ODD VERTEX MAGIC TOTAL LABELING OF SOME 2-REGULAR GRAPHS

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Abstract. A vertex magic total labeling of a graph $G(V,E)$ is defined as one - to - one mapping from the set of integers $\{1,2,3,...,|V|+|E|\}$ to $V \cup E$ with the property that the sum of the label of a vertex and the labels of all edges adjacent to this vertex is the same constant for all vertices of the graph. Such a labeling is called odd if $f(V) = \{1,3,5,...,2n-1\}$. In this paper, we present an odd vertex magic total labeling of of some 2 - regular graphs.

Key words: Magic, Labeling, Odd vertex magic total graph.

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1. Introduction

All graphs considered in this paper are finite, simple and undirected. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$ and we let $n = |V|$ and $m = |E|$. The degree of a vertex v is the number of edges that have v as an end point and the set of neighbours of v is denoted by $N(v)$.

Macdougall et al. [2] introduced the notion of a vertex magic total labeling. The vertex magic total labeling. The vertex magic total labeling $(VMTL)$ of a graph G is a one to one mapping from $V \cup E$ onto the integers $\{1,2,3,...,m+n\}$ such that there is a constant k so that for every vertex u, $w_f(u) = f(u) + \sum_{v \in N(u)} f(uv) = k$, where $N(u)$ is the set of all vertices v that the adjacent to u. The constant k is called the magic constant for f and $w_f(u)$ is called the weight of u under labeling f.

MacDougall et al. [3] introduced the concept of super vertex magic total labeling. They call a vertex magic total labeling is super if $f(V(G)) = \{1, 2, 3, ..., n\}$. In this labeling, the smallest labels are assigned to the vertices.

For more details of vertex magic graphs see the book by wallis [6], and for other types of graph labeling see the dynamic survey by Gallian [1].

Nagaraj et al. [4] introduced the concept of an Odd vertex magic total labeling. A vertex magic total labeling is odd if $f(V(G)) = \{1, 3, 5, ..., 2n-1\}$.

The authors of [4] studied the basic properties of odd vertex magic graphs and showed among other things that C_n and P_n have an odd vertex magic total labelings for $n \geq 3$, that rc_s is an odd vertex magic graph and that (s, t) - kite graph is an odd vertex magic graph iff $s + t$ is an odd.

Nagaraj et al. [5] proved the following results. A star graph $k_{1,r}$ is an odd vertex magic iff $r = 2$. If a tree T is an odd vertex magic then n is odd. If T has s internal vertices and st leaves then T does not admit an odd vertex magic labeling if $t > \frac{(s+1)}{s}$. If $\triangle$ is the largest degree of any vertex in a tree T with n vertices and m edges then T doesnot admit an odd vertex magic labeling whenever $\triangle > \frac{-3+\sqrt{1+16n}}{2}$.

2. Main Results

In this section, we will use the notation $[x_1, x_2, x_3, ..., x_k]$ to denote labels of a cycle of length k in the sequence vertex -edge - vertex - edge and so on. Thus, x_1 is the label of a vertex with weight $x_1 + x_2 + x_{2k}$.

Theorem 2.1. *The graph $C_3 \cup C_{4t}, t > 1$ is an odd vertex magic graph.*

Proof. We label the vertex and edges of C_3 with $[8t+1, 2t+6, 8t+3, 2t+2, 8t+5, 2t+4]$ and then edge label C_{4t} as follows.

$$f(e_i) = \begin{cases} i+3 & \text{if} \quad i \quad \text{odd}, 1 \leq i \leq 2t-3 \\ i+9 & \text{if} \quad i \quad \text{odd}, 2t-1 \leq i \leq 4t-3 \\ 2 & \text{if} \quad i = 4t-1 \\ 4t+6+i & \text{if} \quad i \quad \text{even} \end{cases}$$

The vertex label C_{4t} as follows

$$f(v_i) = \begin{cases} 4t+1 & \text{if} \quad i = 1 \\ 8t+3-2i & \text{if} \quad i = 2, 3, ..., 2t-2 \\ 8t-3-2i & \text{if} \quad 2t-1 \leq i \leq 4t-2 \\ 4t+3 & \text{if} \quad i = 4t \\ 4t+5 & \text{if} \quad i = 4t-1 \end{cases}$$

We show firstly that each edge label appears only once.

Edge Number	Range of labels
$4t-1$	2
i odd, $1 \leq i \leq 2t-3$	$4, 6, 8, ..., 2t$
C_3 edges	$2t+2, 2t+4, 2t+6$
i odd $2t-1 \leq i \leq 4t-3$	$2t+8, 2t+10, ..., 4t+6.$
i even	$4t+8, 4t+10, ..., 8t+6$

Next we show that each vertex label appears only once.

