ON (RW)*-OPEN SETS IN TOPOLOGICAL SPACES

[1] R. S.Wali and [2] Bajirao P Kamble

[1]Department of Mathematics, Bhandari and Rathi College Guledagudd - 587203, Karnataka,INDIA
[2]Research schoolar Department of Mathematics, Rani Channamma University, Belagavi -591156, Karnataka, INDIA.

Abstract

The aim of this paper is to introduce and study the new class of sets called (rw)-open sets are introduced and studied in topological space i.e. A sub set A of a topological space X is said to be (rw)*-open sets , If U⊆ int(cl(A)) ,whenever U⊆A and U is rw-closed sets in X. The new class is properly lies between #rg-open sets and wg-open sets in topological space .Also, as application using properties of (rw)*-open sets and (rw)*-closed stes. We investigate (rw)*-neighbourhoods, (rw)*-derived sets, (rw)*-interior and (rw)*-closure operators and have been discussed some of their basic properties respectively.*

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Keyword: *(rw)*-closed sets, (rw)*-open sets, (rw)*- neighbourhoods, (rw)*-derived sets, (rw)*-interior , (rw)*-closure*.

1. INTRODUCTION

In topological spaces the concept of open sets plays on important rule. The generalization of open sets has been studied in different ways in previous year by many topologists leading to several new ideas. In 1970 Levine first introduced the concept of generalized open (briefly g-open) set [1] and semi-open set and semi continuity [2] in topological space were defined and investigated.

Regular open sets, strong regular open set, α -open sets ,pre-open, wg- open sets and rw- open sets,αrω-open sets, srw-opn sets and RMG-open sets have been introduced and studied by Stone[3] , J. Tong [8] , O. Njastad [4], A.S.Mashur.et.al [5] ,Nagaveni [17] and Benchalli and R.S.Wali [7] R.S.Wali and P. S. Mandalgeri [29], R.S.Wali and Basayya Mathad [30] and R.S.Wali and Nirani Laxmi [31] respectively, and R.S.Wali and Bajirao P.Kamble [32] introduced and studied (rw)*-closed sets.

We introduced and study the (rw)*-open sets, (rw)*- neighbourhoods, (rw)*-derived set, (rw)*-interior and (rw)*-closure in a topological space and obtained some of their basic properties of these concept.

2. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply, X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, cl(A), int(A) and X−A or A^c or X/A denote the closure of A, interior of A and compliment of A in X respectively. (X, τ) will be replaced by X if there is no chance of confusion .Now, we recall the following definitions.

Definition (2.1): A subset A of a topological space X is called a

[1] Regular open set [3] if A = int (cl(A)) and regular closed set [3] if A=cl (int(A)).
[2] Pre-open set [5] if A ⊆ int (cl (A)) and pre-closed set [5] if cl (int (A))⊆A.
[3] Semi-open set [2] if A⊆ cl (int (A)) and semi-closed set [2], if int(cl(A))⊆A.

[4] α-open set [4] if A⊆int (cl (int (A))) and α-closed set [4] if cl(int (cl(A)))⊆ A.
[5] β-open set [9] if A ⊆ cl(int(cl(A))) and β-closed set [9] if int(cl(int(A)))⊆ A.
[6] δ-closed set [10] if A = cl_δ(A), where cl_δ(A) = {x ∈X : int(cl(U)) ∩ A ≠ϕ, U ϵ τ and x ϵ U}
[7] θ-closed set [10] if A=cl_θ (A), where cl_θ (A) = {x ϵ X: (cl(U))∩A ≠ ϕ ,U ϵ τ and x ϵ U}.
[8] Regular semi open set [11] if there is a regular open set U such that U ⊆A ⊆ cl(U).

Definition 2.2: [12] Let X be a topological space. The finite union of regular open sets in X is said to be π-open. The complement of a π-open set is said to be π-closed.

Definition 2.3: Let (X, τ) be a topological space and A⊆ X .The semi-pre-closure (resp. semi-closure, pre-closure and α-closure) of a sub set A of X is the intersection of all semi-pre-closed (resp. semi-closed, pre-closed and α-closed) set containing A and is denoted by spcl(A) (resp. scl(A), pcl(A) and αcl(A)).
It is well know that spcl(A)=A ∪ int(cl(int(A))) , scl(A) =A∪ int(cl(A)), pcl(A) = A ∪ cl(int(A)) and αcl(A) = A∪ cl(int(cl(A))).

Definition 2.4: A subset A of a topological space (X, τ) is called a

[1] Generalized closed set (briefly g-closed) [1], if cl(A) ⊆ U whenever A ⊆ U and U is open in X
[2] Semi-generalized closed set (briefly sg-closed) [13], if scl(A) ⊆ U whenever A⊆U and U is semi-open in X.
[3] Regular generalized closed set (briefly rg-closed)[14], if cl(A)⊆U whenever A⊆U and U is regular open in X.
[4] Generalized semi-pre closed set (briefly gsp-closed) [15], if spcl(A)⊆U whenever A⊆ U and U is open in X.
[5] ω-closed set [6], if cl(A)⊆U whenever A⊆U and U is semi open in X.
[6] Strongly generalized closed set (briefly, g*-closed) [16], if cl(A)⊆U whenever A⊆U and U is g-open in X.
[7] Weakly generalized closed set (briefly, wg-closed) [17], if cl(int(A))⊆U whenever A⊆U and U is open in X.
[8] Regular weakly generalized closed set (briefly, rwg-closed) [17], if cl(int(A))⊆ U whenever A ⊆ U and U is regular open in X.
[9] Regular generalized α-closed set (briefly, rgα-closed) [18], if αcl(A)⊆ U whenever A⊆ U and U is regular α-open in X.
[10] Regular weakly closed (briefly rw -closed) set [19], if cl(A)⊆U whenever A⊆ U and U is regular semi- open in X.
[11] Generalized regular closed (briefly gr–closed) set [20], if rcl(A)⊆ U whenever A⊆ U and U is open in X.
[12] R*- closed (briefly R*-closed) set [21], if rcl(A)⊆U whenever A⊆U and U is regular semi- open in X.
[13] Regular generalized weakly (briefly rgw-closed) set [22], if cl(int(A))⊆ U whenever A⊆ U and U is regular semi-open in X.
[14] Weakly generalized regular α-closed (briefly wgrα-closed) set [23], if cl(int(A))⊆ U whenever A⊆ U and U is regular α-open in X.
[15] Pre generalized pre regular closed (briefly pgpr-closed) set [24], if pcl (A) ⊆ U whenever A⊆ U and U is rg- open in X.
[16] Regular pre semi-closed (briefly rps-closed) set [25], if spcl(A)⊆ whenever A⊆U and U is rg-open in X.
[17] Generalized pre regular weakly closed set (briefly gprw-closed) [26], if pcl(A)⊆U whenever A⊆ U and U is regular semi- open in X.
[18] #rg closed [briefly #rg closed] set [27], if cl(A) ⊆U whenever A⊆ U and U is rw- open in X.

