Multiple integrals Transformation involving a generalized multiple-index Mittag-Leffler function, class of polynomials, multivariable Aleph-function and multivariable I-function

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ABSTRACT
In the present paper we evaluate a generalized multiple integrals transformation involving the product of a generalized multiple-index Mittag-Leffler function, multivariable Aleph-function, the multivariable I-function defined by Prasad [4] and general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.

Keywords:Multivariable Aleph-function, general class of polynomials, multiple integral, generalized multiple-index Mittag-Leffler function, multivariable I-function, multivariable H-function

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1.Introduction and preliminaries.

The function Aleph of several variables generalizes the multivariable I-function recently studied by C.K. Sharma and Ahmad [5] , itself is a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integrals occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $$\aleph(z_1,\cdots,z_r)=\aleph^{0,N:M_1,N_1,\cdots,M_r,N_r}_{P_i,Q_i,\tau_i;R:P_{i^{(1)}},Q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;P_{i^{(r)}},Q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,N}],[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{N+1,P_i}]: \\ \cdot & \\ \cdot & \cdots\cdots\cdots\cdots ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{1,Q_i}]: \\ \cdot & \\ \mathrm{z}_r & \end{array}\right.$$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,N_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,N_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{N_r+1,P_i^{(r)}}] \\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,M_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,M_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{M_r+1,Q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{N}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{Q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k) = \frac{\prod_{j=1}^{M_k}\Gamma(d_j^{(k)} - \delta_j^{(k)}s_k)\prod_{j=1}^{N_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$b_j, j = 1,\cdots,Q; a_j, j = 1,\cdots,P;$

$c_{ji^{(k)}}^{(k)}, j = n_k+1,\cdots,P_{i^{(k)}}; c_j^{(k)}, j = 1,\cdots,N_k;$

$d_{ji^{(k)}}^{(k)}, j = M_k+1,\cdots,Q_{i^{(k)}}; d_j^{(k)}, j = 1,\cdots,M_k;$

with $k = 1\cdots,r, i = 1,\cdots,R\ , i^{(k)} = 1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \tau_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(k)} - \tau_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} + \tau_i\sum_{j=1}^{Q_i}\beta_{ji}^{(k)} - \tau_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)}s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j = 1$ to N and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)}s_k)$ with $j = 1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{N}\alpha_j^{(k)} - \tau_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{Q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{N_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1\cdots,r, i = 1,\cdots,R\ , i^{(k)} = 1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1,\cdots,z_r) = 0(\ |z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\ ), max(\ |z_1|,\cdots,|z_r|\ ) \rightarrow 0$

$\aleph(z_1,\cdots,z_r) = 0(\ |z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\ ), min(\ |z_1|,\cdots,|z_r|\ ) \rightarrow \infty$ and

$\alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1,\cdots,M_k$

$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1,\cdots,N_k$

Series representation of Aleph-function of several variables is given by

$$\aleph(z_1,\cdots,z_r) = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\psi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})z_1^{-\eta_{G_1,g_1}}\cdots z_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(.,\cdots,.), \theta_i(.), i=1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1} = \frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_r,g_r} = \frac{d_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i] \neq \delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)

for $j \neq M_i, M_i = 1,\cdots\eta_{G_i,g_i}; P_i, N_i = 0,1,2,\cdots,; y_i \neq 0, i=1,\cdots,r$ (1.8)

In the document , we will note :

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}) \tag{1.9}$$

where $\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}), \theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are given respectively in (1.2) and (1.3)

We shall note the Aleph-function of r variables $\aleph_{u:w}^{0,N:v}\left(\begin{array}{c} z_1 \\ \cdot\cdot\cdot \\ z_r \end{array}\right)$ (1.10)

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_s) = I_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p',q';\cdots;p^{(s)},q^{(s)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(s)},n^{(s)}}\left(\begin{array}{c|c} z_1 & (a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & (b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (a_{sj};\alpha'_{sj},\cdots,\alpha_{sj}^{(s)})_{1,p_s} : (a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(s)},\alpha_j^{(s)})_{1,p^{(s)}} \\ \\ (b_{sj};\beta'_{sj},\cdots,\beta_{sj}^{(s)})_{1,q_s} : (b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(s)},\beta_j^{(s)})_{1,q^{(s)}} \end{array}\right) \tag{1.11}$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\xi(t_1,\cdots,t_s)\prod_{i=1}^{s}\phi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of

