EVEN VERTEX MAGIC TOTAL LABELING OF SOME 2-REGULAR GRAPHS

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ABSTRACT. A vertex magic total labeling of a graph $G(V, E)$ is defined as one - to - one map f from $V \cup E$ onto the integers $\{1, 2, 3, ..., |V| + |E|\}$ such that $f(u) + \sum f(uv)$ where the sum is over all vertices v adjacent to u, is a constant, independent of the choice of vertex u. Such a labeling is even if $f(V(G)) = \{2, 4, 6, ..., 2n\}$. In this paper, we present an even vertex magic total labeling of some 2 - regular graphs.

Key words: Even vertex magic total labeling, Even vertex magic graph.

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1. INTRODUCTION

All graphs considered in this paper are simple, finite and undirected. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$ and we let $n = |V|$ and $m = |E|$. The degree of a vertex v is the number of edges that have v as an end point and the set of neighbours of v is denoted by $N(v)$.

A labeling of a graph G is a mapping that carries a set of graph elements usually the vertices and / or edges, into a set of numbers, usually integers, called labels. Many kinds of labelings have been studied and an excellent survey of graph labelings can be found in [1]

Macdougall et al. [2] introduced the concept of vertex magic total labeling. If G is a finite simple undirected graph with n vertices and e edges, then a vertex magic total labeling ($VMTL$) is a bijection f from $V(G) \cup E(G)$ to the integers $1, 2, ..., m + n$ with the property that for every u in $V(G)$, $f(u) + \sum_{v \in N(u)} f(uv) = k$, for some constant k (which we call the magic constant for f).

MacDougall et al. [4] introduced the concept of super vertex magic total labeling. They call a vertex magic total labeling is super if $f(V(G)) = \{1, 2, 3, ..., n\}$. In this labeling, the smallest labels are assigned to the vertices.

Nagaraj et al. [3] introduced the concept of an Even vertex magic total labeling. They call vertex magic total labeling is even if $f(V(G)) = \{2, 4, 6, ..., 2n\}$. A graph is called an even vertex magic if the graph has an even vertex magic total labeling.

2. Main Results

In this section, we will use the notation $[x_1, x_2, x_3, ..., x_k]$ to denote labels of a cycle of length k in the sequence vertex -edge - vertex - edge and so on. Thus, x_1 is the label of a vertex with weight $x_1 + x_2 + x_{2k}$.

Theorem 2.1. *The graph $C_3 \cup C_{4t}, t > 1$ is even vertex magic graph.*

Proof. we label the vertex and edges of C_3 with $[8t+2, 2t+5, 8t+4, 2t+1, 8t+6, 2t+3]$ and then edge label C_{4t} as follows.

$$f(e_i) = \begin{cases} i+2 & \text{if} \quad i \quad \text{odd}, 1 \le i \le 2t-3 \\ i+8 & \text{if} \quad i \quad \text{odd}, 2t-1 \le i \le 4t-3 \\ 1 & \text{if} \quad i = 4t-1 \\ 4t+5+i & \text{if} \quad i \quad \text{even} \end{cases}$$

The vertex label C_{4t} as follows

$$f(v_i) = \begin{cases} 4t+2 & i=1 \\ 8t+4-2i & i=2,3,...,2t-2 \\ 8t-2-2i & 2t-1 \leq i \leq 4t-2 \\ 4t+4 & i=4t \\ 4t+6 & i=4t-1. \end{cases}$$

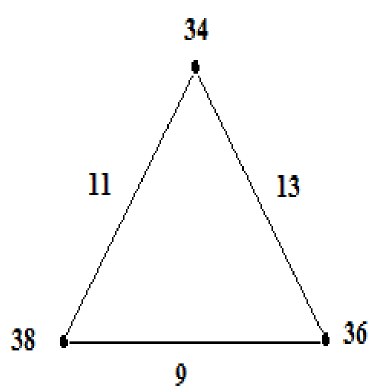
We show firstly that each edge label appears only once.