The complements of the above mentioned closed sets are their respective open sets in same topological space X.

Definition 2.5: A sub set of (X, τ) is called **(**rw)*-closed [32] if cl (int (A)) ⊆U whenever A⊆U and U is rw-open in X. We denote the family of all (rw)*-closed sets, (rw)*-open sets of X by (RW)*C(X), (RW)*O(X) respectively.

3. (RW)*-OPEN SETS IN TOPOLOGICAL SPACES

Definition 3.1: A subset A of X is called (rw)*-open in X. If X−A is closed set in X. The family of all (rw)*-open sets is denoted by (RW)*O(X)

Theorem (3.2): Every #rg-open sets in X is (rw)*-open sets in X, but not conversely.

Proof: Let A #rg open X. Then by X−A is #rg closed by Theorem 3.2 of [32] every #rg closed sets is (rw)*-closed set, X−A is (rw)*-closed set in X. Therefore A is (rw)*-open set in X.

The converse of the above theorem need not be true in generally as seen from the following example.

Example 3.3: Let X= {a, b, c, d} with topology τ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}.Then the set A={a, b, d} is (rw)*-open sets but not #-rg-open set in X.

Theorem 3.4: Every (rw)*-open set in X is wg-open set in X, but not conversely.

Proof: Let A is (rw)*-open X. Then by X−A is (rw)*-closed by Theorem 3.4 of [32] every (rw)*-closed sets wg-closed set, X−A is wg-closed set in X. Therefore A is wg-open set in X.

Example 3.5: Let X= {a, b, c, d} and τ={X, ϕ, {c}, {c, d}, {b, c, d}}.Then the set A= {d} is wg-open set but not (rw)*-open set in X.

Corollary 3.6: A subset A of a topological space (X, τ).

[1] Every open set is (rw)*-open set in X.
[2] Every regular open set is (rw)*-open set in X.
[3] Every θ-open set is (rw)*-open set in X.
[4] Every δ-open set is (rw)*-open set in X.
[5] Every π-open set is (rw)*-open set in X.
[6] Every α -open set is (rw)*-open set in X.
[7] Every pre-open set is (rw)*-open set in X.
[8] Every pgpr-open set is (rw)*-open set in X.
[9] Every ω-open set is (rw)*-open set in X.

Proof:
[1] Let A is open set in X. Then X−A is closed. By Remark 3.7 of [32], every closed set is (rw)*-closed, X−A is (rw)*-closed set .Therefore A is (rw)*-open set in X.
[2] Let A be a regular open set in X. then X−A is regular closed set. By Remark 3.8 of [32], every regular closed set is (rw)*-closed set, X−A is (RW)*-closed set .Therefore A is (rw)*-open set in X.
[3] Let A be a θ-open set in X. then X−A is θ-closed set. By Remark 3.9 of [32], every θ-closed set is (rw)*-closed set, X−A is (RW)*-closed set .Therefore A is (rw)*-open set in X.
[4] Let A be a δ-open set in X. then X−A is δ-closed set. By Remark 3.10 of [32], every δ -closed set is (RW)*- closed set, X−A is (RW)*-closed set .Therefore A is (RW)*-open set in X.
[5] Let A be a π-open set in X. then X−A is π-closed set. By Remark 3.11of [32], every π-closed set is (rw)*-closed set, X−A is (rw)*-closed set .Therefore A is (rw)*-open set in X.
[6] Let A be a α-open set in X. then X−A is α-closed set. By Theorem 3.12 of [32], every α-closed set is (rw)*-closed set, X−A is (rw)*-closed set .Therefore A is (rw)*-open set in X.
[7] Let A be a pre-open set in X. Then X−A is pre-open closed set. By Theorem 3.14 of [32], every pre-closed set is (rw)*-closed set, X−A is (rw)*-closed set .Therefore A is (rw)*-open set in X.
[8] Let A be a pgpr-open set in X. Then X−A is pgpr-closed set. By Theorem 3.16 of [32], every pgpr-closed set is (rw)*-closed set, X−A is (rw)*-closed set .Therefore A is (rw)*-open set in X.

[9] From Sheik Join we know that every ω-open set is pre-open set ,but not conversely and also from corollary 3.6 [7] ,every pre-open set is (rw)*-open set in X, but not conversely .Hence every ω-open set is (rw)*-open set in X,but not conversly.

The converse implication of above theorem need not be true as seen from following example

Example 3.7: Let X= {a, b, c, d} with topology τ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}. Let A= {b, c} is (rw)*-open sets but not open sets, not regular-open sets, , not α-open sets, not pre-open sets, not pgpr-open sets and Let B={a, b, d} is (rw)*-open sets but not θ-open sets, δ-open sets π-open sets

Corollary 3.8: Every (rw)*-open set is rwg-open sets in X. but not conversely.

Proof: Let A be a (rw)*-open set in X. Then X−A is wg-closed set. By Theorem 3.4 of [32] every (rw)*-closed is wg-closed set, X−A is wg-closed set .By the Remark 3.6 of [32] every (rw)*-closed is rwg-closed set, X−A is rgw-closed set. Therefore A is rgw-open set in X.

The converse of the above corollary need not be true in generally as seen from the following example

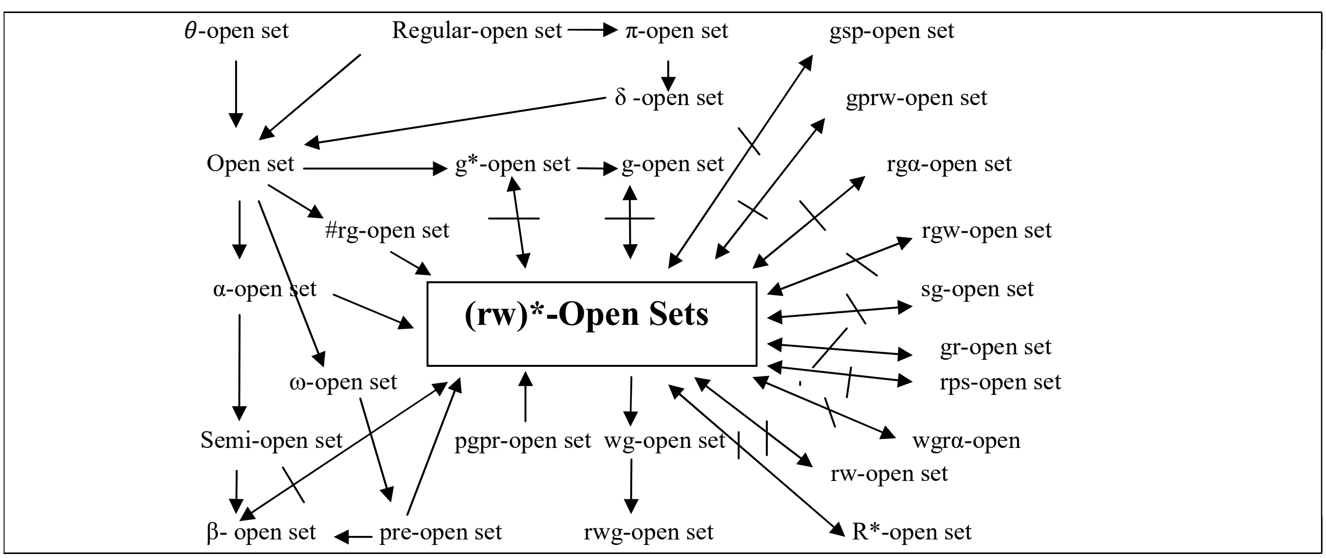
Example 3.9: Let X= {a, b, c, d} with topology τ={X, ϕ, {a},{b},{a, b},{a, b, c}}. Now A={c, d} is rwg-open set, but not (rw)*-open set in X.