the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)} = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)}\right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)}\right) \tag{1.13}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_s) = 0(\,|z_1|^{\alpha_1'}, \cdots, |z_s|^{\alpha_s'}\,), max(\,|z_1|, \cdots, |z_s|\,) \rightarrow 0$$

$$I(z_1, \cdots, z_s) \;= 0(\,|z_1|^{\beta_1'}, \cdots, |z_s|^{\beta_s'}\,), min(\,|z_1|, \cdots, |z_s|\,) \rightarrow \infty$$

where, with $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.14}$$

$$W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \tag{1.15}$$

$$A = (a_{2k}, \alpha_{2k}', \alpha_{2k}'')_{1,p_2}; \cdots; (a_{(s-1)k}), \alpha'_{(s-1)k}, \alpha''_{(s-1)k}, \cdots, \alpha_{(s-1)k}^{(s-1)})_{1,p_{s-1}} \tag{1.16}$$

$$B = (b_{2k}, \beta_{2k}', \beta_{2k}'')_{1,q_2}; \cdots; (b_{(s-1)k}), \beta'_{(s-1)k}, \beta''_{(s-1)k}, \cdots, \beta_{(s-1)k}^{(s-1)})_{1,q_{s-1}} \tag{1.17}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{sk}, \alpha''_{sk}, \cdots, \alpha_{sk}^{(s)})_{p_s,q_s} : \mathfrak{B} = (b_{sk}; \beta'_{sk}, \beta''_{sk}, \cdots, \beta_{sk}^{(s)})_{1,q_s} \tag{1.18}$$

$$A' = (a_k', \alpha_k')_{1,p'}; \cdots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b_k', \beta_k')_{1,q'}; \cdots; (b_k^{(s)}, \beta_k^{(s)})_{1,q^{(s)}} \tag{1.19}$$

The multivariable I-function writes :

$$I(z_1, \cdots, z_s) = I_{U:p_s,q_s;W}^{V;0,n_s;X}\left(\begin{array}{c|c} z_1 & A\ ;\mathfrak{A};A' \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & B;\ \mathfrak{B};\ B' \end{array}\right) \tag{1.20}$$

The generalized polynomials defined by Srivastava [6], is given in the following manner :

$$S_{N_1',\cdots,N_t'}^{M_1',\cdots,M_t'}[y_1,\cdots,y_t] = \sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\frac{(-N_1')_{M_1'K_1}}{K_1!}\cdots\frac{(-N_t')_{M_t'K_t}}{K_t!}$$

$$A[N_1',K_1;\cdots;N_t',K_t]y_1^{K_1}\cdots y_t^{K_t} \tag{1.21}$$

Where $M_1',\cdots,M_s'$ are arbitrary positive integers and the coefficients $A[N_1',K_1;\cdots;N_t',K_t]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1')_{M_1'K_1}}{K_1!}\cdots\frac{(-N_t')_{M_t'K_t}}{K_t!}A[N_1',K_1;\cdots;N_t',K_t] \tag{1.22}$$

2. Generalized multiple-index Mittag-Leffler function

A further generalization of the Mittag-Leffler functions is proposed recently in Paneva-Konovska [2]. These are 3m-parametric Mittag-Leffler type functions generalizing the Prabhakar [3] 3-parametric function , defined as:

$$E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m}(z) = \sum_{k=0}^{\infty}\frac{(\gamma_1)_k\cdots(\gamma_m)_k}{\Gamma(\alpha_1 k+\beta_1)\cdots\Gamma(\alpha_m k+\beta_m)}\frac{z^k}{k!} \tag{2.1}$$

where $\alpha_i,\beta_i,\gamma_i\in\mathbb{C}, i=1,\cdots,m, Re(\alpha_i)>0$

3.Required formula

We have the following multiple integralss transformation, see Marichev et al ([1], 33.5 11 page 595).

$$\int_0^1\cdots\int_0^1 f(x_1\cdots x_n)\,(1-x_1)^{\upsilon_1-1}\prod_{i=2}^{n}x_i^{\upsilon_1+\cdots+\upsilon_i-1}(1-x_i)^{\upsilon_i-1}\mathrm{d}x_1\cdots\mathrm{d}x_n$$

$$=\frac{\Gamma(\upsilon_1)\cdots\Gamma(\upsilon_n)}{\Gamma(\upsilon_1+\cdots+\upsilon_n)}\times\int_0^1 f(x)(1-x)^{\upsilon_1+\cdots+\upsilon_n-1}\,\mathrm{d}x \tag{3.1}$$

where $\upsilon_i>0, i=1,\cdots,n$, provided that the integral of the right hand side converges absolutely.

