentiation. Here, σ is the charge density, U^i is the four velocity vector and ϕ_i is the electromagnetic four potential of the form $\phi_i = (0, m, n, 0)$.

The field equation of zero mass scalar field is,

$$T_{ij} = \frac{1}{4\pi}\left(v_{,i}\ v_{,j} - \frac{1}{2}\ g_{ij}\ v_{,a}\ v^{'a} \right)$$

(5)

which satisfies the relativistically invariant Klein-Gorden equation

$$g^{ij}\ v_{,ij} = 0 .$$

(6)

Here 'v' is the real scalar field with $v_2 = v_3 = 0$

but v_1 *and* $v_4 \neq 0$

(7)

By taking both electromagnetic field and zero mass scalar field as the field of gravitation, we consider the following field equation that

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -k\left(E_{ij} + T_{ij}\right)$$

(8)

The existing values of equation (5) for metric (1) with help of (6) and (7) are

$$\left.\begin{aligned}
T_{11} &= \frac{1}{8\pi}\left(v_1^2 + e^{\lambda-\mu}\, v_4^2\right), \\
T_{14} &= \frac{1}{4\pi}\left(v_1 v_4\right) = T_{41}, \\
T_{22} &= \frac{1}{8\pi}\left(\frac{v_4^2\, r^2}{e^{\mu}} - \frac{v_1^2 r^2}{e^{\lambda}}\right), \\
T_{33} &= \sin^2\theta\ T_{22}, \\
T_{44} &= \frac{1}{8\pi}\left(v_4^2 + v_1^2\ e^{\mu-\lambda}\right) .
\end{aligned}\right\}$$

(9)

The existing values of equation(2) for metric(1) with the help of equation (3) and (4) are

$$\left.\begin{aligned}
E_{11} &= \frac{1}{4\pi}\left(-\frac{3}{2}\frac{m_1^2}{r^2} + \frac{1}{2}\frac{n_1^2}{r^2\sin^2\theta} + \frac{m_4^2}{2r^2}\, e^{\lambda-\mu} + \frac{n_4^2\ e^{\lambda-\mu}}{2r^2\sin^2\theta}\right), \\
E_{14} = E_{41} &= \frac{1}{4\pi}\left(\frac{m_1 m_4}{r^2} + \frac{n_1 n_4}{r^2\sin^2\theta}\right), \\
E_{22} &= \frac{1}{4\pi}\left(\frac{1}{2}\frac{m_1^2}{e^{\lambda}} - \frac{m_4^2}{e^{\mu}} - \frac{n_1^2}{2e^{\lambda}\sin^2\theta} + \frac{m_4^2}{2e^{\mu}} + \frac{n_4^2}{2e^{\mu}\sin^2\theta}\right), \\
E_{23} = E_{32} &= \frac{1}{4\pi}\left(\frac{m_1 n_1}{e^{\lambda}} - \frac{m_4 n_4}{e^{\mu}}\right), \\
E_{33} &= \frac{1}{4\pi}\left(\frac{n_1^2}{e^{\lambda}} - \frac{n_4^2}{e^{\mu}} - \frac{m_1^2\sin^2\theta}{2e^{\lambda}} - \frac{n_1^2}{2e^{\lambda}} + \frac{m_4^2\ \sin^2\theta}{2e^{\mu}} + \frac{n_4^2}{2e^{\mu}}\right), \\
E_{41} = E_{14} &= \frac{1}{4\pi}\left(\frac{m_1 m_4}{r^2} + \frac{n_1 n_4}{r^2\sin^2\theta}\right), \\
E_{44} &= \frac{1}{4\pi}\left(\frac{m_4^2}{r^2}e^{\mu} + \frac{n_4^2}{r^2\sin^2\theta}\, e^{\mu} + \frac{m_1^2}{2r^2}\, e^{\mu-\lambda} + \frac{n_1^2}{2r^2\sin^2\theta}e^{\mu-\lambda} - \frac{m_4^2}{2r^2} - \frac{n_4^2}{2r^2\sin^2\theta}\right).
\end{aligned}\right\} \quad (10)$$