Vertex Number	Range of labels
1	$4t+1$
$2t-1 \leq i \leq 4t-2$	$1, 3, 5, ..., 4t-3, 4t-1$
$4t$	$4t+3$
$4t-1$	$4t+5$
$2 \leq i \leq 2t-2$	$4t+7, 4t+9, ..., 8t-3, 8t-1.$
C_3 vertex	$8t+1, 8t+3, 8t+5$

$\therefore C_3 \cup C_{4t}, t > 1$ is an odd vertex magic graph with magic constant $k = 12t+11$.

□

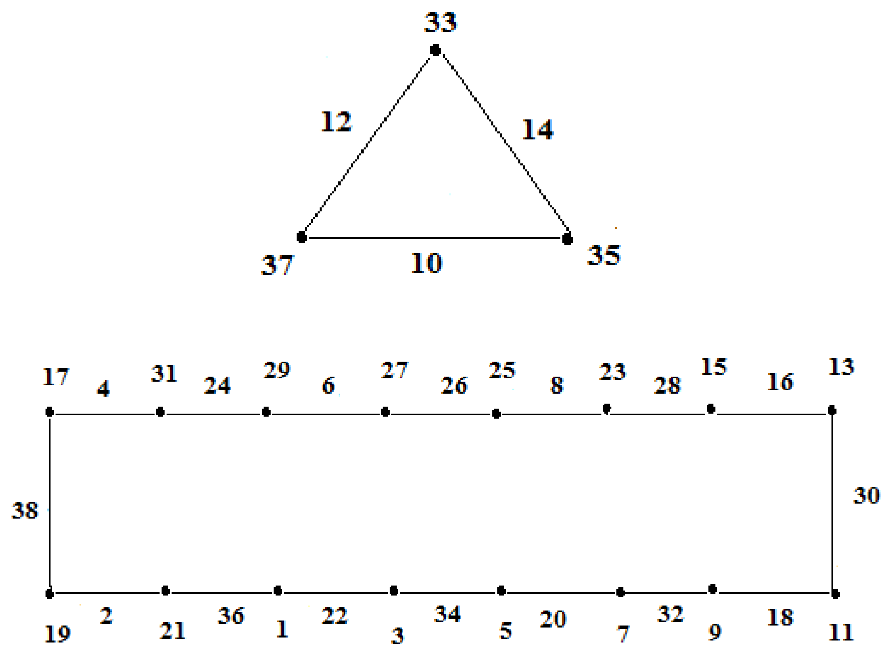


FIGURE 1. $C_3 \cup C_{16} \quad k = 59$

Theorem 2.2. *The graph $C_3 \cup C_{4t+2}, t \geq 1$ is Odd vertex magic graph.*

Proof. For $t = 1$ and $t = 2$ the labelings are
$[15, 4, 17, 8, 11, 10]$ $[9, 2, 13, 14, 3, 12, 1, 16, 7, 6, 5, 16]$ and
$[23, 6, 25, 10, 19, 12]$ $[13, 2, 21, 18, 7, 16, 5, 20, 17, 4, 15, 22, 11, 8, 9, 24, 3, 14, 1, 26]$
For $t \geq 3$, we label the vertex and edges of C_3 with $[8t+7, 2t+2, 8t+9, 2t+6, 8t+3, 2t+8]$ and then vertex label C_{4t+2} as follows.

$$f(v_i) = \begin{cases} 4t+5 & \text{if} \quad i = 1 \\ 8t+5 & \text{if} \quad i = 2 \\ 4t-1 & \text{if} \quad i = 3 \\ 4t-3 & \text{if} \quad i = 4 \\ 8t+11-2i & \text{if} \quad 5 \leq i \leq 2t+2 \\ 4t+3 & \text{if} \quad i = 2t+3 \\ 4t+1 & \text{if} \quad i = 2t+4 \\ 8t+5-2i & \text{if} \quad 2t+5 \leq i \leq 4t+2. \end{cases}$$

The edge label C_{4t+2} as follows

$$f(e_i) = \begin{cases} 2 & \text{if} \quad i = 1 \\ 4t+8 & \text{if} \quad i = 3 \\ i-1 & \text{if} \quad i \quad \text{odd}, 5 \leq i \leq 2t+1 \\ 2t+4 & \text{if} \quad i = 2t+3 \\ i+5 & \text{if} \quad i \quad \text{odd}, 2t+5 \leq i \leq 4t+1 \\ i+4t+8 & \text{if} \quad i \quad \text{even}. \end{cases}$$

It is easily verified that f is an odd vertex magic total labeling.
$C_3 \cup C_{4t+2}, t \geq 1$ is odd vertex magic graph magic constant $k = 12t + 17$.