4. Main integral

We note : $X_{\upsilon_1,\cdots,\upsilon_n}=(1-x_1)^{\upsilon_1}\prod_{l=2}^{n}x_l^{\upsilon_1+\cdots+\upsilon_l}(1-x_l)^{\upsilon_l}$ and $b_k=\frac{(\gamma_1)_k\cdots(\gamma_m)_k}{\Gamma(\alpha_1 k+\beta_1)\cdots\Gamma(\alpha_m k+\beta_m)}$

and we have the following formula :

$$\int_0^1\cdots\int_0^1 f(x_1\cdots x_n)(1-x_1)^{\upsilon_1}\prod_{l=2}^{n}x_i^{\upsilon_1+\cdots+\upsilon_l}(1-x_l)^{\upsilon_l-1}E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m}(zX_{\xi_1,\cdots,\xi_n})$$

$$S_{N_1,\cdots,N_t}^{M_1',\cdots,M_t'}\left(\begin{array}{c}\mathrm{y}_1X_{\gamma_1^{(1)},\cdots\gamma_1^{(n)}}\\ \cdots\\ \mathrm{y}_tX_{\gamma_t^{(1)}\cdots\gamma_t^{(n)}}\end{array}\right)\aleph_{u:w}^{0,N:v}\left(\begin{array}{c}\mathrm{z}_1X_{\alpha_1^{(1)},\cdots\alpha_1^{(n)}}\\ \cdots\\ \mathrm{z}_rX_{\alpha_r^{(1)}\cdots\alpha_r^{(n)}}\end{array}\right)I_{U:p_s,q_s;W}^{V;0,n_s;X}\left(\begin{array}{c}\mathrm{Z}_1X_{\eta_1^{(1)}\cdots\eta_1^{(n)}}\\ \cdots\\ \mathrm{Z}_sX_{\eta_s^{(1)}\cdots\eta_s^{(n)}}\end{array}\right)\mathrm{d}x_1\cdots\mathrm{d}x_n$$

$$=\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\sum_{k=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})a_1\frac{b_kz^k}{k!}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}y_1^{K_1}\cdots y_t^{K_t}\int_0^1(1-x)^{\sum_{i=1}^n(k\xi_i+\sum_{j=1}^tK_j\gamma_j^{(i)}+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^{(i)})-1}f(x)$$

$$I_{U:p_s+n,q_s+1;W}^{V;0,n_s+n;X}\left(\begin{array}{c}\mathrm{Z}_1(1-x)^{\eta_1^{(1)}+\cdots+\eta_1^{(n)}}\\ \cdots\\ \cdots\\ \mathrm{Z}_s(1-x)^{\eta_s^{(1)}+\cdots+\eta_s^{(n)}}\end{array}\left|\begin{array}{c}\mathrm{A}\ ;\\ \\ \\ \mathrm{B};\end{array}\right.\right.$$

$$\left.\begin{array}{c}\left[1-(\upsilon_i+k\xi_i+\sum_{j=1}^tK_i\gamma_j^{(i)}+\sum_{j=1}^r\eta_{G_i,g_i}\alpha_j^{(i)});\eta_1^{(i)},\cdots,\eta_s^{(i)}\right]_{1,n},\mathfrak{A};A'\\ \cdots\\ \mathrm{B}_1,\mathfrak{B};B'\end{array}\right)\mathrm{d}x \qquad (4.1)$$

$$\text{where}\ :\ B_1=\left\{1-\sum_{i=1}^n\left[\upsilon_i+k\xi_i+\sum_{j=1}^tK_j\gamma_j^{(i)}+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^{(i)}\right];\eta_1^{(1)}+\cdots+\eta_1^{(n)},\right.$$

$$\left.\cdots,\eta_s^{(1)}+\cdots+\eta_s^{(n)}\right\} \qquad (4.2)$$

Provided that

a) $min\{\xi_i,\upsilon_i,\gamma_j^{(i)},\alpha_k^{(i)},\eta_l^{(i)},\}>0,i=1,\cdots,n,j=1,\cdots,t,k=1,\cdots,r,l=1,\cdots,s$

b) $Re(\upsilon_i+k\xi_i)+\sum_{j=1}^r\alpha_j^{(i)}\min_{1\leqslant l\leqslant M_j}Re\left(\frac{d_l^{(j)}}{\delta_l^{(j)}}\right)+\sum_{j=1}^s\eta_j^{(i)}\min_{1\leqslant l\leqslant m^{(j)}}Re\left(\frac{b_l^{(j)}}{\beta_l^{(j)}}\right)>0$, $i=1,\cdots,n$

c) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.5) ; $i=1,\cdots,r$

d) $|argZ_k|<\frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is defined by (1.11) ; $i=1,\cdots,s$

e) $\alpha_i,\beta_i,\gamma_i\in\mathbb{C},i=1,\cdots,m,Re(\alpha_i)>0$

f)The integral of the right hand side converges absolutely.

g) The series occurring on the right-hand side of (4.1) is absolutely and uniformly convergent.