Remark 3.10: The following example shows that (rw)*- open sets are independent of semi- open sets, g- open sets, g*- open sets, β - open sets, gsp- open sets, gprw- open sets, rgα- open sets, wgrα- open sets, rw- open sets . R*- open sets, rgw- open sets, sg- open sets, gr- open sets and rps- open sets.

Example 3.11: Let X= {a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}},then (RW)*O(X)={ X, ϕ,{a},{b},{c},{a, c}, {b, c}, {a, b}, {a, b, c}, {a, b, d}}. However it can be verified that the Sets, let set {b, c} is (rw)*-open sets , not gprw- open sets, not R*- open sets, not rw-open set, not rgα- open sets & not wgrα- open sets ,but sets {d} is gprw- open sets, R*- open sets, rw-open set, rgα- open sets & wgrα- open sets ,it's not (rw)*-open sets. Let sets {c}is (rw)*-open sets is not semi- open sets, not β - open sets, not rps- open sets but sets {a, c, d} is semi- open sets, β - open sets, & rps- open sets its not (rw)*-open sets. Also {b, d} is both gr-open sets and grp-open sets but it not (rw)*-open sets.

Example 3.12: Let X= {a, b, c, d} and τ ={X, ϕ, {c}, {c, d}, {b, c, d}}.Then set (RW)*O(X)={ X, ϕ,{c},{a, c}, {b, c},{c, d},{a, b, c},{a, c, d}, { b, c, d}}. Let {b, d} is g- open sets, g*- open sets but not (rw)*-open sets.

Remark 3.13: From the above discussion and know results we have the following implications in following diagram.



A ⟷ B means A & B are independent of each other

A —|→ B means A implies B but not conversely

Remark 3.14: The intersection of two (rw)*-open sets in X is generally not a (rw)*-open in X.

Examples 3.15: Let X= {a, b, c, d} and τ ={X, ϕ, {a}, {c, d}, {a, c, d}} then the set A= {a, b, c} & B = {a, b, d} are (rw)*-open set in X, but A∩B = {a, b} is not (rw)*-open set in X.

Remark 3.16: The union of two (rw)*-open subsets of X is generally not a (rw)*-open set in X.

Example 3.17: Let X= {a, b, c, d} and τ ={X, ϕ, {b}, {c}, {b, c}}, then the set A= {a, b} & B ={d} are (rw)*-open set in X, but A∪B= {a, b, d} is not (rw)*-open set in X.

Theorem 3.18: A subset of A of a topological space X is (rw)*-open set, iff U ⊆ int(cl(A)),whenever U is rw-closed and U ⊆ A

Proof: Assume that A is (rw)*-open set in X and U is rw-closed set of (X, τ), such that U ⊆ A. Then X−A= A^c is a (rw)*-closed set in (X, τ).Also X −A ⊆ X− U and X− U is rw-open set of (X, τ).This implies that cl(int(X−A)) ⊆ X−U.But cl(int(X−A)) = X−int(cl(A)).Thus X−int(cl(A)) ⊆ X ⊆ U ,so consequently U⊆ int(cl(A)).
Conversely: Suppose U ⊆ int (cl (A)) whenever U is rw-closed set and U ⊆ A. To prove that A is open set, let U be rw-open set of (X, τ) such that X−A ⊆ U. Then X− U ⊆A. Now X− U is rw-closed set containing A. So X−U⊆ int(cl(A)), X−int(cl(A)) ⊆ F but cl(int(X−A))=X – int(cl(A)) ⊆ U. Thus cl (int(X−A) ⊆ U .This is prove that X- A is (rw)*-closed set & hence A is (rw)*-open set.

Theorem 3.19: If int ((cl (A)) ⊆ B ⊆ A and A is (rw)*-open set, then B is (rw)*-open set.

Proof: Let int(cl(A)) ⊆ B ⊆ A ,Thus X− A ⊆ X−B ⊆ X−int(cl(A)), implies that cl(int(X−A). since X− A is set (rw)*-closed, by Theorem 3.23 of [32] X−B is (rw)*-closed .This prove that B in (rw)*-open set.

Remark 3.20: Converse of the above theorem need not be true in generally as shown in below example

Example: Let X= {a, b, c, d} with topology τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}. Now A= {a, c} and B={c}. Now A and B both are (RW)*-open sets. But int (cl(A)⊈B⊆A

Theorem 3.21: If A⊆X is (rw)*- closed set in X, then cl (int (A)) − A is (rw)*- open set in X.

Proof: Let A⊆X is (rw)*-closed and let F be a rw-closed set such that F⊆ cl (int (A)) − A. Then by Theorem 3.28 of [32], F = ϕ, that implies F⊆int (cl (cl (int(X − A)))) − A. This proves that cl(int (A)) − A is (rw)*-open set in X.

Theorem 3.22: Every singleton point set in a space X is either (rw)* -open or (rw)*-closed

Proof: Let X be a topological space. Let x ∈ X, Tso prove {x} is either (rw)*-open or (rw)*-closed. That is to prove X-{x} is either (rw)*-open or (rw)*-closed which follows from theorem 3.26 of [32]

Remark 3.23: Complement of (rw)*-open need not be (rw)*-open set in X. As show in following example.

Example: Let X= {a, b, c, d} with topology τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}}, if A= {a, b, c} is (rw)*-open but X −{a, b, c}= {d} is not (rw)*-open set in X.

Theorem 3.24: If a subset A is (rw)*-open in X and if U is rw-open in X with int (cl(A)) ∪ (X−A)⊆U then U=X.