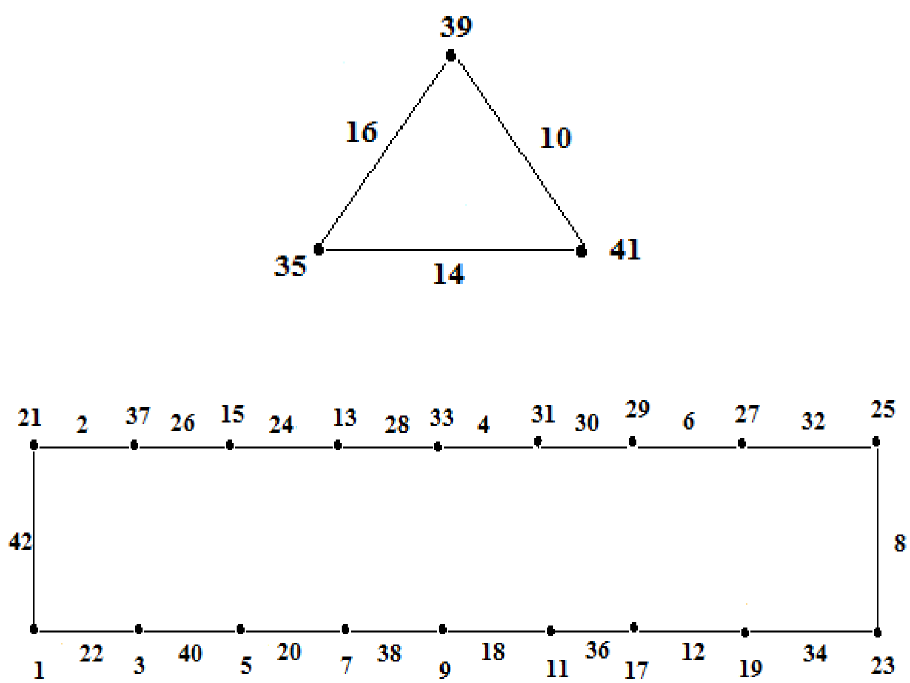


FIGURE 2. $C_3 \cup C_{18} \quad k = 65$

□

Theorem 2.3. *The graph $C_4 \cup C_{4t-1}$ for $t > 1$ is an odd vertex magic graph.*

Proof. We label the vertex and edges of C_4 consecutively as $[1, 4t+4, 3, 8t+4, 7, 4t, 5, 8t+6]$

We label the edges of C_{4t-1} consecutively as follows

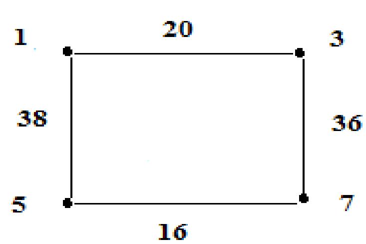
$$f(e_i) = \begin{cases} i+3 & \text{if} \quad i \equiv 1mod4, i < 4t-3 \\ 4t+6 & \text{if} \quad i = 2 \\ i+4t+2 & \text{if} \quad i \equiv 2mod4, i > 2 \\ i-1 & \text{if} \quad i \equiv 3mod4, i < 4t-1 \\ i+4t+6 & \text{if} \quad i \equiv 0mod4 \\ 4t-2 & \text{if} \quad i = 4t-3 \\ 4t-2 & \text{if} \quad i = 4t-1` \end{cases}$$

We label the vertex of C_{4t-1} consecutively as follows

$$f(v_i) = \begin{cases} 8t+5 & \text{if} \quad i = 1 \\ 8t-7 & \text{if} \quad i = 5, t \neq 2 \\ 11 & \text{if} \quad t = 2, i = 5 \\ 8t+1 & \text{if} \quad i = 2 \\ 8t-2i+7 & \text{if} \quad i \equiv 6mod4 \\ 13 & \text{if} \quad i = 4t-2 \\ 9 & \text{if} \quad i = 4t-1 \\ 8t+3 & \text{if} \quad i = 3 \\ 8t-2i+11 & \text{if} \quad 7 \leq i \leq 4t-5, i \equiv 3mod4 \\ 8t-2i+7 & \text{if} \quad i \equiv 0mod4 \\ 8t-2i+3 & \text{if} \quad i \equiv 5mod4, i \leq 4t-7 \\ 11 & \text{if} \quad i = 4t-3 \end{cases}$$

It is easily verified that f is an odd vertex magic total labeling.