Edge Number	Range of labels
$4t-1$	1
i odd, $1 \leq i \leq 2t-3$	$3,5,7,...,2t-1$
C_3 edges	$2t+1, 2t+3, 2t+5$
i odd, $2t-1 \leq i \leq 4t-3$	$2t+7, 2t+9, ..., 4t+5$
i even	$4t+7, 4t+9, ..., 8t+5.$

Next we show that each vertex label appear only once

Vertex Number	Range of labels
1	$4t+2$
$2t-1 \leq i \leq 4t-2$	$2,4,6,...,4t-2,4t$
$4t$	$4t+4$
$4t-1$	$4t+6$
$2 \leq i \leq 2t-2$	$4t+8, 4t+10, ..., 8t-2, 8t.$
C_3 vertex	$8t+2, 8t+4, 8t+6$

$\therefore C_3 \cup C_{4t}, t > 1$ is an even vertex magic graph with magic constant $k = 12t + 10$.



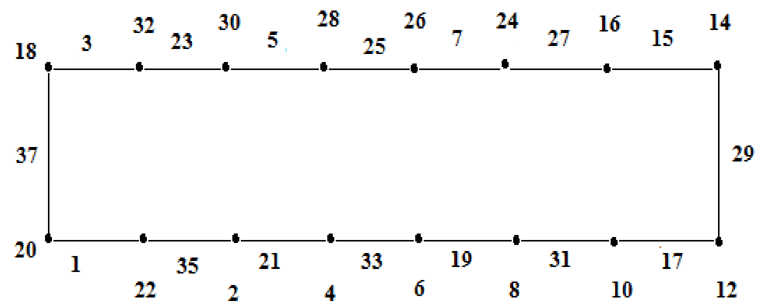


FIGURE 1. $C_3 \cup C_{16}, Magic \quad constant \quad k = 58$

Theorem 2.2. *The graph $C_3 \cup C_{4t+2}, t \geq 1$ is an even vertex magic graph.*

Proof. For $t = 1$ and $t = 2$ the labelings are
$[16, 3, 18, 7, 12, 9]$ $[10, 1, 14, 13, 4, 11, 2, 15, 8, 5, 6, 17]$ and
$[24, 5, 26, 9, 20, 11]$ $[14, 1, 22, 17, 8, 15, 6, 19, 18, 3, 16, 21, 12, 7, 10, 23, 4, 13, 2, 25]$
For $t \geq 3$, we label the vertex and edge of C_3 with $[8t+8, 2t+1, 8t+10, 2t+5, 8t+4, 2t+7]$ and then vertex label C_{4t+2} as follows.

$$f(v_i) = \begin{cases} 4t+6 & \text{if} \quad i = 1 \\ 8t+6 & \text{if} \quad i = 2 \\ 4t & \text{if} \quad i = 3 \\ 4t-2 & \text{if} \quad i = 4 \\ 8t+12-2i & \text{if} \quad 5 \leq i \leq 2t+2 \\ 4t+4 & \text{if} \quad i = 2t+3 \\ 4t+2 & \text{if} \quad i = 2t+4 \\ 8t+6-2i & \text{if} \quad 2t+5 \leq i \leq 4t+2. \end{cases}$$

The edge label C_{4t+2} as follows

$$f(e_i) = \begin{cases} 1 & \text{if} \quad i = 1 \\ 4t+7 & \text{if} \quad i = 3 \\ i-2 & \text{if} \quad i \quad \text{odd}, 5 \leq i \leq 2t+1 \\ 2t+3 & \text{if} \quad i = 2t+3 \\ i+4 & \text{if} \quad i \quad \text{odd}, 2t+5 \leq i \leq 4t+1 \\ i+4t+7 & \text{if} \quad i \quad \text{even}. \end{cases}$$

$\therefore C_3 \cup C_{4t+2}, t \geq 1$ is an even vertex magic graph magic constant $k = 12t + 16$.

