Multiple integral of the sequence of functions, a general class of polynomials, the multivariable Aleph-function and the multivariable I-function I

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ABSTRACT
In the present paper we evaluate a multiple integrals involving the product of a sequence of functions, the multivariable Aleph-function, the multivariable I-function defined by Prasad and general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.

Keywords:Multivariable Aleph-function, general class of polynomial, sequence of functions, multivariable I-function, multivariable H-function

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1.Introduction and preliminaries.

The function Aleph of several variables is an extension of the multivariable I-function recently studied by C.K. Sharma and Ahmad [7] , itself is a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integrals occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,N:M_1,N_1,\cdots,M_r,N_r}_{P_i,Q_i,\tau_i;R:P_{i(1)},Q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;P_{i(r)},Q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c} \mathrm{y}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_r \end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,N}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{N+1,P_i}]:$
$\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{1,Q_i}]:$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,N_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,N_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{N_r+1,P_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,M_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,M_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{M_r+1,Q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^N\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{Q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{M_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{N_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$b_j, j = 1, \cdots, Q; a_j, j = 1, \cdots, P;$

$c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, P_{i^{(k)}}; c_j^{(k)}, j = 1, \cdots, N_k;$

$d_{ji^{(k)}}^{(k)}, j = M_k + 1, \cdots, Q_{i^{(k)}}; d_j^{(k)}, j = 1, \cdots, M_k;$

with $k = 1 \cdots, r, i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \tau_i \sum_{j=N+1}^{P_i} \alpha_{ji}^{(k)} - \tau_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} + \tau_i \sum_{j=1}^{Q_i} \beta_{ji}^{(k)} - \tau_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to N and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{N} \alpha_j^{(k)} - \tau_i \sum_{j=N+1}^{P_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{Q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{N_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{M_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R \text{ , } i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\alpha_1}, \cdots, |y_r|^{\alpha_r}), max(|y_1|, \cdots, |y_r|) \rightarrow 0$

$\aleph(y_1, \cdots, y_r) = 0(|y_1|^{\beta_1}, \cdots, |y_r|^{\beta_r}), min(|y_1|, \cdots, |y_r|) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, M_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, N_k$$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1, \cdots, y_r) = \sum_{G_1, \cdots, G_r = 0}^{\infty} \sum_{g_1=0}^{M_1} \cdots \sum_{g_r=0}^{M_r} \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} \psi(\eta_{G_1, g_1}, \cdots, \eta_{G_r, g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})y_1^{-\eta_{G_1,g_1}}\cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(.,\cdots,.), \theta_i(.), i=1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1}=\frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_r,g_r}=\frac{d_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i]\neq\delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)

for $j\neq M_i, M_i=1,\cdots\eta_{G_i,g_i}; P_i,N_i=0,1,2,\cdots,; y_i\neq 0, i=1,\cdots,r$ (1.8)

In the document , we will note :

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})=\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}) \tag{1.9}$$

where $\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),\theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are given respectively in (1.2) and (1.3)

We shall note the Aleph-function of r variables $\aleph_{u:w}^{0,N:v}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_r\end{array}\right)$ (1.10)

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_s)=I_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p',q';\cdots;p^{(s)},q^{(s)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(s)},n^{(s)}}\left(\begin{array}{c|c}\mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;\\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;\end{array}\right.$$

$$\left.\begin{array}{c}(\mathrm{a}_{sj};\alpha'_{sj},\cdots,\alpha_{sj}^{(s)})_{1,p_s}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(s)},\alpha_j^{(s)})_{1,p^{(s)}}\\ \\ (\mathrm{b}_{sj};\beta'_{sj},\cdots,\beta_{sj}^{(s)})_{1,q_s}:(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(s)},\beta_j^{(s)})_{1,q^{(s)}}\end{array}\right) \tag{1.11}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\xi(t_1,\cdots,t_s)\prod_{i=1}^{s}\phi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)} = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.13}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_s) = 0(|z_1|^{\alpha'_1}, \cdots, |z_s|^{\alpha'_s}), max(|z_1|, \cdots, |z_s|) \rightarrow 0$$

$$\aleph(z_1, \cdots, z_s) = 0(|z_1|^{\beta'_1}, \cdots, |z_s|^{\beta'_s}), min(|z_1|, \cdots, |z_s|) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We shall use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.14}$$