Using equation (9) and (10) in equation (8), the existing field equations are

$$G_{11} \equiv -\frac{1}{4}\lambda_4^2 e^{\lambda-\mu} - \frac{\lambda_4\mu_4}{8} e^{\lambda-\mu} - \frac{1}{r^2} + \frac{e^{\lambda}}{r^2} =$$
$$-k\left[\frac{1}{4\pi}\left(-\frac{3}{2}\frac{m_1^2}{r^2} + \frac{1}{2}\frac{n_1^2}{r^2\sin^2\theta} + \frac{m_4^2}{2r^2}e^{\lambda-\mu} + \frac{n_4^2}{2r^2\sin^2\theta}e^{\lambda-\mu}\right) + \frac{1}{8\pi}\left(v_1^2 + e^{\lambda-\mu} v_4^2\right)\right]$$

(11)

$$G_{22} \equiv \frac{r^2}{2e^{\mu}}\left(\lambda_{44} + \lambda_4^2 - \frac{\lambda_4\mu_4}{4}\right) =$$
$$-k\left[\frac{1}{4\pi}\left(\frac{m_1^2}{2e^{\lambda}} - \frac{m_4^2}{2e^{\mu}} - \frac{n_1^2}{2e^{\lambda}\sin^2\theta} + \frac{n_4^2}{2e^{\mu}\sin^2\theta}\right) + \frac{r^2}{8\pi}\left(\frac{-v_1^2}{e^{\lambda}} + \frac{v_4^2}{e^{\mu}}\right)\right]$$

(12)

$$G_{33} \equiv \frac{r^2\sin^2\theta}{2e^{\mu}}\left(\lambda_{44} + \lambda_4^2 - \frac{\lambda_4\mu_4}{4}\right)$$
$$= -k\left[\frac{1}{4\pi}\left(\frac{n_1^2}{2e^{\lambda}} - \frac{n_4^2}{2e^{\mu}} - \frac{m_1^2\sin^2\theta}{2e^{\lambda}} + \frac{m_4^2\sin^2\theta}{2e^{\mu}}\right) + \frac{r^2\sin^2\theta}{8\pi}\left(\frac{-v_1^2}{e^{\lambda}} + \frac{v_4^2}{e^{\mu}}\right)\right]$$

(13)

$$G_{44} \equiv -\frac{\lambda_4^2}{4} - \frac{\lambda_4\mu_4}{8} + \frac{e^{\mu-\lambda}}{r^2} - \frac{e^{\mu}}{r^2}$$
$$= -k\left[\frac{1}{8\pi}\left(v_4^2 + v_1^2 e^{\mu-\lambda}\right) + \frac{1}{4\pi}\left\{\frac{m_4^2}{r_2}\left(e^{\mu} - \frac{1}{2}\right) + \frac{n_4^2}{r^2\sin^2\theta}\left(e^{\mu} - \frac{1}{2}\right) + \frac{m_1^2 e^{\mu-\lambda}}{2r^2} + \frac{n_1^2 e^{\mu-\lambda}}{2r^2\sin^2\theta}\right\}\right]$$

(14)

$$G_{14} = G_{41} \equiv -\frac{k}{4\pi}\left(\frac{m_1 m_4}{r^2} + \frac{n_1 n_4}{r^2\sin^2\theta} + v_1 v_4\right) = 0$$

(15)

$$G_{23} = G_{32} \equiv \frac{-k}{4\pi}\left(\frac{m_1 n_1}{e^{\lambda}} - \frac{m_4 n_4}{e^{\mu}}\right) = 0$$

(16)

III. SOLUTION TO FIELD EQUATIONS (11) TO EQUATION (16):

We can find the solution in three different cases.