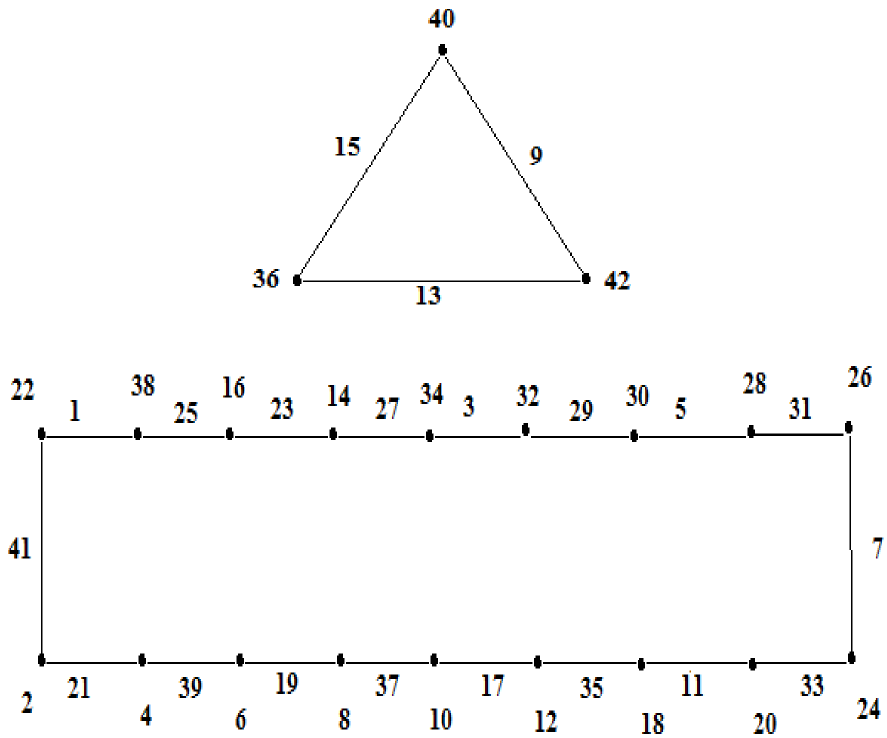


FIGURE 2. $C_3 \cup C_{18} \quad k = 64$

$\square$

Theorem 2.3. *The graph $C_4 \cup C_{4t-1}$ for $t > 1$ is an even vertex magic graph.*

Proof. We label the vertex and edges of C_4 consecutively as $[2, 4t+3, 4, 8t+3, 8, 4t-1, 6, 8t+5]$

We label the edges of C_{4t-1} consecutively as follows

$$f(e_i) = \begin{cases} i+2 & \text{if} \quad i \equiv 1 mod 4, i < 4t-3 \\ 4t+5 & \text{if} \quad i = 2 \\ i+4t+1 & \text{if} \quad i \equiv 2 mod 4, i > 2 \\ i-2 & \text{if} \quad i \equiv 3 mod 4, i < 4t-1 \\ i+4t+5 & \text{if} \quad i \equiv 0 mod 4 \\ 4t-3 & \text{if} \quad i = 4t-3 \\ 4t-1 & \text{if} \quad i = 4t-1` \end{cases}$$

We label the vertex of C_{4t-1} consecutively as follows

$$f(v_i) = \begin{cases} 8t+6 & \text{if} \quad i=1 \\ 8t-6 & \text{if} \quad i=5, t \neq 2 \\ 12 & \text{if} \quad t=2, i=5 \\ 8t+2 & \text{if} \quad i=2 \\ 8t-2i+8 & \text{if} \quad i \equiv 6 mod 4 \\ 14 & \text{if} \quad i=4t-2 \\ 10 & \text{if} \quad i=4t-1 \\ 8t+4 & \text{if} \quad i=3 \\ 8t-2i+12 & \text{if} \quad 7 \leq i \leq 4t-5, i \equiv 3 mod 4 \\ 8t-2i+8 & \text{if} \quad i \equiv 0 mod 4 \\ 8t-2i+4 & \text{if} \quad i \equiv 5 mod 4, i \leq 4t-7 \\ 12 & \text{if} \quad i=4t-3 \end{cases}$$

$\therefore$ The graph $C_4 \cup C_{4t-1}, t > 1$ is an even vertex magic graph with magic constant $k = 12t + 10$.