$\therefore$ The graph $C_4 \cup C_{4t-1}, t > 1$ is odd vertex magic graph with magic constant $k = 12t + 11$.



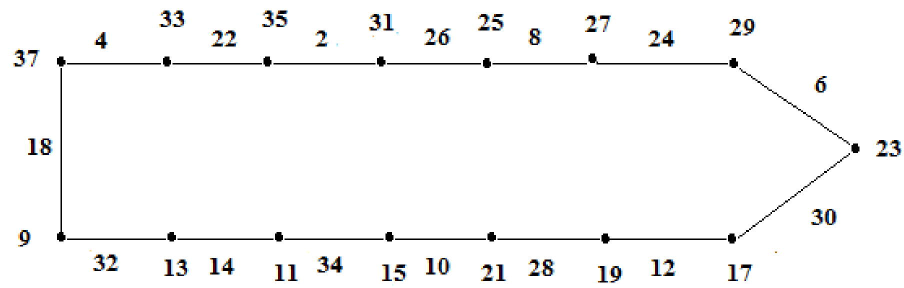


FIGURE 3. $C_4 \cup C_{15} \quad k = 59$

□

Theorem 2.4. *The graph $c_4 \cup C_{4t-3}, t > 1$ is odd vertex magic graph.*

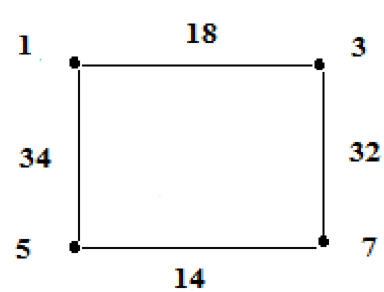
Proof. We label the edges of C_4 consecutively as $[1, 4t+2, 3, 8t, 7, 4t-2, 5, 8t+2]$ and then edge label C_{4t-3} as follows

$$f(e_i) = \begin{cases} i+3 & \text{if} \quad i \equiv 1mod4 \\ i+4t+4 & \text{if} \quad i \equiv 2mod4 \\ i-1 & \text{if} \quad i \equiv 3mod4 \\ i+4t & \text{if} \quad i \equiv 0mod4 \end{cases}$$

The vertex label C_{4t-3} as follows

$$f(v_i) = \begin{cases} 8t-2i+3 & \text{if} \quad i \equiv 1,3mod4 \\ 8t-2i-1 & \text{if} \quad i \equiv 2mod4 \\ 8t-2i+7 & \text{if} \quad i \equiv 0mod4 \end{cases}$$

It is easily verified that f is an odd vertex magic total labeling.
$\therefore$ The graph $C_4 \cup C_{4t-3}, t > 1$ is an odd vertex magic graph.



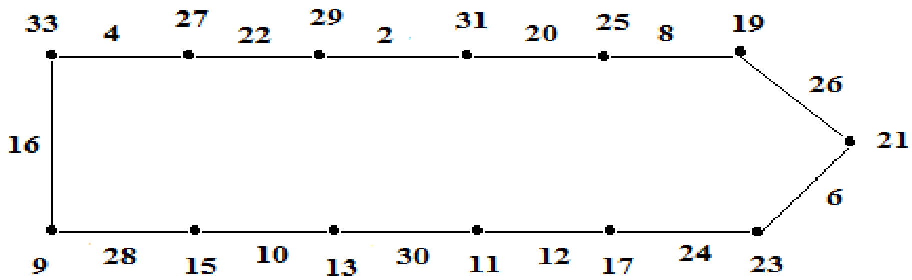


FIGURE 4. $C_4 \cup C_{13} \quad k = 53$

REFERENCES

[1] J.A. Gallian, *A dynamic survey of graph labeling electronic*, J.Combinatorics 5(2002) # D56.

[2] J.A. MacDougall, M.Miller, Slamin, W.D.Wallis, *Vertex magic total labeling of graphs*, Util.Math 61 (2002) 3-21.

[3] J.A.MacDougall, M.Miller,K.A.Sugeng,*Super vertex magic total labelling of graphs*, in: Proc.of the 15^th Australian Workshop on Combinatorial Algorithms,2004,pp.222-229.

[4] CT.Nagaraj, C.Y.Ponnappan, G.Prabakaran, *Odd vertex magic total labeling of some graphs*, Communicated

[5] CT.Nagaraj, C.Y.Ponnappan, G.Prabakaran, *Odd vertex magic total labeling of Trees, International Journal of Mathematics Trends and Technology*, Volume 52,No.6,December(2017).

[6] W.D.Wallis, *magic Graphs*, Birkhauser, Barel, 2001.