Case-I : $m \neq 0,\ n \neq 0,\ v \neq 0$

Case-II: $m = 0,\ n \neq 0,\ v \neq 0$

Case-III: $m \neq 0,\ n = 0,\ v \neq 0$

Case-I:

This case have three sub-cases:

Sub-Case(a): $m = f(r,t),\ n = f(r,t), v \neq 0.$

From equation (16), we get

$$\frac{m_1 n_1}{m_4 n_4} = e^{\lambda-\mu}.$$

(17)

Substracting equation (14), with the help of the reciprocal of equation (17) multiplied with $\frac{m_1 n_1}{m_4 n_4}$, from equation (11) with the help of equation (17), we get

$$\frac{k}{4\pi}\frac{n_4^2}{r^2\sin^2\theta}\frac{m_1 n_1}{m_4 n_4} - \frac{k}{4\pi}v_4^2\frac{m_1 n_1}{m_4 n_4} + \frac{2e^{\lambda}}{r^2} - \frac{2}{r^2} - \frac{km_1^2}{2\pi r^2} + \frac{km_4 m_1 n_1}{4\pi r^2 n_4} +$$

$$\frac{e^{\mu}}{r^2}\frac{m_1 n_1}{m_4 n_4}\left(1 - \frac{km_4^2 n_4}{4\pi m_1 n_1} - \frac{kn_4^3 m_4}{4\pi\sin^2\theta\, m_1 n_1}\right) = 0$$

(18)

Dividing by $\frac{m_4 n_4 r^2}{m_1 n_1}$ and taking

$$P = \frac{-2m_4 n_4}{m_1 n_1} - \frac{k}{2\pi}\frac{m_1 m_4 n_4}{n_1} + \frac{k}{4\pi}m_4^2 + \frac{k}{4\pi}\frac{n^2}{\sin^2\theta} - \frac{k}{4\pi}v_4^2 r^2$$

and

$$Q = 1 - \frac{k}{4\pi}\frac{m_4^3 n_4}{m_1 n_1} - \frac{k}{4\pi}\sin^2\theta\frac{n_4^3 m_4}{m_1 n_1},$$ equation (10) yields

$$\frac{2m_4 n_4}{m_1 n_1}e^{\lambda} + P = -Qe^{\mu}.$$

(19)

By solving equation (18) after using equation (17), we get

$$\mu = \ln\left(\frac{-P}{Q+P}\right).$$

(20)

Using equation (20) in equation (17), we get

$$\lambda = \ln\left[\frac{m_1 n_1}{m_4 n_4}\left(\frac{-P}{2+Q}\right)\right].$$

(21)

Putting the values of 'P' and 'Q', equation (20) and equation (21) becomes

$$\lambda = \ln\left[\frac{\frac{4\pi}{k} + m_1^2 - \frac{2m_1n_1m_4}{n_4} - \frac{2m_1n_1n_4}{m_4\sin^2\theta} + \frac{2m_1n_1v_4^2r^2}{m_4n_4}}{\frac{6\pi}{k} - \frac{2m_4^3\,n_4}{m_1n_1} - \frac{2n_4^3\,m_4}{\sin^2\theta\, m_1n_1}}\right]$$

(22)

$$\mu = \ln\left[\frac{\frac{4\pi m_4n_4}{km_1n_1} + \frac{m_1n_4m_4}{n_1} - 2m_4^2 - \frac{2n_4^2}{\sin^2\theta} + 2v_4^2\,r^2}{\frac{6\pi}{k} - \frac{2m_4^3\,n_4}{m_1n_1} - \frac{2n_4^3m_4}{m_1n_1}}\right].$$

(23)

Taking 'm' and 'n' as function of 'r' and 't', we find that the scalar field is also function of r and t.

Sub-Case(b): $m = f(r);\, n = f(t)\; and\; v \neq 0$:

For this case, equation (22) and equation (23) yields

$$\lambda = \ln\left[\frac{k}{12\pi^2}\left(4\pi + km_1^2\right)\right] = \ln\left(\frac{4\pi k + k^2\, f(r)}{12\pi^2}\right).$$

From which we get

$$\lambda_4 = 0\; and\; m_1^2 = \frac{12\pi^2 e^{\lambda} - 4\pi k}{k^2}.$$

The value of cosmic parameter μ is found to be

$$\mu = \ln\left[\frac{-kn_4^2 + kv_4^2\, r^2\, \sin^2\theta}{12\pi\, \sin^2\theta}\right] = \ln\left[\frac{kv_4^2\, r^2\, \sin^2\theta - kf(t)}{12\pi\, \sin^2\theta}\right].$$