$$W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \tag{1.15}$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k})_{1,p_2}; \cdots; (a_{(s-1)k}), \alpha'_{(s-1)k}, \alpha''_{(s-1)k}, \cdots, \alpha_{(s-1)k}^{(s-1)})_{1,p_{s-1}} \tag{1.16}$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k})_{1,q_2}; \cdots; (b_{(s-1)k}), \beta'_{(s-1)k}, \beta''_{(s-1)k}, \cdots, \beta_{(s-1)k}^{(s-1)})_{1,q_{s-1}} \tag{1.17}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{sk}, \alpha''_{sk}, \cdots, \alpha_{sk}^{(s)})_{p_s,q_s} : \mathfrak{B} = (b_{sk}; \beta'_{sk}, \beta''_{sk}, \cdots, \beta_{sk}^{(s)})_{1,q_s} \tag{1.18}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b'_k, \beta'_k)_{1,q'}; \cdots; (b_k^{(s)}, \beta_k^{(s)})_{1,q^{(s)}} \tag{1.19}$$

The multivariable I-function write :

$$I(z_1, \cdots, z_s) = I_{U:p_s,q_s;W}^{V;0,n_s;X} \left(\begin{array}{c|c} z_1 & \text{A ;}\mathfrak{A}\text{;A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & \text{B; }\mathfrak{B}\text{; B'} \end{array} \right) \tag{1.20}$$

The generalized polynomials defined by Srivastava [8], is given in the following manner :

$$S_{N_1',\cdots,N_t'}^{M_1',\cdots,M_t'}[y_1,\cdots,y_t] = \sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\frac{(-N_1')_{M_1'K_1}}{K_1!}\cdots\frac{(-N_t')_{M_t'K_t}}{K_t!}$$

$$A[N_1',K_1;\cdots;N_t',K_t]y_1^{K_1}\cdots y_t^{K_t} \quad (1.21)$$

Where $M_1',\cdots,M_s'$ are arbitrary positive integers and the coefficients $A[N_1',K_1;\cdots;N_t',K_t]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1')_{M_1'K_1}}{K_1!}\cdots\frac{(-N_t')_{M_t'K_t}}{K_t!}A[N_1',K_1;\cdots;N_t',K_t] \quad (1.22)$$

2. Sequence of functions

Agarwal and Chaubey [1], Salim [6] and several others have studied a general sequence of functions. In the present document we shall study the following useful series formula for a general sequence of functions.

$$R_n^{\alpha,\beta}[x;E,F,g,h;p,q;\gamma;\delta;e^{-sx^r}] = \sum_{w,v,u,t,e,k_1,k_2,}\psi(w,v,u,t,e,k_1,k_2)x^R \quad (2.1)$$

where $$\sum_{w,v,u,t,e,k_1,k_2} = \sum_{w=0}^{\infty}\sum_{v=0}^{n}\sum_{u=0}^{v}\sum_{t=0}^{n}\sum_{c=0}^{t}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} \quad (2.2)$$

and the infinite series on the right side (2.1) is absolutely convergent, $R = ln+qv+pt+rw+k_1r+k_2q$

and $$\psi(w,v,u,t,e,k_1,k_2) = \frac{(-)^{t+w+k_2}(-v)_u(-t)_e(\alpha)_t l^n}{w!v!u!t!e!K_nk_1!k_2!}\frac{s^{w+k_1}F^{\gamma n-t}}{(1-\alpha-t)_e}(\alpha-\gamma n)_e$$

$$(-\beta-\delta n)_v g^{v+k_2}h^{\delta n-v-k_2}(v-\delta n)_{k_2}E^t\left(\frac{pe+rw+\lambda+qn}{l}\right)_n \quad (2.3)$$

where K_n is a sequence of constants.

By suitably specializing the parameters involving in (2.1), a general sequence of functions reduced to generalized polynomials set studied by Raizada [5], a class of polynomials introduced by Fujiwara [2] and several others authors.