Proof: Suppose that U is an (rw)*-open and int(cl(A)) ∪ (X−A) ⊆ U . Now X−A ⊆ (X−int (cl(A))∩X − (X−A) implies that (X−U) ⊆ cl(int (X−A))∩A. Suppose A is (rw)*-open. Since X−U is rw-closed and X−A is (rw)*-closed, then by Theorem 3.28 of [32], X−U=∅ and hence U=X.

The converse of the above Theorem need not be true in general as shown in example.

Example 3.25: Let X={a, b, c, d} with topology τ={X, ϕ,{a},{c},{c, d},{a, c, d}}.Then (rw)*O(X) = {X , ϕ,{a}, {c},{ d}, {a, c}, {c, d}, {a, b, c}, { a, b, d },{a, c, d}} and RWO(X) ={X, ∅, a , b , {c, d}, {a, d},{a, c, d}, {a, c}, {b, d}, {b, c} {d}, {c}, {b}, {c}, {a}}. Let A = {a, b} is not an (rw)*-open set in X. However int (cl(A)) ∪ X−A ={a}∪{c, d} ={a, c, d}⊂U. So for some rw-open set U, gives U=X but A is not (rw)*-open set in X.

Theorem 3.26: Let X be a topological space and A, B ⊆X. If B is (rw)*-open and int (cl(B))⊆A, then A ∩B is (rw)*-open in X.

Proof: Since B is (rw)*-open and int (cl (B))⊆ A, then int (cl (B)) ⊆ A∩B ⊆ B, then by Theorem 3.23 of [32], A∩B is (rw)*-open set in X.

4. PROPERTIES OF (RW)*-NEIGHBOURHOOD AND (RW)*-DERIVED SET

Definition 4.1: (i) Let (X ,τ) be a topological space and Let x ∈ X ,A subset of N of X is said to be (rw)*-neighbourhood of x if there exists a (rw)*-open set G such that x ∈ G⊂N

(ii) The collection of all (rw)*-neighbourhood of x ∈ X is called (rw)*-neighbourhood system at x and shall be denoted by (rw)*-N(x).

Theorem 4.2: Every neighbourhood N of x ∈ X is a (rw)*-neighbourhood of x.

Proof: Let N be neighbourhood of point x ∈ X .To prove that N is a (rw)*-neighbourhood of x by definition of neighbourhood ∃ an open set G s.t. x∈G⊂ N. As every open set is (rw)*- open, G is an (rw)*-open in X. Then there exists an open set G such that x∈G⊂ N.Hence N is (rw)*-neighbourhood of x.

Remark 4.3: In general, a (rw)*-nbhd N of x∈X need not be a neighbourhood of x in X, as seen from the following example.

Example 4.4 : Let X={a, b, c, d}with topology τ={X,∅, {a},{b},{a, b},{a, b, c}}.Then (rw)*-O(X) = {X, ∅,{a},{b},{c},{a, b},{b, c},{a, c},{a, b, c},{a, b, d}}.The set {c, d} is (rw)*-nbd of the point c, since the (rw)*-open set {c} is such that c∈{c}⊂{c, d}. However, the set {c, d} is not a neighbourhood of the point c, since no open set G exists such that c∈G⊂ {c, d}.

Theorem 4.5: If a subset N of a space X is (rw)*-open, then N is a (rw)*-nbhd of each of its points.

Proof: Suppose N is (rw)*-open. Let x ∈N, we claim that N is (rw)*-nbhd of x. For N is a (rw)*-open set such that x∈N⊂N. Since x is an arbitrary point of N, it follows that N is a (rw)*-nbhd of each of its points.

Remark 4.6: The converse of the above theorem is not true in general as seen from the following example.

Example 4.7: Let X= {a, b, c, d} with topology τ={X,∅, {b}, {c}, {b, c}}. Then (rw)*O(X) ={X ,∅,{a},{b},{c},{d},{a, b},{b, c},{a, c},{b, d},{c, d},{b, c, d}}. The set {a, d} is (rw)*-nbd of the point a, since the (rw)*-open set {a} is such that a∈{a}⊂ {a, d}. Also the set {a, d} is a (rw)*-nbhd of the point d. Since the (rw)*-open set {d} is such that d ∈{d}⊂ {a, d}.That is {a, d} is a (rw)*-nbhd of each of its points. However the set {a, d} is not a (rw)*-open set in X.

Theorem 4.8: Let X be a topological space. If F is a (rw)*-closed subset of X, and x ∈ F^c. Prove that there exists a (rw)*-nbhd N of x such that N∩F =ϕ.

Proof: Let F be (rw)*-closed subset of X and $x \in F^c$. Then F^c is (rw)*-open set of X. So by Theorem 4.5, F^c contains a (rw)*-nbhd of each of its points. Hence there exists a (rw)*-nbhd N of x such that $N \subset F^c$. That is $N \cap F = \phi$.

Theorem 4.9: Let X be a topological space and for each $x \in X$. Let (rw)*-N(x) be the collection of all (rw)*-nbhds of x. Then we have the following results

(I) $\forall x \in X$, (rw)*-N(x) $\neq \phi$.
(II) N $\in$ (rw)*-N (x) $\Rightarrow x \in N$.
(III) N$\in$ (rw)*-N (x), $M \supset N \Rightarrow$ M$\in$ (rw)*-N(x)
(IV) N$\in$ (rw)*-N(x) $\Rightarrow$ there exists M $\in$ (rw)*-N(x) such that $M \subset N$ and M$\in$ (rw)*-N(y) for every $y \in M$.

Proof:

(I) Since X is a (rw)*-open set, it is a (rw)*-nbhd of every $x \in X$. Hence there exists at least one (rw)*-nbhd for each $x \in X$. Hence (rw)*-N (x) $\neq \phi$ for every $x \in X$.
(II) If N$\in$(rw)*-N (x),then N is a (rw)*-nbhd of x. So by definition of (rw)*-nbhd,$x \in N$.
(III) Let N$\in$(rw)*-N (x) and $M \supset N$. Then there is a (rw)*-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is (rw)*-nbhd of x. Hence M$\in$(rw)*-N(x).
(IV) If N$\in$(rw)*-N (x), then there exists a (rw)*-open set M such that $x \in M \subset N$. Since M is a (rw)*-open set, it is (rw)*-nbhd of each of its points. Therefore M $\in$(rw)*-N(y) for every $y \in M$.

Definition 4.10: Let A be a subset of a topological space (X, τ). A point $x \in X$ is said to be (rw)*-limit point of A if and only if every nbd of x contains a point of A of X. i.e., $(N-\{x\}) \cap A \neq \phi$.$\forall$ (rw)*-nhd N of x

Or, $(x \in G \Rightarrow G \cap (A-\{x\}) \neq \phi)$,$\forall$G is (rw)*-open set containing x contains point of A other than x. The set of all (rw)*-limit points of A is called derived set of A and is denoted by $D_{(rw)^*}(A)$.