Thus $n_4^2 = \frac{12\pi\, \sin^2\theta}{k}\, e^{\mu}$. Equation (15) yields $v_1v_4 = 0$, taking $v_4 = 0$ and $v_1 \neq 0$

$$\mu = \ln\left[\frac{-kf(t)}{12\pi\, \sin^2\,\theta}\right].$$

Putting $\lambda_4 = 0, n_1 = 0, v_4 = 0, m_4 = 0$ in equation (11), we get

$$v_1^2 = \frac{-8\pi}{kf(r)}\left(e^{\lambda} - 1\right) + \frac{3k}{4\pi}.$$

Thus we conclude that

$$\lambda = f(r), \mu = f(t), v = f(r)$$
$$m = f(r),\, n = f(t).$$

Sub-Case(c): $m = f(t),\ n = f(r)\ and\ v \neq 0$**:**

For this sub-case, equation (22) and (23) yields

$$\mu = \ln\left[\frac{kv_4^2\ n_1^2 - km_4^2}{12\pi}\right] = \ln\left[\frac{kv_4^2 f(r) - kf(t)}{12\pi}\right].$$

Thus $m_4^2 = \frac{-4\pi}{k} e^{\mu}$. Equation (15) yields, $v_1 v_4 = 0$, taking $v_4 = 0\ and\ v_1 \neq 0$

$$\mu = \ln\left(\frac{-kf(t)}{12\pi}\right).$$

Taking $m_1 = 0, n_4 = 0, v_4 = 0, \lambda_4 = 0$ in equation (11), we get

$$v_1^2 = \frac{-8\pi}{kr^2}\left(e^{\lambda} - 1\right) - \frac{t^2}{r^2} e^{\lambda - \mu}.$$

This contradicts to my assumption that $v_4 = 0$. Hence this case is not considered.

Case-II $(m = 0,\ n \neq 0,\ v \neq 0)$**:**

Putting $m = 0$ in equations from (11) to (16), we get

$$G_{11} \equiv -\frac{1}{4}\lambda_4^2 e^{\lambda - \mu} - \frac{\lambda_4 \mu_4}{8} e^{\lambda - \mu} - \frac{1}{r^2} + \frac{e^{\lambda}}{r^2}$$

$$= -\frac{k\ n_1^2}{8r^2 \sin^2\theta} - \frac{kn_4^2\ e^{\lambda - \mu}}{8\pi r^2 \sin^2\theta} - \frac{k}{8\pi}\left(v_1^2 + v_4^2\ e^{\lambda - \mu}\right)$$

(24)

$$G_{22} \equiv \frac{1}{2} r^2 e^{-\mu}\left(\lambda_{44} + \lambda_4^2 - \frac{\lambda_4 \mu_4}{4}\right)$$

$$= \frac{k}{8\pi} \frac{n_1^2}{e^{\lambda} \sin^2\theta} - \frac{kn_4^2}{8\pi e^{\mu} \sin^2\theta} + \frac{kr^2 v_1^2}{8\pi e^{\lambda}} - \frac{kr^2 v_4^2}{8\pi e^{\mu}}$$

(25)

$$G_{33} = G_{22}\, r^2 \sin^2\theta$$

(26)

$$G_{44} \equiv \frac{-\lambda_4^2}{4} - \frac{\lambda_4 \mu_4}{8} + \frac{e^{\mu - \lambda}}{r^2} - \frac{e^{\mu}}{r^2}$$

$$= -\frac{k}{8\pi}\left(v_4^2 + v_1^2 e^{\mu - \lambda}\right) - \frac{kn_4^2\left(e^{\mu} - \frac{1}{2}\right)}{8\pi r^2 \sin^2\theta} - \frac{kn_1^2}{8\pi r^2 \sin^2\theta} e^{\mu - \lambda}$$

(27)

$$G_{14} = G_{41} \equiv \frac{-k}{4\pi}\left(\frac{n_1 n_4}{r^2 \sin^2\theta} + v_1 v_4\right) = 0.$$

(28)

Solution of Field Equations from (24) to (28):

Subtracting equation (25) by multiplying e^{λ} from equation (24) by multiplying r^2, we get

$$-\frac{3}{4}\lambda_4^2 r^2 e^{\lambda-\mu} + e^{\lambda} - \frac{r^2}{2}\lambda_{44}\, e^{\lambda-\mu}$$

$$= \frac{k}{4\pi}\frac{n_1^2}{\sin^2\theta} - \frac{k}{4\pi} r^2 v_1^2 \;.$$

(29)

Subtracting equation (25) multiplied by $e^{\lambda-\mu}$ from equation (23), we get

$$e^{\lambda} - 1 = \frac{k n_4^2}{32\pi \sin^2\theta}\, e^{\lambda-\mu}.$$

(30)

But this case can have two sub-cases.