3.Required formula

We have the following multiple integrals ,see Marichev et al ([3], 3.3.3 Eq.6, page 589)

Lemma

$$\int_{x_1\geqslant 0}\cdots\int_{x_n\geqslant 0}\frac{x_1^{p_1-1}\cdots x_n^{p_n-1}}{(x_1^{\alpha_1}+\cdots+x_n^{\alpha_n})^{\mu}}\mathrm{d}x_1\cdots\mathrm{d}x_n = \frac{\Gamma\left(\frac{p_1}{\alpha_1}\right)\cdots\Gamma\left(\frac{p_n}{\alpha_n}\right)}{\Gamma\left(\frac{p_1}{\alpha_1}+\cdots+\frac{p_n}{\alpha_n}\right)}\times$$

$$\frac{\alpha_1^{-1}\cdots\alpha_n^{-1}}{\mu-\left(\frac{p_1}{\alpha_1}\right)-\cdots-\left(\frac{p_r}{\alpha_r}\right)} \tag{3.1}$$

where $x_1^{\alpha_1}+\cdots+x_n^{\alpha_n}\geqslant 1,\alpha_i>0,p_i\in\mathbb{R},i=1,\cdots,n$ and $Re(\mu)>\frac{p_1}{\alpha_1}+\cdots+\frac{p_n}{\alpha_n}$,

4. Main integral

We note $X_{p_1,\cdots,p_n,\mu}=\frac{x_1^{p_1}\cdots x_n^{p_n}}{(x_1^{\alpha_1}+\cdots+x_n^{\alpha_n})^{\mu}}$

We have the following formula

Theorem

$$\int_{x_1\geqslant 0}\cdots\int_{x_n\geqslant 0}\frac{x_1^{p_1-1}\cdots x_n^{p_n-1}}{(x_1^{\alpha_1}+\cdots+x_n^{\alpha_n})^{\mu}}R_n^{\alpha,\beta}[zX_{a_1,\cdots,a_n,b};E,F,g,h;p,q;\gamma;\delta;e^{-s(zX_{a_1,\cdots,a_n,b})^r}]$$

$$S_{N_1,\cdots,N_t}^{M_1',\cdots,M_t'}\begin{pmatrix}\mathrm{y}_1X_{\gamma_1^1,\cdots\gamma_1^n,\mu_1}\\ \cdots\\ \mathrm{y}_tX_{\gamma_t^1\cdots\gamma_t^n,\mu_t}\end{pmatrix}\aleph_{u:w}^{0,N:v}\begin{pmatrix}\mathrm{z}_1X_{\alpha_1^1,\cdots\alpha_1^n,\beta_1}\\ \cdots\\ \mathrm{z}_rX_{\alpha_r^1\cdots\alpha_r^n,\beta_r}\end{pmatrix}I_{U:p_s,q_s;W}^{V;0,n_s;X}\begin{pmatrix}\mathrm{Z}_1X_{\eta_1^1\cdots\eta_1^n,\epsilon_1}\\ \cdots\\ \mathrm{Z}_sX_{\eta_s^1\cdots\eta_s^n,\epsilon_s}\end{pmatrix}\mathrm{d}x_1\cdots\mathrm{d}x_n$$

$$=\frac{1}{\alpha_1\cdots\alpha_n}\sum_{w,v,u,t,e,k_1,k_2}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}y_1^{K_1}\cdots y_t^{K_t}$$

$$z^R\,G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}\,\psi(w,v,u,t,e,k_1,k_2)I_{U:p_s+n+1,q_s+2;W}^{V;0,n_s+n+1;X}\left(\begin{array}{c|l}\mathrm{Z}_1 & \mathrm{A}\ ;\\ \cdots & \\ \cdots & \\ \mathrm{Z}_s & \mathrm{B};\end{array}\right.$$

$$\left.\begin{array}{c}\left[1-\frac{p_i+Ra_i+\sum_{j=1}^t K_j\gamma_j^i+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^i}{\alpha_j};\frac{\eta_1^i}{\alpha_i},\cdots,\frac{\eta_s^i}{\alpha_i}\right]_{1,n},\mathfrak{A},A_1;A'\\ \cdots\\ \mathrm{B}_1,B_2,\mathfrak{B};B'\end{array}\right) \tag{4.1}$$