Note that for sub set A of X, a point x$\in$is not a (rw)*-limit point of A if and only if there exists a (rw)*-open set G in X such that $x \in G$ and $G \cap (A-\{x\})=\phi$ or, equivalently, $x \in G$ and $G \cap A=\phi$ or $G \cap A = \{x\}$. Or, equivalently $x \in G$ and $G \cap A \subseteq \{x\}$.

Theorem 4.11: If A is a subset of a discrete topological space (X, τ), then $D_{(rw)^*}(A) = \phi$

Proof: Let x be any point of X in topological space (X, τ).As every subset of X is open. We know that every open set is (rw)*-open set in X. In particular the singleton set G:=$\{x\}$ is (rw)*-open. But $G \cap A = \{x\} \cap A \subseteq \{x\}$. Hence x is not a (rw)*-limit of A i.e., $D_{(rw)^*}(A) = \phi$.

Theorem 4.12: Let A is subset of a indiscrete topological space (X, τ), then $D_{(rw)^*}(A) = X$ and $D_{(rw)^*}(A) = \phi$.

Proof: Let A be a subset of X containing two or more point of X. As every point of $x \in X$ is (rw)*-limit of A. Since every (rw)*-open set containing x is X which contains (rw)*-limit point of A other than A. Hence $D_{(rw)^*}(A) = X$ and if A= ϕ, then evidently no point of X can be (rw)*-limit point of A and so, $D_{(rw)^*}(A) = \phi$.

Theorem 4.13: For any subset A and B of (X, τ), then

(i) $D_{(rw)^*}(\phi) = \phi$
(ii) If $A \subseteq B$, then $D_{(rw)^*}(A) \subseteq D_{(rw)^*}(A)$
(iii) $D_{(rw)^*}(A \cap B) \subseteq D_{(rw)^*}(A) \cap D_{(rw)^*}(B)$.
(iv) $D_{(rw)^*}(A \cup B) \subseteq D_{(rw)^*}(A) \cup D_{(rw)^*}(B)$.

Proof:

(I) since ϕ is closed : $D_{(rw)^*}(\phi) \subset \phi$---(1).But ϕ is a subset of every set and so $\phi \subset D_{(rw)^*}(\phi)$---(2).From (1) and (2) , we have $D_{(rw)^*}(\phi) = \phi$.

(II) Let $x \in D_{(rw)^*}(A)$ and let $G \in$(rw)*-open in X with $x \in G$. Then $(G \cap A) - \{x\} \neq \phi$, since $A \subseteq B$, it follows that $(G \cap B) - \{x\} \neq \phi$. So that $x \in D_{(rw)^*}(B)$.

(III) Obviously proof is follows from (ii), since $D_{(rw)^*}(A \cap B) \subseteq D_{(rw)^*}(A)$ and $D_{(rw)^*}(A \cap B) \subseteq D_{(rw)^*}(B)$ and consequently, $D_{(rw)^*}(A \cap B) \subseteq D_{(rw)^*}(A) \cap D_{(rw)^*}(B)$.

(IV) Obviously proof is follows from (ii), since $D_{(rw)^*}(A \cup B) \subseteq D_{(rw)^*}(A)$ and $D_{(rw)^*}(A \cup B) \subseteq D_{(rw)^*}(B)$ and consequently $D_{(rw)^*}(A \cup B) \subseteq D_{(rw)^*}(A) \cup D_{(rw)^*}(B)$.

Remark 4.14: The converse of the above Theorem 4.13(ii) need not be true in general as shown in example.

Example 4.15: Let X={a, b, c, d}with topology τ={X, ∅, {a},{b},{c},{a, b},{b, c},{a, c},{b, c},{a, b, c},{a, b, d}}.Then (rw)*-O(X) = {X, ∅,{a},{b},{c},{b, c},{a, c},{a, b, c},{a, b, d}}.Let A={a, b, c} and B={b, c, d},then $D_{(rw)^*}(A)=\{d\}= D_{(rw)^*}(B)$. But $A \not\subset B$

Theorem 4.16: For any subset A of (X, τ), then (rw)*-cl(A)=A∪ $D_{(rw)^*}(A)$

Proof: Let $x \in$(rw)*-cl(A). Assume that $x \notin A$ and let G is (rw)*-open with $x \in G$. Then $G \cap (A - \{x\}) \neq \phi$ and $x \in D_{(rw)^*}(A)$. Hence (rw)*-cl(A)$\subseteq$ A∪ $D_{(rw)^*}(A)$.The converse inclusion is by A$\subseteq$ (rw)*-cl(A) and hence A∪ $D_{(rw)^*}(A)$.

Theorem 4.17: For any subset A of (X, τ), we have A is (rw)*-closed if and only if $D_{(rw)^*}(A) \subseteq A$.

Proof: Assume that A is (rw)*-closed. Let $x \notin A$, i.e., $x \in X - A$, since X−A is (rw)*-open, x is not (rw)*-limit point of A. i.e., $x \in D_{(rw)^*}(A)$,because $(X-A) \cap (A - \{x\}) = \phi$. Hence $D_{(rw)^*}(A) \subseteq A$. **Conversely**: Let $D_{(rw)^*}(A) \subseteq A$ and let $x \in X - A$ .then $x \notin A$. Since $D_{(rw)^*}(A) \subseteq A$, $x \in D_{(rw)^*}(A)$ such that G is (rw)*-open set with $x \in G$, then $G \cap A = \phi$ and so, X−A is (rw)*-open that is A is (rw)*-closed.

Corollary 4.18: Let A be a subset of (X, τ). If F is a (rw)*-closed super set of A, then $D_{(rw)^*}(A) \subseteq F$.

Proof: It is follows by the Theorem 4.13(ii) and Theorem 4.17, implies that $D_{(rw)^*}(A) \subseteq D_{(rw)^*}(F) \subseteq F$.

5. PROPERTIES OF (RW)*- INTERIOR AND (RW)*-CLOSURE OPERATOR

In this section the notation of (rw)*-interior is defined and some of its properties are studied. Also, we introduce the concept of (rw)*-closure in topological spaces by using the notation of (rw)*-closed sets and obtain some of their results. We define $\tau_{(rw)^*}$ and prove that it forms a topology on X. For any $A \subset X$, it is proved that the complement of (rw)*-interior of A is the (rw)*-closure of the complement of A.

Definition 5.1: For a subset A of (X, τ), (rw)*-interior of A is denoted by (rw)*-int(A) and defined as (rw)*-int(A)=∪{G:G$\subseteq$A and G is (rw)*-open in X} or ∪{G:G$\subseteq$A and G$\in$(rw)*-O(X)} i.e.,(rw)*-int(A) is the union of all (rw)*-open set contained in A.