Sub-Case(a): $n = f(t),\ v \neq 0$

Sub-Case(b): $n = f(r),\ v \neq 0$

Sub-Case(a) $n = f(t)$**:**

Equation (29) yields

$$-\frac{3}{4}\lambda_4^2\, r^2\, e^{\lambda-\mu} + e^{\lambda} - \frac{r^2}{2}\lambda_{44}\, e^{\lambda-\mu} = \frac{-k}{4\pi} r^2\, v_1^2 .$$

(31)

Equation(28) yields

$$v_1 v_4 = 0, \text{let } v_1 = 0 \text{ and } v_4 \neq 0 .$$

(32)

Putting equation (32) in equation (31), we get

$$-\frac{3}{2}\lambda_4^2 + \frac{2}{r^2}e^{\mu} - \lambda_{44} = 0.$$

Using $\lambda = \ln(t)$

(33)

in the above equation and solving, we get

$$\mu = \ln\left(\frac{r^2}{4t^2}\right).$$

(34)

Equation (30) reduces to

$$n_4^2 = \left(e^{\mu} - e^{\mu-\lambda}\right) \frac{32\pi \sin^2 \theta}{k}.$$

Putting equation (33) and (34) in the above equation, we get

$$n_4 = \left\{ \left(\frac{t-1}{4r^3 k} \right) 32\pi r^2 \sin^2 \theta \right\}^{\frac{1}{2}}.$$

(35)

Putting $v_1 = 0,\ n_1 = 0$ and the values of $\lambda\ and\ \mu$ and using equation (33) and (34) in equation (32), we get

$$v_4^2 = \frac{8\pi}{kr^2} e^{-\ln\left(\frac{4t^3}{r}\right)} \left(1 - e^{l_n t + 2 l_n \frac{4t^3}{r^2}} \right) - \frac{2\pi}{k} \left(\frac{1}{t^2} + 1 \right) - l_n \left(\frac{r^2}{4t^3} \right)^{\frac{\cos u^2 \theta}{r^2}}$$

(36)

Conclusion $\lambda = f(t),\ \mu = f(r,t),\ v_4 = f(r,t),\ n_4 = f(r,t).$

Sub Case(b): $n = f(r)$**:**

Equation (28) yields $v_1 v_4 = 0$. Let $v_1 = 0\ and\ v_4 \neq 0$ and putting $v_1 = 0$, equation(29) yields

$$r^2 e^{\lambda-\mu} \left(\frac{-3}{4} \lambda_4^2 + \frac{e^{\mu}}{r^2} - \frac{\lambda_4}{2} \right) = \frac{k}{4\pi} \frac{n_1^2}{\sin^2 \theta}$$

(37)

Equation (30) becomes

$$e^{\lambda} - 1 = 0 \Rightarrow \lambda = \ln 1 = c \text{ (constant)}$$

(38)

Using the value of λ in equation (37), we get

$$n_1 = 2 \sin\theta\ e^{c/2} \sqrt{\frac{\pi}{k}}.$$

(39)

Putting $n_4 = 0,\ v_1 = 0,\ \lambda_4 = \lambda_{44} = 0$ in equation(25), we get $\mu = \dfrac{l_n\ kr^2 v_4^2}{4\pi}$.