$$B_1=\left\{1-\sum_{i=1}^{n}\left[\frac{p_i+Ra_i+\sum_{j=1}^t K_j\gamma_j^i+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^i}{\alpha_i}\right]:\frac{\eta_1^1}{\alpha_1}+\cdots+\frac{\eta_1^n}{\alpha_n}\right.$$

$$\cdots, \frac{\eta_s^1}{\alpha_1}+\cdots+\frac{\eta_s^n}{\alpha_n} \Big\} \quad (4.2)$$

$$A_1 = \Big\{ 1-[\mu+Rb+\sum_{j=1}^{t} K_j\mu_j+\sum_{j=1}^{r}\eta_{G_j,g_j}\alpha_j]+\sum_{i=1}^{n}\left[\frac{p_i+Ra_i+\sum_{j=1}^{t}K_j\gamma_j^i+\sum_{j=1}^{\eta}{}_{G_j,g_j}\alpha_j^i}{\alpha_i}\right]$$

$$;\epsilon_1-\frac{\eta_1^1}{\alpha_1}-\cdots-\frac{\eta_1^n}{\alpha_n},\cdots,\epsilon_s-\frac{\eta_s^1}{\alpha_1}-\cdots-\frac{\eta_s^n}{\alpha_n}\Big\} \quad (4.3)$$

and $B_2 = \Big\{ -[\mu+Rb+\sum_{j=1}^{t} K_j\mu_j+\sum_{j=1}^{r}\eta_{G_j,g_j}\alpha_j]+\sum_{i=1}^{n}\left[\frac{p_i+Ra_i+\sum_{j=1}^{t}K_j\gamma_j^i+\sum_{j=1}^{\eta}{}_{G_j,g_j}\alpha_j^i}{\alpha_i}\right]$

$$;\epsilon_1-\frac{\eta_1^1}{\alpha_1}-\cdots-\frac{\eta_1^n}{\alpha_n},\cdots,\epsilon_s-\frac{\eta_s^1}{\alpha_1}-\cdots-\frac{\eta_s^n}{\alpha_n}\Big\} \quad (4.4)$$

We note : $u' = Re(\mu+Rb)+\sum_{j=1}^{r}\beta_j \min_{1\leqslant l\leqslant M_j} Re\left(\frac{d_l^{(j)}}{\delta_l^{(j)}}\right)+\sum_{j=1}^{s}\epsilon_j \min_{1\leqslant l\leqslant m^{(j)}} Re\left(\frac{b_l^{(j)}}{\beta_l^{(j)}}\right)$ (4.5)

and

$$\mu_i = Re(p_i+Ra_i)+\sum_{j=1}^{r}\alpha_j^i \min_{1\leqslant l\leqslant M_j} Re\left(\frac{d_l^{(j)}}{\delta_l^{(j)}}\right)+\sum_{j=1}^{s}\eta_j^i \min_{1\leqslant j\leqslant m^{(j)}} Re\left(\frac{b_l^{(j)}}{\beta_l^{(j)}}\right) \quad (4.6)$$

Provided that

a) $min\{\xi_i, \upsilon_i, \gamma_j^{(i)}, \alpha_k^{(i)}, \eta_l^{(i)},\} > 0, i = 1,\cdots,n, j = 1,\cdots,t, k = 1,\cdots,r, l = 1,\cdots,s$

b) $min\{\xi, \gamma_j, \alpha_k, \eta_l,\} > 0, j = 1,\cdots,t, k = 1,\cdots,r, l = 1,\cdots,s$

c) $\mu_i > 0, i = 1,\cdots,n$

d) $\mu' > \sum_{i=1}^{n}\mu_i > 0$

e) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) ; $i = 1,\cdots,r$

f) $|argZ_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is defined by (1.11) ; $i = 1,\cdots,s$

g) $x_1^{\alpha_1}+\cdots+x_n^{\alpha_n} \geqslant 1, \alpha_i > 0, p_i \in \mathbb{R}, i = 1,\cdots,n$

h) The series occuring on the right-hand side of (4.1) is absolutely and uniformly convergent.