Theorem 5.2: Let A is a subset of (X, τ), then (rw)*-int (A) =∪ {G: G is (rw)*-open, G$\subseteq$A}.

Proof: Let A be a subset of (X, τ). x is (rw)*-int(A)
⇔ x is a (rw)*-interior point of A
⇔A is a (rw)*-nhd of point x.
⇔There exists an (rw)*-open set G such that $x \in G \subseteq A$
⇔$x \in$∪ {G: G is (rw)*-open, G $\subseteq$A}.Hence (rw)*-int (A) =∪ {G: G is (rw)*-open G $\subseteq$A}.

Theorem 5.3: Let A and B be subset of space X then,

(I) ((rw)*-int(X) =X and (rw)*-int(ϕ)=ϕ.

(II) (rw)*-int(A)⊆A
(III) If B is any (rw)*-open set contained in A, then B ⊆ (rw)*-int(A).
(IV) If A ⊆ B then (rw)*-int (A) ⊆(rw)*-int (B).
(V) (rw)*-int(A) = (rw)*-int((rw)*-int(A)).

Proof:

(I) By definition of (rw)*-interior of A,it is obvious that.
(II) Let x∈(rw)*-int(A) ⇒ x is a (rw)*-interior point of A ⇒A is a (rw)*-nhd of x
⇒Thus x ∈(rw)*-int(A)
⇒ x ∈A. Hence (rw)*-int(A)⊆A.
(III) Let B be a any(rw)*-open set such that B⊆A. Let x∈ B. Then since B is an (rw)*- open set contained in A. x is an (rw)*-interior point of A. That is x∈ (rw)*-int(A).Hence B⊆(rw)*-int(A).
(IV) Let A and B are subsets of X such that A⊆B. Let x∈ (rw)*-int(A). Then x is an (rw)*-interior point of A and so A is a (rw)*-nhd of x. since A⊆B, B is also a (rw)*-nhd of x. This implies that x (rw)*-int(B). Thus we have show that x∈(rw)*-int(A)⇒ x∈ (rw)*-int(B). Hence (rw)*-int(A)⊆(rw)*-int(B).
(V) Since (rw)*-int(A) is a (rw)*-open set in X, it follows that (rw)*-int((rw)*-int(A)) =(rw)*-int(A

Theorem 5.4 : If a subset A of X is (rw)*-open then (rw)*-int(A) = A

Proof: Let A be (rw)*-open subset of X. We know that (rw)*-int(A)⊆A---(1). SinceA is (rw)*-open set contained in A from Theorem 5.3.(iii), A⊆(rw)*-int(A)----(2).Hence from (1) and (2) (rw)*-int(A) = A.

The converse of the above theorem need not be true as seen form the following example.

Example 5.5: Let X= {a, b, c, d} with topology τ={X, ∅, {b},{c},{b, c}}. Then (rw)*-O(X) ={X , ∅,{a},{b},{c},{d},{a, b},{b, c},{a, c},{b, d},{c, d},{b, c, d}}. Let A={a, d},then set (rw)*-int(A)= {a}∪{d} = {a, d}, but {a, d} is not (rw)*-open set in X.

Theorem 5.6: If A and B are subsets of space X, then (rw)*-int (A)∪(rw)*-int (B) ⊆ (rw)*-int (A∪B).

Proof: We know that A⊆A∪B and B⊆A∪B. We have Theorem 5.3 (iv) (rw)*-int(A)⊆(rw)*-int(A∪ B) and (rw)*-int(B)⊆(rw)*-int(A∪B).This implies that (rw)*-int(A)∪(rw)*-int(B)⊆(rw)*-int(A∪ B).

Theorem 5.7: If A and B are subsets of space X , then (rw)*-int(A ∩ B) ⊆ (rw)*-int(A) ∩ (rw)*-int(B)

Proof: We know that A and B are subsets of space X. Clearly A ∩ B⊆A and A ∩ B⊆B. We have, by Thereom 5.3. (iv) (rw)*-int(A ∩ B) ⊆ (rw)*-int(A) and (rw)*-int(A ∩ B) ⊆ (rw)*-int(B). Hence implies that (rw)*-int(A ∩ B) ⊆ (rw)*-int(A) ⊆ (rw)*-int(B).

Theorem 5.8: If A is a subset of X, then int(A) ⊆ (rw)*-int(A).

Proof: Let A be a subset of a space X. Let x∈ int(A) ⇒ x∈∪{G: G open, G⊆A}.
⇒ There exists an open set G such that x∈ G⊆ A
⇒There exists an (rw)*-open set G such that x∈G⊆A. From corollary 3.6 (1), As every open set is an (rw)*-open set in X
⇒x∈∪{G:G is (rw)*-open, G⊆A}.
⇒x∈(rw)*-int(A).Thus, x∈ int(A) ⇒ x∈(rw)*-int(A). Hence int(A)⊆(rw)*-int(A).

Remark 5.9: Containment relation in the above Theorem 5.8 may be proper as seen from the following example.

Example 5.10: Let X = {a, b, c, d} with topology τ = {X, ∅, b, {a, d}, {a, b, d}}. Then (RW)*-O(X) = {X, ∅, { a }, {d}, {b}, {a, d} , {a, b} , {b, d}, {a, b, c} , {a, b, d} ,{b, c, d} . Let A ={b, d}. Now (rw)*-int(A) ={b, d} and int(A) ={b}. It follows that int(A)⊆(rw)*-int(A).

Theorem 5.11: If A is a subset of X, then #rg-int(A)⊆(rw)*-int(A). Where #rg-int(A) is given by #rg- int(A) = x∈∪{G: G #rg-open, G⊆A}.

Proof: Let A be a subset of a space X. Let x∈#rg- int(A)
⇒ x∈∪{G: G #rg-pen, G⊆A}.
⇒There exists an (rw)*-open set G such that x∈ G⊆A. From Theorem 3.2 , As every #rg-open set is an (rw)*-open set in X.
⇒ x ∈∪{G:G ⊆X : G is (rw)*-open, G⊆A}.Thus, x∈ #rg-int(A)
⇒ x∈(rw)*-int(A). Hence #rg-int(A) ⊆ (rw)*- int(A).

Remark 5.12: Containment relation in the above Theorem 5.11 may be proper as seen from the following example.

Example 5.13: Let X = {a, b, c, d} with topology τ = {X, ∅, {a}, {c},{a, c, d},{a, b, c}}. Then (RW)*-O(X) = {X, ∅, { a }, {c}, {a, c}, {a, c, d},{a, b, c}} and #rg-O(X) = {X, ∅, { a }, {c}, {a, c}, {a, c, d}}. Let A ={a, b, c}. Now (rw)*-int(A) ={a, b, c} and #rg-int (A) ={a, c}. It follows that #rg- int(A)⊂(rw)*-int(A) and #rg- int(A) ≠(rw)*-int(A).