By taking $v = \pi t^2,\ v_4 = 2\pi t$

$$\therefore\ \mu = \ln\left(kr^2\ \pi t^2\right).$$

(40)

We conclude that $\lambda = const,\ \mu = f(r,t),\ v_4 = f(t),\ n \neq f(r,t).$

Case-III $(m \neq 0,\ n = 0, v \neq 0)$**:**

Putting n = 0 in equations from (11) to (15), we get

$$G_{11} \equiv \frac{-\lambda_4^2}{4} e^{\lambda-\mu} - \frac{\lambda_4\mu_4}{8} e^{\lambda-\mu} - \frac{1}{r^2} + \frac{e^{\lambda}}{r^2}$$

$$= \frac{-k}{4\pi}\left(\frac{-3n_1^2}{2r^2} + \frac{m_4^2}{2r^2} e^{\lambda-\mu} + \frac{1}{2}v_1^2 + \frac{v_4^2}{2} e^{\lambda-\mu} \right),$$

(41)

$$G_{22} \equiv \frac{r^2}{2e^{\mu}}\left(\lambda_{44} + \lambda_4^2 - \frac{\lambda_4\mu_4}{4} \right)$$

$$= \frac{-k}{4\pi}\left(\frac{m_1^2}{2e^{\lambda}} - \frac{m_4^2}{2e^{\mu}} - \frac{r^2 v_1^2}{2e^{\lambda}} + \frac{r^2 v_4^2}{2e^{\mu}} \right),$$

(42)

$$G_{33} = G_{22} \sin^2\theta,$$

(43)

$$G_{44} \equiv \frac{-\lambda_4^2}{4} - \frac{\lambda_4\,\mu_4}{8} + \frac{e^{\mu-\lambda}}{r^2} - \frac{e^{\mu}}{r^2}$$

$$= \frac{-k}{4\pi}\left[\frac{v_4^2}{2} + \frac{v_1^2}{2} e^{\mu-\lambda} + \frac{m_4^2}{r^2}\left(e^{\mu} - \frac{1}{2} \right) + \frac{m_1^2}{2r^2} e^{\mu-\lambda} \right]$$

(44)

and

$$G_{14} = G_{41} \equiv \frac{-k}{4\pi}\left(\frac{m_1 m_4}{r^2} + v_1 v_4 \right) = 0.$$

(45)

IV.SOLUTION OF FIELD EQUATIONS FROM (41) TO (45):

Substracting equation (44) multiplied by $e^{\lambda-\mu}$ from equation (41), we get

$$e^{\lambda}\left(8\pi + k m_4^2\, e^{-\mu} - k m_4^2\right) = 2km^2 + 8\pi.$$

(46)

This case can have two possible sub-cases.

Sub-Case $(a): m = f(t),\ v \neq 0$**:**

Equation(45) yields $v_1 v_4 = 0$. Taking $v_1 = 0$ *and* $v_4 \neq 0$, equation (46) yields

$$e^{\lambda}\left(2 + \frac{k m_4^2}{4\pi} e^{-\mu} - \frac{k m_4^2}{4\pi} \right) = 2.$$

Taking $\lambda = \ln t$ *and* $e^{\lambda} = t$ in the above equation, we get

$$\mu = \ln \frac{km_4^2\ t}{8\pi - 8\pi t + km_4^2 t}.$$

Let

$$m_4 = f(t) \Rightarrow \mu = \ln\left(\frac{4kf(t)}{8\pi - 8\pi t + 4kf(t)}\right).$$

Putting $n = 0, m_1 = 0,\ v_1 = 0, m_4 = f(t)$ and $\lambda = \ln\ t$ in equation (41) we get,

$$v_4^2 = \frac{-2\pi\lambda_4^2}{k} - \frac{\pi}{k}\lambda_4\,\mu_4 - \frac{8\pi}{kr^2}\ e^{\mu-\lambda} + \frac{8\pi}{kr^2}\ e^{\mu} - \frac{f(t)}{r^2}.$$

Since λ and μ are function of 't' and hence V_4 is also found to be function of 't'.

We conclude that $\lambda = f(t),\ \mu = f(t),\ m_4 = f(t), v_4 = f(r,t).$

Sub-Case-b: $m = f(r), v \neq 0$

Equation (42) yields $\lambda = \ln\left(1 + \frac{km_1^2}{4\pi}\right).$

Taking $m_1 = f(r),\ \lambda = \ln\left(1 + \frac{kf(r)}{4\pi}\right) \Rightarrow \lambda_4 = 0$,

from equation (42), we get

$$e^{\mu} = \frac{km_4^2}{\left(2km_1^2 + 8\pi\right)e^{-\lambda} - 8\pi + km_4^2}.$$

Now putting $n = 0.m_4 = 0,\ v_1 = 0,\ \lambda_4 = 0$ in equation (41), we get

$$v_4^2 = \frac{8\pi}{k}\ e^{-\ln\left(1+\frac{kr^2}{4\pi}\right)}\left(\frac{8\pi - 2kr^2 t}{8\pi r^2}\right).$$

Hence v_4 is also a function of 't'and 'r'.