The quantities $U,V,W,X,A,B,\mathfrak{A},\mathfrak{B},A'$ and B' are defined by the equations (1.14) to (1;19) respectively.

Proof of (4.1) : Let $M\{\} = \frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\xi(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)\{\}$. We have :

$$E^{(\gamma_i),m}_{(\alpha_i),(\beta_i)}\left(zX_{\xi_1,\cdots,\xi_n}\right) S^{M_1',\cdots,M_t'}_{N_1,\cdots,N_t}\begin{pmatrix} \mathrm{y}_1X_{\gamma_1^{(1)},\cdots\gamma_1^{(n)}} \\ \cdots \\ \mathrm{y}_tX_{\gamma_t^{(1)}\cdots\gamma_t^{(n)}}\end{pmatrix}\aleph^{0,N:v}_{u:w}\begin{pmatrix} \mathrm{z}_1X_{\alpha_1^{(1)},\cdots\alpha_1^{(n)}} \\ \cdots \\ \mathrm{z}_rX_{\alpha_r^{(1)}\cdots\alpha_r^{(n)}}\end{pmatrix}$$

$$I^{V;0,n_s;X}_{U:p_s,q_s;W}\begin{pmatrix} \mathrm{Z}_1X_{\eta_1^{(1)}\cdots\eta_1^{(n)}} \\ \cdots \\ \mathrm{Z}_sX_{\eta_s^{(1)}\cdots\eta_s^{(n)}}\end{pmatrix} = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\sum_{k=0}^{\infty}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\, a_1\frac{b_kz^k}{k!}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}y_1^{K_1}\cdots y_t^{K_t}\prod_{j=1}^{t}X^{K_j}_{\gamma_j^{(1)},\cdots,\gamma_j^{(n)}}X^k_{\xi_{(1)},\cdots,\xi_{(n)}}\prod_{j=1}^{r}X^{\eta_{G_j,g_j}}_{\alpha_j^{(1)},\cdots,\alpha_j^{(n)}}$$

$$M\left[\prod_{j=1}^{s}Z_j^{t_j}X^{t_j}_{\eta_j^{(1)},\cdots,\eta_j^{(n)}}\right]\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{4.3}$$

Multiplying both sides of (4.3) by $f(x_1\cdots x_n)\;(1-x_1)^{\upsilon_1}\prod_{l=2}^{n}x_l^{\upsilon_1+\cdots+\upsilon_l}(1-x_l)^{\upsilon_l-1}$and integrating with respect to $x_1,\cdots,x_s$verifying the conditions e), changing the order of integrations ans summations (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we obtain :

$$\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\sum_{k=0}^{\infty}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})a_1\frac{b_kz^k}{k!}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}y_1^{K_1}\cdots y_t^{K_t}\int_0^1\cdots\int_0^1\prod_{j=1}^{t}X^{K_j}_{\gamma_j^{(1)},\cdots,\gamma_j^{(n)}}X^k_{\xi_1,\cdots,\xi_n}\prod_{j=1}^{r}X^{\eta_{G_j,g_j}}_{\alpha_j^{(1)},\cdots,\alpha_j^{(n)}}$$

$$\left\{M\left[\prod_{j=1}^{s}Z_j^{t_j}X^{t_j}_{\eta_j^{(1)},\cdots,\eta_j^{(n)}}\right]\mathrm{d}t_1\cdots\mathrm{d}t_s\right\}f(x_1\cdots x_n)\;(1-x_1)^{\upsilon_1}\prod_{l=2}^{n}x_l^{\upsilon_1+\cdots+\upsilon_l}(1-x_l)^{\upsilon_l-1}$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_n \tag{4.4}$$