Remark 5.14: Let A is a subject of X. Then
(I) p-int(A)⊆(rw)*-int(A).
(II) α-int(A)⊆(rw)*-int(A).
(III) ω- int(A)⊆(rw)*-int(A).

Proof : Obviously result is proved by following proof of Theorem 5.8 or 5.11.

Theorem 5.15: If A is a subset of X, then (rw)*-int(A)⊂rwg-int(A).

Proof: Let A be a subset of a space X. Let x∈(rw)*-int(A)
⇒ x∈∪{G: G (rw)*-pen, G ⊆A}.
⇒ There exists an (rw)*- open set G such that x∈ G⊆A
⇒ There exists an rwg-open set G such that x∈ G⊆A. From corollory 3.8 , As every (rw)*-open set is an rwg-open set in X.
⇒ x ∈∪{G:G is rwg-open, G⊆A}.Thus, x∈(rw)*-int(A)
⇒ x ∈ rwg-int(A). Hence (rw)*-int(A)⊆ rwg-int(A).

Definition 5.16: For a subset A of (X, τ), (rw)*-closure of A is denoted by (rw)*-cl(A) and defined as (rw)*-cl(A)=∩{G: A⊆G,G is (rw)*-closed in (X,τ} or ∩{G:A⊆G, G∈(rw)*-C(X)}.

Theorem 5.17: If A and B are subsets of space (X, τ), then

(I) (rw)*-cl(X)=X, and (rw)*-cl(ϕ)=ϕ.
(II) A⊆(rw)*-cl(A).
(III) If B is any (rw)*-closed set containing A, then (rw)*-cl(A)⊆B.
(IV) If A⊆B then (rw)*-cl(A)⊆(rw)*-cl(B).
(V) (rw)*-cl(A) = (rw)*-cl((rw)*-cl(A)).

Proof:

(I) By definition of (rw)*-closure, X is only (rw)*-closed set containing X. Therefore (rw)*-cl(X) = Intersection of all the (rw)*-closed set containing X = $\cap$\{X\} =X, therefore (rw)*-cl (X) = X and again by definition of (rw)*-closure. (rw)*-cl(ϕ) = Intersection of all (rw)*-closed sets containing ϕ = $\phi\cap$any (rw)*-closed set containing ϕ=ϕ. i.e. (rw)*-cl(ϕ)=ϕ.

(II) By definition of (rw)*-closure of A, it is obvious that A$\subseteq$ (rw)*-cl(A).

(III) Let B be any (rw)*-closed set containing A. Since (rw)*-cl(A) is the intersection of all (rw)*-closed set containing A, (rw)*-cl(A) is contained in every (rw)*-closed set containing A. Hence in particular (rw)*-cl(A)$\subseteq$B.

(IV) Let A and B be subsets of (X,τ) such that A$\subseteq$B, by definition of (rw)*-closure, (rw)*-cl(B) = $\cap$\{F: B$\subseteq$F$\in$(rw)*-C(X)\}. If B$\subseteq$F$\in$(rw)*-C(X), then (rw)*-cl(B)$\subseteq$F. Since A$\subseteq$B, A$\subseteq$B$\subseteq$F$\in$(rw)*-C(X). We have (rw)*-cl(A)$\subseteq$F, (rw)*-cl(A)$\subseteq\cap$\{F:B$\subseteq$F$\in$(rw)*-C(X)\}= (rw)*-cl(B).Therefore (rw)*-cl(A)$\subseteq$(rw)*-cl(B).

(V) Let A be any subset of X. By definition of (rw)*-closure,(rw)*-cl(A)=$\cap$\{F:A$\subseteq$F$\in$(rw)*-C(X)\}. If A$\subseteq$F$\in$(rw)*-C(X) then (rw)*-cl(A)$\subseteq$F, since F is (rw)*-closed set containing (rw)*-cl(A) by (iii) (rw)*-cl((rw)*-cl(A))$\subseteq$F.Hence-(rw)*-cl((rw)*-cl(A))=$\cap$\{F:A$\subseteq$F$\in$(rw)*-C(X)\}=(rw)*-cl(A). Therefore, (rw)*-cl((rw)*-cl(A))= (rw)*-cl(A).

Theorem 5.18: If A $\subseteq$X is (rw)*-closed set then (rw)*-cl(A) =A.

Proof: Let A be (rw)*-closed subset of X. We know that A$\subseteq$(rw)*-cl(A)--(1). Also A$\subseteq$A and A is (rw)*-closed set by theorem 5.17. (iii) (rw)*-cl(A)$\subseteq$A---(2).Hence (rw)*-cl(A) =A.

The Converse of the above need not be true as seen from the following example.

Example 5.19: Let X= \{a, b, c, d\}, τ=\{X, ϕ, \{b\}, \{c\}, \{b, c\}\}. Here (RW)*-C(X) = \{X, ϕ, \{a\}, \{d\}, \{a , b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}. Let A= \{c\}, (rw)*-cl(A) =\{c\}= A, then A is not (rw)*-closed set.

Theorem 5.20: If A and B are subsets of space X then (rw)*-cl(A$\cap$B)$\subseteq$(rw)*-cl(A)$\cap$(rw)*-cl(B).

Proof: Let A and B be subsets of X, Clearly A$\cap$B$\subseteq$A and A$\cap$B$\subseteq$B by Theorem 5.17 (iv) (rw)*-cl(A$\cap$B)$\subseteq$(rw)*-cl(A) and (rw)*-cl(A$\cap$B)$\subseteq$(rw)*-cl(B).Hence it implies that (rw)*-cl(A$\cap$B)$\subseteq$ (rw)*-cl(A)$\cap$(rw)*-cl(B).

Remark 5.21: In-general, (rw)*-cl(A)$\cap$(rw)*-cl(B) $\nsubseteq$ (rw)*-cl(A$\cap$B) as seen from the following example

Example 5.22: Consider X= \{a, b, c, d\}, τ=\{X, ϕ, \{a\}, \{c, d\},\{a, c, d\}\}, Let A=\{c, d\} and B= \{a, d\}, A$\cap$B= \{d\}. (rw)*-cl(A)=\{b, c, d\} and (rw)*-cl(B)=\{a, b, d\}, (rw)*-cl(A$\cap$B)=\{d\} and (rw)*-cl(A)$\cap$(rw)*-cl(B)=\{b, d\}. Therefore (rw)*-cl(A)$\cap$(rw)*-cl(B) $\nsubseteq$(rw)*-cl(A$\cap$B).