We conclude that $\lambda = f(r),\quad v = f(r,t),\ m = f(r).$

V. STUDY OF SOME PHYSICAL PROPERTIES AND ENERGY CONDITIONS:

(a). Energy density of electromagnetic field is found to be

$$E_{44} = \frac{1}{4\pi}\left(\frac{e^{\mu}n_4^2}{r^2} + \frac{e^{\mu}n_4^2}{r^2\sin^2\theta} + \frac{e^{\mu-\lambda}n_1^2}{2r^2} + \frac{n_1^2\,e^{\mu-\lambda}}{2r^2\sin^2\theta} - \frac{n_4^2}{2r^2} - \frac{n_4^2}{2r^2\sin^2\theta}\right)$$

- At 't' constant hyper surface, we have $E_{44} \to \infty$ as $r \to \infty$ and .. for all sub cases but $E_{44} \to 0\ as\ r \to \infty$ for Case-III.

- At 'r' constant hyper surface, $E_{44} \to \infty$ as $r \to \infty$ and $E_{44} \to \infty \; as \; t \to \infty$ and $E_{44} \to 0 \; as \; t \to 0$ for all sub-cases except Case-II(b) and Case- III(b) which remain unaffected.

(b). Energy condition of Scalar field is found to be

$$T_{44} = \frac{1}{8\pi}\left(v_4^2 + e^{\mu-\lambda}\; v_1^2\right).$$

At 't' constant hyper surface, $T_{44} \to \infty \; as \; t \to \infty \; and \; T_{44} \to 0 \; as \; t \to 0$, for all cases and at 'r' constant hyper surface, also $T_{44} \to \infty \; and \; r \to \infty \; and \; T_{44} \to 0 \; as \; r \to 0$ for all sub cases except Case-III(b) which remains unaffected.

GRAVITATIONAL RADIATION:

The scalar field has gravitation field radiation, if $T \neq 0$. In our investigation for the spherically symmetric space-time,

$$T = \frac{1}{8\pi}\left[v_4^2\left(1 + e^{\lambda-\mu} + r^2 e^{-\mu} + r^2 e^{-\mu}\sin^2\theta\right) + v_1^2\left(e^{\mu-\lambda} + 1 - r^2 e^{-\lambda} - r^2 e^{-\lambda}\sin^2\theta\right) + 2v_1 v_4\right]$$

$\neq 0$

Hence the scalar field possesses gravitational field radiation.

UNIFORMITY:

The electromagnetic field is said to be uniform, if $F_{ij,k} = 0$.

In the present investigation, $F_{ij,k} \neq 0$ in all the cases because the existing values of F_{ij} and

$$F_{12} = m_1 = -F_{12} \qquad\qquad F_{42} = m_4 = -F_{24}$$

$$F_{13} = n_1 = -F_{31} \qquad\qquad F_{43} = n_4 = -F_{34}$$

and the electromagnetic four potentials m and n are found to be functions of r and t except Case-II(b). Hence the field is uniform in this particular case except all cases.

NEW FORM OF THE SPACE TIME:

According to the various of λ and μ in different cases of our investigation , the structure of the universe can be of different types as follows:

Case-I:

Sub-Case-a:

$$ds^2 = -\left[\frac{\frac{4\pi}{k} + m_1^2 - \frac{2m_1 n_1 m_4}{n_4} - \frac{2m_1 n_1 n_4}{m_4 \sin^2\theta} + \frac{2m_1 n_1 v_4^2 r^2}{m_4 n_4}}{\frac{6\pi}{k} - \frac{2m_4^3 n_4}{m_1 n_1} - \frac{2n_4^3 m_4}{\sin^2\theta\, m_1 n_1}}\right] dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 +$$

$$\left[\frac{\dfrac{4\pi m_4 n_4}{k m_1 n_1}+\dfrac{m_1 n_4 m_4}{n_1}-2m_4^2-\dfrac{2n_4^2}{\sin^2\theta}+2v_4^2\ r^2}{\dfrac{6\pi}{k}-\dfrac{2m_4^3\ n_4}{m_1 n_1}-\dfrac{2n_4^3\ m_4}{m_1 n_1}}\right]dt^2 .$$