Proof

First, expressing the generalized the sequence of functions $R_n^{\alpha,\beta}[x;E,F,g,h;p,q;\gamma;\delta;e^{-sx^r}]$ in multiple series with the help of equation (2.1), the Aleph-function of r variables in series with the help of equation (1.6), the general class of polynomial of several variables $S_{N_1',\cdots,N_t'}^{M_1',\cdots,M_t'}[y_1,\cdots,y_t]$ with the help of equation (1.21) and the I-function of s variables in defined by Prasad [4] in Mellin-Barnes contour integrals with the help of equation (1.12), changing the order of integrations and summations (which is easily seen to be justified due to the absolute convergence of the integrals and the summations involved in the process) and then evaluating the resulting multiple $(x_1,\cdots,x_n)$-integrals with the help of equation (3.1). Use the relation $\frac{1}{\mu-\frac{p_1}{\alpha_1}-\cdots-\frac{p_n}{\alpha_n}}=\frac{\Gamma\left(\mu-\frac{p_1}{\alpha_1}-\cdots-\frac{p_n}{\alpha_n}\right)}{\Gamma\left(1+\left(\mu-\frac{p_1}{\alpha_1}-\cdots-\frac{p_n}{\alpha_n}\right)\right)}$ and interpreting the result thus obtained with the Mellin-barnes contour integrals, we arrive at the desired result.

5. Particular case

If $U=V=A=B=0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava Panda [9]. We have the following result.

Corollary.

$$\int_{x_1\geqslant 0}\cdots\int_{x_n\geqslant 0}\frac{x_1^{p_1-1}\cdots x_n^{p_n-1}}{(x_1^{\alpha_1}+\cdots+x_n^{\alpha_n})^{\mu}}R_n^{\alpha,\beta}[zX_{a_1,\cdots,a_n,b};E,F,g,h;p,q;\gamma;\delta;e^{-s(zX_{a_1,\cdots,a_n,b})^r}]$$

$$S_{N_1,\cdots,N_t}^{M_1',\cdots,M_t'}\begin{pmatrix}\mathrm{y}_1X_{\gamma_1^1,\cdots\gamma_1^n,\mu_1}\\ \cdot\cdot\cdot\\ \mathrm{y}_tX_{\gamma_t^1\cdots\gamma_t^n,\mu_t}\end{pmatrix}\aleph_{u:w}^{0,N:v}\begin{pmatrix}\mathrm{z}_1X_{\alpha_1^1,\cdots\alpha_1^n,\beta_1}\\ \cdot\cdot\cdot\\ \mathrm{z}_rX_{\alpha_r^1\cdots\alpha_r^n,\beta_r}\end{pmatrix}H_{p_s,q_s;W}^{0,n_s;X}\begin{pmatrix}\mathrm{Z}_1X_{\eta_1^1\cdots\eta_1^n,\epsilon_1}\\ \cdot\cdot\cdot\\ \mathrm{Z}_sX_{\eta_s^1\cdots\eta_s^n,\epsilon_s}\end{pmatrix}\mathrm{d}x_1\cdots\mathrm{d}x_n$$

$$=\frac{1}{\alpha_1\cdots\alpha_n}\sum_{w,v,u,t,e,k_1,k_2}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]}a_1\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}y_1^{K_1}\cdots y_t^{K_t}$$

$$z^R\,G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}\psi(w,v,u,t,e,k_1,k_2)H_{p_s+n+1,q_s+2;W}^{0,n_s+n+1;X}\left(\begin{matrix}\mathrm{Z}_1\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \mathrm{Z}_s\end{matrix}\ \middle|\ \begin{matrix}\left[1-\frac{p_i+Ra_i+\sum_{j=1}^{t}K_j\gamma_j^i+\sum_{j=1}^{r}\eta_{G_j,g_j}\alpha_j^i}{\alpha_j};\frac{\eta_1^i}{\alpha_i},\cdots,\frac{\eta_s^i}{\alpha_i}\right]_{1,n},\mathfrak{A},A_1;A'\\ \cdot\cdot\cdot\\ \mathrm{B}_1,B_2,\mathfrak{B};B'\end{matrix}\right)\tag{5.1}$$

under the same notations and conditions that (4.1) with $U=V=A=B=0$ .A_1,B_1 and B_2 is defined by (4.2)

6. Conclusion

In this paper we have evaluated a generalized multiple integral involving the multivariable Aleph-function, a class of polynomials of several variables a sequence of functions and the multivariable I-function defined by Prasad. The integral established in this paper is of very general nature as it contains Multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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