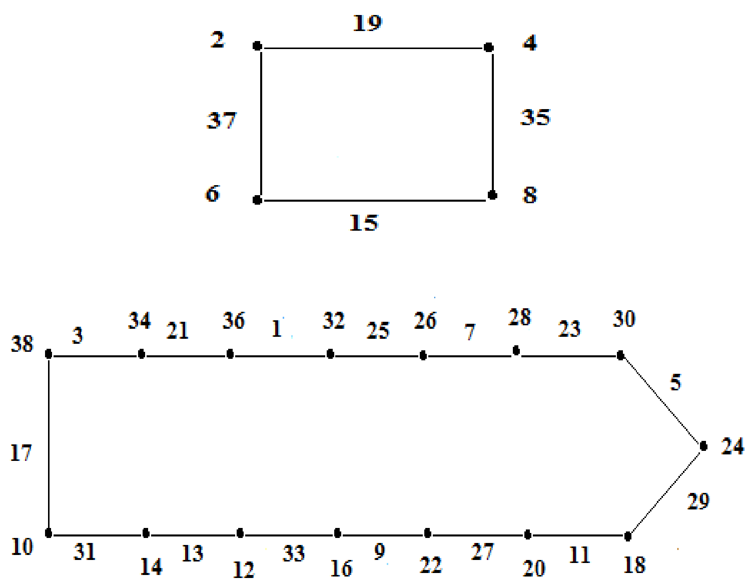


FIGURE 3. $C_4 \cup C_{15} \quad k = 58$

Theorem 2.4. *The graph $c_4 \cup C_{4t-3}, t > 1$ is an even vertex magic graph.*

Proof. We label the edges of C_4 consecutively as $[2, 4t+1, 8t-1, 8, 4t-3, 6, 8t+1]$ and then edge label C_{4t-3} as follows

$$f(e_i) = \begin{cases} i+2 & \text{if} \quad i \equiv 1 mod 4 \\ i+4t+3 & \text{if} \quad i \equiv 2 mod 4 \\ i-2 & \text{if} \quad i \equiv 3 mod 4 \\ i+4t-1 & \text{if} \quad i \equiv 0 mod 4 \end{cases}$$

The vertex label C_{4t-3} as follows

$$f(v_i) = \begin{cases} 8t-2i+4 & \text{if} \quad i \equiv 1, 3 mod 4 \\ 8t-2i & \text{if} \quad i \equiv 2 mod 4 \\ 8t-2i+8 & \text{if} \quad i \equiv 0 mod 4 \end{cases}$$

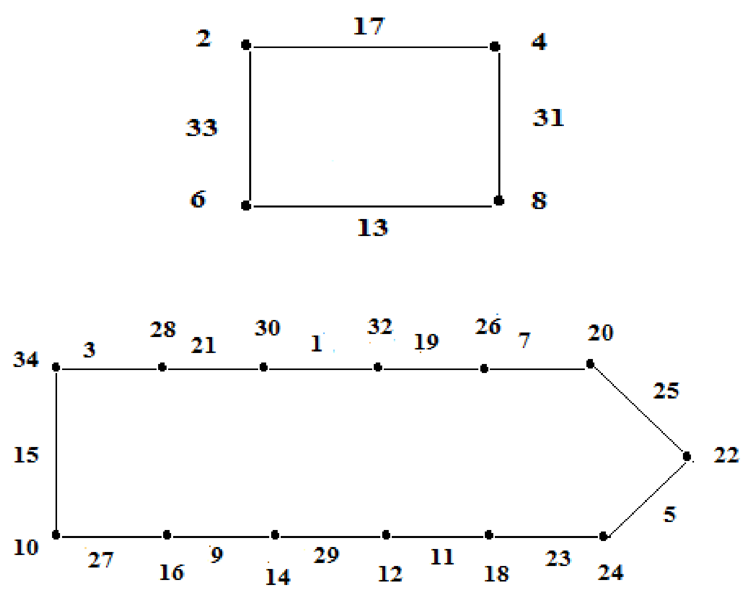


FIGURE 4. $C_4 \cup C_{13} \quad k = 52$

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