Sub-Case-b:

$$ds^2=-\frac{4\pi k+k^2 f(r)}{12\pi^2}dr^2-r^2 d\theta^2-r^2\sin^2\theta d\phi^2+\frac{-kf(t)+3kr^2v_4^2\sin^2\theta}{12\pi^2}dt^2$$

Sub-Case-c:

$$ds^2=-\frac{k}{3\pi}dr^2-r^2 d\theta^2-r^2\sin^2\theta d\phi^2+\left(\frac{-kf(t)}{12\pi}\right)dt^2,$$

Case-II:

Sub-Case-a:

$$ds^2=-t\ dr^2-r^2 d\theta^2-r^2\sin^2\theta d\phi^2+\frac{r^2}{4t^2}dt^2.$$

Sub-Case-b:

$$ds^2=-e^c\ dr^2-r^2 d\theta^2-r^2\sin^2\theta d\phi^2+\frac{kv_4^2 f(r)}{4\pi}dt^2.$$

Case-III:

Sub Case-a:

$$ds^2=-t\ dr^2-r^2\ d\theta^2-r^2\ \sin^2\ d\phi^2+\frac{kf(t)}{8\pi-8\pi t+kf(t)}dt^2.$$

Sub-Case-b:

$$ds^2=-\left(1+\frac{km_1^2}{4\pi}\right)dr^2-r^2\ d\theta^2-r^2\ \sin^2\ d\phi^2+\frac{km_4^2}{\left(2km_1^2+8\pi\right)e^{-\lambda}-8\pi+km_4^2}dt^2.$$

VI. CONCLUSION:

In this paper, we have instructed a spherically symmetric space time with the mutual affect of electromagnetic field and zero mass scalar field. We got solution of the field equations in three cases and their sub cases and various models of the universe has been constructed. The energy conditions of the gravitational fields has been studied at ‘t’ constant and ‘r’ constant, hyper surfaces.

The gravitational field radiation is found to exist as $T\neq 0$. The electromagnetic field is found to be uniform only in Case-II(b) as $F_{ij,k}=0$ but in all other cases the field is found to be non uniform.

REFERENCES:

[1]. Ayangar, B.R.N. and G. Mohanty(1985), Interacting mass less scalar and source free

electromagnetic field, Acta. Phys. Polon B916), pp.13-20.
[2]. Bonnor, W.B (1954), Magnetic field in general relativity. Proc. Phys. E0c.67, pp.225-232.
[3]. Kashyap, J.N.S(1979), Coupled electromagnetic and scalar fields in cylindrically symmetric space-time. Act. Phys. Acad. Ecl. Hung. 45(4), p.309.
[4]. Rao, J.R., Tiwari, R.N. and Roy, A.R.(1973), A Class of exact solutions for coupled electromagnetic and scalar fields for Einstein-Rosen metric Part IA. Ann. Phys. 78, p.533.
[5]. Datta, C.R.(1975), Some scalar and electromagnetic distributions in general theory of relativity. Indian Institute of Technology, Kanpur, India, P.80.
[6]. S.N.Jena, R.N.patra, R.R.Swain(2015), An isotropic radiating model as a mutual effect of perfect fluid, cloud massive string and magnetic field in Bimetric Theory-International Journal of Mathematics Research, Vol.7(10, pp.69-73.
[7]. Jena, S.N., Patra, R.N., Swain, R.R(2015), Existence of stift fluid model as a coupling effect of perfect fluid and magnetic field in Bimetric Relativity-International Journal of Physics and Mathematical Sciences, Vol.5(3), pp.14-17.
[8]. S.N.Jena, R.N.Patra, R.R.Swain(2015), Non-existence of string cosmological models in presence of magnetic field in Bimetric theory, IJISET, Vol.2(6), pp.721-727.