Change the order of the $(x_1,\cdots,x_n)$-integrals and $(t_1,\cdots,t_s)$-integrals ,we get :

$$\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\sum_{k=0}^{\infty}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})a_1\frac{b_k z^k}{k!}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}y_1^{K_1}\cdots y_t^{K_t}M\left\{\prod_{j=1}^{s}Z_j^{t_j}\int_0^1\cdots\int_0^1(1-x_1)^{\upsilon_1+\xi_1 k+\sum_{j=1}^t K_j\gamma_j^{(1)}+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^{(1)}+\sum_{j=1}^s t_j\eta_j^{(1)}}\right.$$

$$\prod_{l=2}^{n}(1-x_l)^{\upsilon_l+k\xi_l+\sum_{j=1}^t K_j\gamma_j^{(l)}+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^{(l)}+\sum_{j=1}^s t_j\eta_j^{(l)})-1}\ f(x_1\cdots x_n)$$

$$\left.\prod_{l=2}^{n}x_l^{\sum_{i=1}^l(\xi_i k+\sum_{j=1}^t K_j\gamma_j^{(i)}+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^{(i)}+\sum_{j=1}^s t_j\eta_j^{(i)}+\upsilon_i)}\ \mathrm{d}x_1\cdots\mathrm{d}x_n\right]\mathrm{d}t_1\cdots\mathrm{d}t_s\Big\} \tag{4.5}$$

Use the equation (3.1) and interpreting the result thus obtained in the Mellin-barnes contour integral (1.12), we arrive at the desired result.

5. Particular case

If $U=V=A=B=0$, the multivariable I-function defined by Prasad reduces in multivariable H-function defined by Srivastava and panda [7]. We have the following result.

Corollary.

$$\int_0^1\cdots\int_0^1 f(x_1\cdots x_n)(1-x_1)^{\upsilon_1-1}\prod_{i=2}^{n}x_i^{\upsilon_1+\cdots+\upsilon_k-1}(1-\upsilon_k)^{\upsilon_k-1}E^{(\gamma_i),m}_{(\alpha_i),(\beta_i)}(zX_{\xi_1,\cdots,\xi_n})$$

$$S^{M_1',\cdots,M_t'}_{N_1,\cdots,N_t}\begin{pmatrix}\mathrm{y}_1X_{\gamma_1^{(1)},\cdots\gamma_1^{(n)}}\\ \cdots\\ \mathrm{y}_tX_{\gamma_t^{(1)}\cdots\gamma_t^{(n)}}\end{pmatrix}\aleph^{0,N:v}_{u:w}\begin{pmatrix}\mathrm{z}_1X_{\alpha_1^{(1)},\cdots\alpha_1^{(n)}}\\ \cdots\\ \mathrm{z}_rX_{\alpha_r^{(1)}\cdots\alpha_r^{(n)}}\end{pmatrix}H^{0,n_s;X}_{p_s,q_s;W}\begin{pmatrix}\mathrm{Z}_1X_{\eta_1^{(1)}\cdots\eta_1^{(n)}}\\ \cdots\\ \mathrm{Z}_sX_{\eta_s^{(1)}\cdots\eta_s^{(n)}}\end{pmatrix}\mathrm{d}x_1\cdots\mathrm{d}x_n$$

$$=\sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\sum_{k=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})a_1\frac{b_k z^k}{k!}$$

$$z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}y_1^{K_1}\cdots y_t^{K_t}\int_0^1\ (1-x)^{\sum_{i=1}^n(k\xi_i+\sum_{j=1}^t K_j\gamma_j^{(i)}+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^{(i)})-1}\ f(x)$$

$$H^{0,n_s+n;X}_{p_s+n,q_s+1;W}\left(\begin{array}{c|}\mathrm{Z}_1(1-x)^{\eta_1^{(1)}+\cdots+\eta_1^{(n)}}\\ \cdots\\ \cdots\\ \mathrm{Z}_s(1-x)^{\eta_s^{(1)}+\cdots+\eta_s^{(n)}}\end{array}\right.$$

$$\left[1-(\upsilon_i+k\xi_i+\sum_{j=1}^{t}K_i\gamma_j^{(i)}+\sum_{j=1}^{r}\eta_{G_i,g_i}\alpha_j^{(i)});\eta_1^{(i)},\cdots,\eta_s^{(i)}\right]_{1,n},\mathfrak{A};A' \atop \cdots \atop \mathrm{B}_1,\mathfrak{B};B' \Bigg)\mathrm{d}x \tag{5.1}$$

under the same notations and exisrence conditions that (4.1) with $U=V=A=B=0$ and B_1 is defined by (4.2)

6. Conclusion

In this paper we have evaluated a generalized multiple integrals transformation involving the multivariable Aleph-function, a class of polynomials of several variables a generalized multiple-index Mittag-Leffler function, and the multivariable I-function defined by Prasad. The multiple integrals transformation established in this paper is of very general character as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, this relation established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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