Theorem 5.23: For an xϵX, x $\in$(rw)*-cl (A) and if A$\cap$V$\neq\phi$ for every (rw)*-open set V containing x.

Proof: Let x $\in$(rw)*-cl (A).To prove A$\cap$V$\neq$ ϕ for every (rw)*-open set V containing x by contradiction. Suppose there exist (rw)*-open set V containing x s.t A$\cap$V=ϕ. Then A$\subset$X-V, X-V is (rw)*-closed set, (rw)*-cl(A)$\subset$X-V. This shows that x $\notin$(rw)*-cl(A) which is contradiction. Hence A$\cap$V $\neq$ ϕ for every (rw)*-open set V containing x. **Conversely:** Let A$\cap$V $\neq$ ϕ for every (rw)*-open set V containing x. To prove xϵ(rw)*-cl(A).We prove the result by contradiction. Suppose x$\notin$(rw)*-cl(A) then there exist a (rw)*-closed subset F Containing A s.t x$\notin$F.Then x$\in$X-F & X-F is (rw)*-open. Also (X-F)$\cap$A=ϕ which is contradiction. Hence x $\in$(rw)*-cl (A).

Theorem 5.24: Let A be a (rw)*-open set and B be any open set in X. If A$\cap$B= $\emptyset$, then A$\cap$(rw)*-cl(B) = $\emptyset$.

Proof: Suppose A $\cap$ (rw)*-cl(B)$\neq$ $\emptyset$ and x $\in$A $\cap$ (rw)*-cl(B). Then x$\in$ A and x$\in$A $\cap$ (rw)*-(B). By above Theorem 5.23. A $\cap$ B$\neq\emptyset$, which is contrary to the hypothesis. Hence A $\cap$ (rw)*-(B) $\neq$ $\emptyset$.

Theorem 5.25: If A is subset of space X , then

(i) (rw)*-cl(A) $\subseteq$ cl(A). (ii) (rw)*-cl(A) $\subseteq$ #rg-cl(A).

Proof:

(I) Let A be subset of space X, by definition of Closure, Cl(A)=$\cap$\{F:A$\subseteq$F$\in$C(X)\}. If A$\subseteq$F$\in$C(X) then A$\subseteq$F$\in$(rw)*-C(X), because every closed set is (rw)*-closed that is (rw)*-cl(A)$\subseteq$F, therefore (rw)*-cl(A)$\subseteq\cap$\{F:A$\subseteq$F$\in$C(X)\}=cl(A).Hence, (rw)*-cl(A)$\subseteq$cl(A).

(II) Let A be subset of space X. By definition of #rg-closure, #rg-cl(A) = $\cap$ \{F:A$\subseteq$F$\in$ #rg-C(X)\}. If A$\subseteq$F$\in$ #rg-C(X) then A$\subseteq$F$\in$(rw)*-C(X), because every #rg-closed set is (rw)*-closed that is (rw)*-cl(A)$\subseteq$F, therefore (rw)*-cl(A)$\subseteq\cap$\{F$\in$ #rg-C(X)\}= #rg-cl(A).Hence (rw)*-cl(A)$\subseteq$ #rg-cl(A).

Remark 5.26: Containment relation in the above Theorem 5.25 may be proper as seen from following example**.**

Example 5.27: Let X=\{a, b, c, d\},τ=\{X, ϕ, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}, Let A=\{a,\}, cl(A)=\{a, c, d\}, (rw)*-cl(A)=\{a, d\}, #rg-cl(A)= \{a, c, d\}. It follows that (rw)*-cl(A)$\subset$cl(A) and (rw)*-cl(A)$\subset$ #rg-cl(A).

corollory 5.28: If A is subset of space X, Then

[I] (rw)*-cl(A) $\subseteq$ α-cl(A).
[II] (rw)*-cl(A) $\subseteq$ p-cl(A).
[III] (rw)*-cl(A) $\subseteq$ pgpr-cl(A
[IV] (rw)*-cl(A) $\subseteq$ ω-cl(A)

Proof: Obviously result is proved by following proof of Theorem 5.25

Theorem 5.29: If A is subset of space X, then rwg-cl(A)$\subseteq$(rw)*-cl(A), where rwg-cl(A)= $\cap$ \{F:A $\subseteq$ F$\in$ RWG -C(X)\}.

Proof: Let A be a subset of X, by definition of (rw)*-closure, (rw)*-cl(A)=$\cap$\{F:A$\subseteq$F$\in$(rw)*-C(X)\}. If A$\subseteq$F $\in$(rw)*-C(X) then A$\subseteq$F$\in$RWG-C(X). Remark 3.6 of [32], every (rw)*-closed is rwg-closed i.e. rwg-cl(A) $\subseteq$ F. Therefore rwg-cl(A)$\subseteq\cap$\{F:A$\subseteq$F$\in$(rw)*-C(X)\}= (rw)*-cl(A). Hence rwg-cl(A)$\subseteq$(rw)*-cl(A).

Theorem 5.30: For any subset A of X. Then

(i) X–(rw)*-int(A) = (rw)*-cl(X–A).
(ii) (rw)*-int(A)=X–((rw)*-cl(X–A)).
(iii) (rw)*-cl(A)=X–(rw)*-int(X–A).
(iv) X–(rw)*-cl(A) = (rw)*-int(X–A).

Proof:

(I) x$\in$X-(rw)*-int(A),then x is not in (rw)*-int(A) i.e. every (rw)*-open set G containing x s.t. G $\not\subseteq$ A.This implies every (rw)*-open set G containing x intersects (X – A) i.e. $G\cap(X-A)\neq\phi$.Then by Theorem (5.23), x$\in$ (rw)*-cl(X-A), therefore X-(rw)*-int(A)$\subset$(rw)*-cl(X-A)--(1) and Let x$\in$(rw)*-cl(X-A),then every x$\in$(rw)*-open set G containing x, intersects X – A i.e. $G\cap(X-A)\neq\phi$,i.e. every (rw)*-open G containing x s.t. $G\subseteq A$.Then by Definition 5.1 , where x is not in (rw)*-int(A), i.e . x$\in$X-(rw)*-int(A) and so (rw)*-cl(X-A)$\subset$X- (rw)*-int(A)-- (2).Thus X – (rw)*-int(A) = (rw)*-cl(X –A).

(II) Follows by taking complements in (i).

(III) Follows by replacing A by X-A in (i)

(IV) Follows by taking complements in (iii).

CONCLUSION

In this article we have focused on (rw)*-open sets, (rw)*-neighbourhood, (rw)*-derived sets, (rw)*-interior and (rw)*-closure in topological space. With the help of these properties, we will be investigated (rw)*-continuous and (rw)*-irresolute function in topological spaces and fuzzy topological space.

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